

# METHODOLOGY IN CRITICAL MATHEMATICS EDUCATION: A CASE ANALYSIS<sup>1</sup>

Alexandre Pais, Elsa Fernandes, João Filipe Matos, and Ana Sofia Alves

Learning Technology Mathematics and Society Research Group,  
University of Lisbon

*In this article we engage in a critical analysis on how the notion of “critique” is being used in mathematics education research. After clarifying our theoretical and methodological position – which assumes the need for a systematic critical reflection on our own research – we argue that the notion of critique suffers from a process of “domestication” resulting from a superficial deployment of the radical ideas that emerged in the middle of the last century. After providing clarification of the notion of “critique”, we jump into the critical analysis of a case of research in critical mathematics education using data collected in a typical mathematics education research environment: teachers in the classrooms working with students.*

## INTRODUCTION

In order to reproduce itself the present capitalist<sup>2</sup> society demands for perpetual reforms by means of integrating what could be new and potential emancipatory acts into well established social structures. The word “critique” usually becomes a common place in educational research and curricular documents, being used as a signifier implicitly conveying different ideologies about what it means to be critical. Today we find notions of ‘critique’ in a variety of contexts such as school curricula

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<sup>2</sup> The word capitalism to refer the current mode of living seems to be falling into disuse. In the social sciences’ discourse it is usually absent, as it is in mathematics education literature. Dowling (1998) refers to it as “a word which should remain unspoken” (p. 19). Indeed, even in mathematics education research that takes a social and political dimension, society is usually described as being “neo-liberal”, “market economy”, “imperialism”, “pos-colonialist”, “post-modern”, “consumerist” and other euphemisms. If we recall how ideology works (Žižek, 1988), the obliteration of the word capitalism from mathematics education discourse (but also in general in social sciences discourse) is a symptom of the way we “naturalize” the way we live – we do not need to mention that we live in a capitalist society because we actually live in a capitalist society, without any other imaginable alternative. Nobody seriously considers possible alternatives to capitalism any longer. This is, of course, ideology functioning at its best. This is why we opted for using the word capitalism (instead of liberalism, for instance). Although we criticize the consumerist society, the neo-liberal ideas that takes to its extremes the individualization of social life, and so on, we think that we can go further and realize how behind all these epithets relies the capitalist system. For instance, Chinese society is not organized around liberal tenets, nevertheless it is profoundly capitalist. Capitalism dresses diverse clothes in order to keep reproducing, and no matter how different the “philosophies” of political organization could be around the world (monarchy, socialism, religious fundamentalism, dictatorship, neo-liberalism, etc.) what is common in all them is that, despite the different apparent “clothes”, the human relations are based on capital. By explicitly mentioning capitalism we want to point to the very core of the problem – this (so often) unaddressed reality that permeates all social relations and for which we seem to have no alternatives.

(“educate people to become critical citizens”<sup>3</sup>), in teacher education (“Tips for teaching critical thinking skills”<sup>4</sup>), professional education (“Education and Knowledge in Safety-Critical Software”<sup>5</sup>), online education (“Role of critical thinking in online education”<sup>6</sup>), etc. One consequence of this extensive use is an obvious loss of meaning. That is, words begin to function as empty signifiers, representing no more than a way of joining the apparent mainstream talk instead of directing the audience into specific and soundful shared meanings. Very often, the use of these words lacks a deeper concern for understanding what could be the ideologies filling the empty space conveyed by them.

We will argue that the notion of critique suffered from a kind of “domestication” in the field of education by focusing on the case of mathematics education. In the last twenty years a strong critical emphasis has emerged, particularly from the work of Ole Skovsmose. By considering a typical research environment in this field – teachers with their students learning mathematics in the classroom – and the will of the teacher to promote a critical mathematics education in her classroom, we explore, firstly, how the notion of critique can be lost when inserted in social frames (like schools) that aim not at emancipation but reproduction; secondly, we highlight what could be a methodology that specifically addresses the emerging tensions and avoids to “clean” the research from them. The exploration of this example will allow us to make visible how ideology is effective in integrating what is presented as emancipatory actions into existing social structures, such as the capitalist mode of production. Our goal is to address the radicalism involved in a critical educational methodology by confronting it with methodologies that reclaim themselves as “critical” while they do not seem to keep all the substance of such a notion. From a research project involving teachers and students working with issues of critical mathematics education, we explore what could be a methodology that takes seriously the notion of critique, in contrast with one that suspends it for the sake of research. We conclude that when adopting a critical methodology research needs to bring into the research practice the ways in which the “domestication” is achieved.

