(2) Optimization

Study questions

1. Write down the definition of a smooth multivariate real-valued function.
2. Write down the definition of the gradient of a multivariate real-valued function.
3. Write down the definition of the Hessian of a multivariate real-valued function.
4. Write down Taylor's theorem in the mean value theorem form.
5. Write down the definition of an unconstrained minimization problem.
6. Why does it suffice to consider minimization problems in optimization?
7. Write down the definition of global, local, and strict local function minimizers.
8. State the first-order necessary condition for a local minimum.
9. State the second-order necessary conditions for a local minimum.
10. State the second-order sufficient conditions for a local minimum.
11. Write down the general form of a gradient descent algorithm and explain its components.
12. Why is the direction of the negative gradient a sensible direction for function minimization?
13. Write down the general form of a line search algorithm and explain its components.
14. Write down the general form of a Newton descent algorithm and explain its components.
15. What motivates the choice of the Newton descent update?
16. Write down the general form of a trust region algorithm and explain its components.

Exercises

1. Implement a gradient descent algorithm for the minmization of the Rosenbrock function

$$f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto f(x, y) := (1 - x)^2 + 100(y - x^2)^2. \tag{1}$$

   Document and visualize your results.

2. Implement a Newton descent algorithm for the minmization of the Himmelblau function

$$f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto f(x, y) := (x^2 + y - 11)^2 + (x + y^2 - 7)^2 \tag{2}$$

   Document and visualize your results.