Bayesian inference

(12) Foundations and conjugate inference

Study questions

1. Write down the definition of a probabilistic model, a generative model, a prior distribution, and a likelihood.
2. Write down the distribution of n conditionally independent and identically distributed random variables $X_i, i = 1, ..., n$ given a parameter random variable θ.
3. Write down the posterior distribution for a probabilistic model $p(x_{1:n}, \theta)$.
4. Describe the difference and similarities between batch and recursive Bayesian estimation.
5. Write down the definition of the marginal data likelihood (model evidence).
6. Given two probabilistic models and a set of data observations, write down the Bayes factor.
7. Write down the definition of the posterior predictive distribution.
8. Write down the definition of a loss function, the expected posterior loss, and a Bayes estimator.
9. Write down the Bayes estimator under a quadratic loss function.
10. Write down the Bayes estimator under zero-one loss function.
11. Write down the definition of a conjugate family of distributions.

Exercises (Theory)

1. Derive the posterior distribution as well as the MMSE and MAP estimators for the Beta-Binomial model (Lecture slides).
2. Derive the posterior distribution as well as the MMSE and MAP estimators for the univariate Gaussian-Gaussian model (Lecture slides).
3. Derive the posterior distribution as well as the MMSE and MAP estimators for the Dirichlet-Multinomial model (Held and Sabanés Bové (2014, Example 6.20)).
4. Derive the posterior distribution as well as the MMSE and MAP estimators for the Gaussian-Gamma-Gaussian model (Held and Sabanés Bové (2014, Example 6.21)).
5. Show that the Bayes estimator under an absolute error loss function is given by the median (DeGroot and Schervish, 2012, Corollary 7.4.2, Theorem 4.5.3).

Exercises (Programming)

1. For $n = 10$, implement batch and recursive Bayesian estimation for the Beta-Binomial model. Compare the results based on identical samples.
2. Using simulation, study the bias and consistency properties of the posterior expected value of the Beta-Binomial model.
3. Using simulation, study the bias and consistency properties of the posterior expected value of the Gaussian-Gaussian model.