

**(4) Gaussian models**

## STUDY QUESTIONS

1. Write down the definition of a linear Gaussian model (LGM).
2. Write down an LGM in hierarchical form and name its components.
3. Write down the marginal observed data distribution of an LGM.
4. What are the central differences between factor analysis, probabilistic PCA, and PCA from the perspective of LGMs?
5. Which question does “inference” in LGMs address?
6. Which question does “learning” in LGMs address?
7. How can the conditional distribution of the latent random vector  $x$  given the observed random vector  $y$  be evaluated in LGMs?
8. Write down the formula of the evidence lower bound  $\text{ELBO}(q(X), \theta)$  and name its components.
9. Write down the (general) expectation-maximization algorithm.
10. Write down the exact expectation-maximization algorithm.
11. What is the purpose of the application of the exact EM algorithm in LGMs?
12. Describe how the evidence lower bound changes with respect to the log marginal likelihood during the E and M Step of the exact expectation-maximization algorithm.
13. Write down the definition of a factor analysis model and name its components.
14. How does a factor analysis model explain covariance between factors and unique variance of individual factors?
15. Write down the definition of a probabilistic principal component analysis model.
16. How are the  $B$  parameter and the orthonormal decomposition  $\mathbb{C}(y) = Q\Lambda Q^T$  of the data covariance matrix of a probabilistic principal component analysis model related?
17. Write down the definition of a principal component analysis model.
18. How are the  $B$  parameter and the orthonormal decomposition  $\mathbb{C}(y) = Q\Lambda Q^T$  of the data covariance matrix of a principal component analysis model related?
19. Write down the definition of an independent component analysis analysis model.
20. Write down an independent component analysis model in hierarchical form and name its components.

## EXERCISES

1. Implement the exact EM algorithm for factor analysis.
2. Implement the exact EM algorithm for probabilistic principal component analysis.
3. Implement the infomax algorithm for ICA parameter estimation.