## **THEORETICAL AND METHODOLOGICAL STANDPOINT**

We share the idea of Valero (2009) that mathematics education as a research field needs to develop research where its own principles and practices are put under scrutiny. She argues that “developing awareness on the research perspectives that I adopt has, therefore, been as central to me as generating particular understandings and interpretations of the practices of teaching and learning in mathematics classrooms” (p. 2). Therefore we claim the need of a constant critical analysis of the way we engage with research and how we understand its results. This kind of analysis

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<sup>3</sup> Portuguese, Colombian, South African curriculums.

<sup>4</sup> <http://www.modernghana.com/news/203119/1/tips-for-teaching-critical-thinking-skills.html>

<sup>5</sup> <http://ercim-news.ercim.org/content/view/446/699/>

<sup>6</sup> [http://www.masternewmedia.org/education/critical\\_thinking/educational\\_role\\_of\\_critical\\_thinking.htm](http://www.masternewmedia.org/education/critical_thinking/educational_role_of_critical_thinking.htm)

demands looking at research from a socio-political perspective (Valero, 2004) that explicitly searches for connecting the role of research – in particular in mathematics education – to the discourses and ideologies that fuel our current society. In order to understand the dynamics of the teaching and learning of mathematics and the way research results influence what is happening in mathematics classrooms, we need to contextualize these practices and the social modes of living that characterizes the world today.

We take the standpoint that a critical methodological approach in research in education has not just to do with the way the researcher engages with the participants, but also the way the researcher makes sense of the empirical reality addressed. Reality is seen as contradictory, full of curves and spins, and a critical methodology is the one that tries to find a language to express these contradictions in a way that does not neglect them, nor clean the research from them, but takes them as part of the core focus. In order to enlighten this tension between a research that “cleans” reality from contradictions and a critical one, we will bring in the example of the work of Ana, a mathematics teacher in a Portuguese secondary school. In the empirical part of her study Ana was confronted with several difficulties while trying to implement critical mathematics education in the context of a mathematics class. For the sake of the research, she decided to obliterate them from the final report (the Master thesis), concluding that despite all the constraints she felt, it is possible and fruitful to bring critical mathematics education into the mathematics classroom. We see the difficulties faced by Ana not as marginalities, things to be avoided, details of a school system, but as core problems of the current school systems and societies that keep suspending what could be a radical emancipatory mathematics education. Therefore we assume that difficulties and constraints of research are not things to avoid but central issues of the research.

## **RECOVERING THE MEANING OF “CRITIQUE” IN CRITICAL MATHEMATICS EDUCATION**

Although we are aware that in our days “critical mathematics education” is a trend in mathematics education research with several ramifications<sup>7</sup>, we base our analysis on the ideas of Ole Skovsmose, for two reasons. Firstly because his work is one where the notion of critique is used with a philosophical background based on the Critical Theory as it was developed by some of the Frankfurt scholars (especially the further development undertaken by Jürgen Habermas). Secondly because his theory on critical mathematics education was the one used by Ana in her work.

Put briefly, Skovsmose (1994) understands critical education as one that addresses the conflicts and crisis in society: “critical education must disclose inequalities and oppression of whatever kind” (p. 22). In such a task, there is a desire for

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<sup>7</sup> For instance the work of Marylin Frankenstein in the United States; or the work of the *Critical Mathematics Education Group* at Sheffield Hallam University, United Kingdom.

emancipation, where a critical education must not simply contribute to the prolonging of existing social relations. Skovsmose substantiates his idea of emancipation in the work of Habermas, who connected emancipation with a critique of the positivist way of researching in social sciences, and the need for social sciences to be founded on an interest in emancipation.

