(14) **Numerical methods**

**Study questions**

1. Name two quantities in Bayesian inference that often necessitate numerical integration.

2. Name an example for a quadrature rule.

3. What is the difference between the right rule and the midpoint rule in Riemann sum-based numerical integration?


5. Write down the Laplace approximation of a posterior expectation of the form $E_{p(\theta|X_1:n)}(f(\theta))$.

6. Write down the definition of the Monte Carlo estimator of an integral $I = \int_X f(x)p(x) \, dx$.

7. State the importance sampling identity.

8. Write down the acceptance-rejection algorithm.

**Exercises (Theory)**

1. Develop the Laplace approximation as discussed in the lecture ([Lecture slides](#)).

2. Apply the Laplace approximation for the evaluation of the posterior expected value in a Beta-Bernoulli model ([Held and Sabanés Bové, 2014, Example 8.3](#)).

3. Show the validity of the acceptance-rejection algorithm ([?, Section 8.3.3, lecture slides](#)).

**Exercises (Programming)**

1. Estimate the expected value of a Beta($\alpha, \beta$) for varying values of $\alpha$ and $\beta$ by means of Monte Carlo integration by using a Beta distribution random number generator. Compare the results to the true expected values.

2. Estimate the expected value of a Beta($\alpha, \beta$) for varying values of $\alpha$ and $\beta$ by means of Monte Carlo integration using an importance sampling scheme and a uniform random number generator.

3. Use an acceptance-rejection algorithm to sample random numbers from Beta(5, 1).