(14) Numerical methods

Study questions

1. Name two quantities in Bayesian inference that often necessitate numerical integration.
2. Name an example for a quadrature rule.
3. What is the difference between the right rule and the midpoint rule in Riemann sum-based numerical integration?
4. State Laplace's integral approximation method.
5. Write down the Laplace approximation of a posterior expectation of the form $\mathbb{E}_{p(\theta|x_{1:n})}(f(\theta))$.
6. Write down the definition of the Monte Carlo estimator of an integral $I = \int_{\mathcal{X}} f(x)p(x)\,dx$.
7. State the importance sampling identity.
8. Write down the acceptance-rejection algorithm.

Exercises (Theory)

1. Develop the Laplace approximation as discussed in the lecture (Lecture slides).
2. Apply the Laplace approximation for the evaluation of the posterior expected value in a Beta-Bernoulli model (Held and Sabanés Bové, 2014, Example 8.3).
3. Show the validity of the acceptance-rejection algorithm (**?**, Section 8.3.3,lecture slides)

Exercises (Programming)

1. Estimate the expected value of a Beta(α, β) for varying values of α and β by means of Monte Carlo integration by using a Beta distribution random number generator. Compare the results to the true expected values.
2. Estimate the expected value of a Beta(α, β) for varying values of α and β by means of Monte Carlo integration using an importance sampling scheme and a uniform random number generator.
3. Use an acceptance-rejection algorithm to sample random numbers from Beta$(5, 1)$.