By so doing [fall into the trap of logical positivism], social sciences will be colonised by the technical-manipulative research paradigm, according to Habermas. It is not possible to find any platform of neutrality. Social sciences must be 'committed'. A pretended neutral registration of facts will result in an acceptance of the social status quo. (Skovsmose, 1994, p. 12)

But how did Critical Theory understand the "existing social relations" or the "social status quo"? In other words, which was the core focus of the social and political critique developed by Critical Theory? The answer is capitalism. Despite major differences between members of the Frankfurt School in their assessment of the development of capitalism, it may be noted from the outset that their respective analysis were informed by Marxian tenets (Held, 1980). According to Benhabib (1994) the core feature of critical theory, as it emerged in the works of Horkheimer, Adorno, Marcuse, Löwenthal, Pollock and Benjamin, was the realisation that a revolutionary transformation of capitalism from within capitalism itself was doomed to fail. Critical theory was confronted with the enterprise of thinking a "radical alternative".

Although initially the critique was focused on political economy, with time it gave place to a critique of instrumental reason, as a response to a positivist paradigm which restricted research to the activity of outlining correlations between well-defined phenomena. These two critiques did not coincide; rather the critique of instrumental reason surpassed the critique on political economy:

The transformation of the critique of political economy into the critique of instrumental reason signals not only a shift in the *object* of critique, but, more significantly, in the *logic* of critique. (Benhabib, 1994, p. 79)

The work of Habermas exemplifies this shift on the object of critique. In his work political economy is not just a matter of *superstructure*, of class struggle, but a matter of administration and technique, due to a change in which politics becomes the sphere for the technical elimination of dysfunctions and the avoidance of risks threatening "the system" (Held, 1980). This split provoked a displacement of the way the political was conceived: capitalism became "naturalized" and accepted, and transformation started to be conceived inside capitalism. Capitalism is no longer seen as the fundamental core of the problem, as the system we have to emancipate from, but the social and political background in which emancipation can take place.

Despite the apparent fall of capitalist principles in the last two years, we are facing the emergence of reforms that keep unaddressed the core of the societal problems. All emancipatory actions are thought and put into action within capitalism. If we recover

the critique of political economy developed in the first years of the Frankfurt School, we can say that emancipation from capitalism failed completely. No radical alternative was made. But this fact contrasts with the proliferation of the idea of critique, especially in education. We can read in the curricula all around the world the word critique, how important it is to allow students a critical education, to become critical citizens. It is in that sense that we argue that the word critique has become “domesticated”, it has lost its most radical meaning. It is a case of what Žižek (2005) calls “progressive amnesia” (p. 9): we recover critical theory but deprived from its true transformative core. It is fine to take a critical stance as long as you do not raise questions that could undermine the foundations of society – we are allowed to be critical as long as we do not criticize the capitalist system itself.

## **A RESEARCH IN CRITICAL MATHEMATICS EDUCATION**

We will now look at a piece of research in critical mathematics education, trying to make visible how a potentially emancipatory theory can end up reproducing the same ideologies that it tries to criticize. For this purpose, we understand schools in the Althusserian way as crucial ideological state apparatus in the reproduction of capitalism (Althusser, 1994).

The interest of Ana into critical mathematics education is partly the result of a concern with the way mathematics is traditionally taught in schools: as something disconnected from students’ reality. Being committed to pupils’ education for citizenship, Ana sees her role as a mathematics teacher as an important factor in allowing her students to become participative, active, competent, critical citizens. The ways she found to accomplish this aim are diverse, being one the development of activities with students where they can uncover and understand the role of mathematics in different social situations. She adopts Skovsmose’ idea of *mathemacy*, as the competence to analyse and reflect upon the mathematics behind a world strongly structured around mathematical modelling (Skovsmose, 1994). As a final product of Ana’s experience (which took place during the first period of 2006, with a class of 9th graders), she developed a Master thesis where she explored the implementation of this critical mathematics education experience.

The Portuguese curriculum gives her space to work with such topics in the classroom, by explicitly mentioning that “mathematics education has the purpose of helping students to uncover the mathematics behind the more diverse situations, promoting the education of participative, critical and confident citizens” (ME-DEB, 2001, p.58). Everything seems prepared and even willing to implement a critical mathematics education in the classroom. What issues are involved here?

The first issue is the decision of Ana of not implementing her critical mathematics education experience in the regular schedule of the mathematics class. She decided to invite some students and form a club, outside the hours destined to mathematics,

where they could work with topics of critical mathematics education<sup>8</sup>. Ana justifies this decision because students of the 9th grade will have a final exam at the end of the year, on which their final grades will depend and the approval to enrol in 10<sup>th</sup> grade next year. Here we can notice the contradiction between the official discourse (present in the curriculum – involving students with topics of critical mathematics education) and the real practice where it is the exam which delineates the teaching content and form. Ana is well aware of this contradiction:

É assim visível que, mesmo sendo uma professora com preocupações ligadas à educação matemática crítica e ciente de que a desocultação das estruturas matemáticas presentes em fenómenos sociais constitui uma forma de aprendizagem potencialmente mais significativa para a maioria dos alunos, a pressão do sistema organizacional envolvente (escola, pais e alunos) levou-me a tomar esta opção, o que ilustra as primeiras dificuldades que um professor tem de enfrentar quando se pretende implementar este tipo de trabalho no contexto de aula de Matemática. (Alves, 2007 p. 57, 58)<sup>9</sup>

The idea conveyed here is that it is good and innovative to implement such topics but there is an inner and rather invisible pressing into conformity that the teacher is aware of and that makes her to put in practice activities that do not directly challenge the school system (and do not change any core features of the school structuring activities). On the other hand, it turns explicit that critical mathematics education is not part of the curriculum and in pupils' minds creates the idea that perhaps is not really mathematics.

Another aspect of the research of Ana that we want to highlight is the criteria that she used to choose the students to interview. She opted for those who had shown more interest and enthusiasm along the sessions, and justifies this choice by mentioning the visibility – “choosing those who appeared more involved and participative in the sessions was a way of guarantee the collection of data (...) I choose the students who gave more visibility to their involvement” (Alves, 2007, p. 66, our translation). This is an option that most researchers do (finding the ‘best’ informants) as they need to provide clear evidence of their claims. In the case of Ana, what did she want to make visible in her research? She wanted to highlight the potentialities of critical mathematics education for developing citizenship. Therefore, it was not appropriate to chose students who in a way or another did not engage so enthusiastically which such experiences. On the other hand, the selection of the students was also related to the aim of her research. This type of “selection” is a case of what Vithal & Valero (2003) call the “cleaning” of research – putting aside the conflicts and the constrains so that research is presented in a harmonious and positive way.

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<sup>8</sup> Ana explored with students two situations: “Supermarket promotions” and “A taxi trip”.

<sup>9</sup> Translation: It is though visible that, even being a teacher concerned with critical mathematics education and conscious that uncovering mathematical structures present in social phenomena is a way of learning potentially more meaningful to the majority of pupils, the pression of the school system (school, parents, pupils) took me to decide for this alternative [critical mathematics education developed not in the regular class but in a club] showing the first kind of difficulties that a teacher faces when one wants to implement this kind of work in the context of school mathematics.

What about if what Ana wanted to turn visible was precisely the artificiality of bringing to the structure of the school system ideas that conflict with that structure? What would be the more visible things in this case? The issue of visibility has always to do with what we want to make visible and ultimately with the research problem we formulate.

Finally, Ana justifies the lower involvement of some students because, on the one hand, they were not familiarised with the way they work in the club (which was more unstructured and free than in the classroom environment) and, on the other hand, they were still attached to a vision of mathematics as a static science having nothing to do with real life situations. Although these arguments could be true, we suggest that other issues are at stake. We take the risk of saying that the lower involvement of the students could be due to the fact that they knew that these activities would not contribute directly to prepare them to the tests and to get a good mark at the end of the year. Using Vinner's (2007) description of school as a credit system, we could say that students felt that those activities will not give them much credit.<sup>10</sup> Just remember how many times a teacher who wants to flower some explanation (a little bit of history, an application, a connection with other themes, a more insightful explanation) heard the students promptly ask "will that show in the test, teacher?" And the teacher is forced to say "well, yes" if maintaining students' attention is in the agenda.

## **OPENING POSSIBILITIES FOR A CRITICAL METHODOLOGY**

The research developed by Ana shows methodological concerns that are characteristic of a critical methodology. The most evident one is the assumption by the researcher of her subjectivity. Ana expresses her concerns about the difficulties of implementing critical mathematics education in schools, the resistance of the students to such topics, the pressure to fulfil the entire disciplinary program, the need to prepare students for the final exam. This is an example of what Valero (2004) calls "making the researcher visible" (p. 19), which opens to the critical examination of the reader the products of the research process, the intentionality of the researcher, and the paths that the researcher decided. It was this openness in the work of Ana that allowed us to develop such a critique on her work.

But, despite all the difficulties, Ana assumes that it is possible and desirable to develop with students tasks of critical mathematics education, and suggests that this could be a way of promoting a bigger societal transformation. Ana conceives transformation within the school structure. What we think remains problematic in such approach is the absence of a critical analysis of schools as institutions of reproduction which tend to incorporate all potential emancipatory reforms into the mainstream ideology fuelling schools which we identify as capitalism. When we say,

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<sup>10</sup> It is always useful to remember the research made by Baldino & Cabral (1999), where they show how students in school are primarily worried to pass (and not necessarily to learn).

like Ana does, that students should become active, interventive, competent, critical citizens we should also ask what it means to educate people to be participative in a more and more consumerist society? When we abstract these desirable features for the students from the concrete social spaces in which they are made operational, we take the risk of just being part of the language game with empty words ready to be filled with the dominant ideology.

We argue that a critical methodology in mathematics education research needs to bring to light what Žižek (2005) calls the symptoms – the points at which the hidden truths of a system emerge, and to avoid engaging in salvation discourses which, by blindly misunderstanding the true problems facing mathematics education, only perpetuate existing realities. In the case of the research developed by Ana one of these symptoms is the fact that critical mathematics education collides with the assessment system, which forced her to implement the critical mathematics tasks outside the mathematics classroom. Ana sees this contradiction as a difficulty, as a problem she had to surpass in order to open a space to promote critical education to her students. But what this contradiction shows is that what the system points as the most important role of the teacher is in fact to prepare students for the final exam. It is good to work with students on these “radical” topics as long as they do not change the smooth functioning of schools as credit systems (Vinner, 2007). This way a potentially emancipatory attempt to educate students according to a critical education is completely inserted and transformed into a small change. Maybe this is a case of what Paulo Freire called the “superficial transformations”: when he suggests that the elites are anxious to maintain the status quo by allowing only superficial transformations designed to prevent any real change in their power of prescription.

From a critical theory stance this ‘marginal’ problem makes visible the inconsistency of the system itself and may force the radical teacher to face the challenge of being an ‘outsider’ within the system and the Trojan horse inside.

This realisation is well acknowledged in critical mathematics education research, although we continue to act as if it were not. It is common to acknowledge that critical mathematics education research requires social and political approaches that commonly situate the problem of “change” in a broader context than the classroom or schools (Gutstein, 2003; Gates & Zevenbergen, 2009; Valero, 2004). Although studies in a critical trend acknowledge this social and political dimension of emancipation (especially in the beginning and the end of the texts), we argue that they manifest signs of persistence as if the problem of allowing students a “critical education” could be understood and solved within mathematics education. It is as if we realize that the problem has a social and political nature well beyond the classroom, but, since we are mathematics educators, we should investigate it in the classroom.

We understand critical methodology as explicitly addressing these borderline problems, which truly connects mathematics education to the political sphere in which we live. Our claim is that a critical methodology should imply the

responsibility for the researcher to develop a critical stance towards his/her own work and results, by framing his/her research in the social and political discourses in which he/she moves.

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