(15) Variational inference

Study questions

1. Define the variational inference problem.
2. Write down the log model evidence decomposition.
3. Write down the definition of the evidence lower bound.
4. Write down the definition of the Kullback-Leibler divergence.
5. Name two properties of the Kullback-Leilber divergence.
6. State the evidence lower bound theorem.
7. Describe two approaches of using the evidence lower bound theorem for solving the variational inference problem.
8. Define the concept of a mean-field approximation in variational inference.
9. State the free-form mean-field variational inference theorem.
10. Write down the general CAVI algorithm.
11. Define the concept of fixed-form variational inference.

Exercises (Theory)

1. Show that $\mathrm{KL}(q(x)||p(x)) = 0$ for $q(x) = p(x)$.
2. Prove Jensen's inequality by induction (Jensen inequality notes).
3. Prove the non-negativity of the Kullback-Leibler divergence (Murphy, 2012, Section 2.8.2).

Exercises (Programming)

1. Evaluate and visualize the KL divergence between two Gaussian PDFs as well as between two Gamma PDFs for varying parameter settings of the distributions. Closed form solutions for these KL divergence are available from the literature.
2. Implement the free-form CAVI algorithm for the Gaussian-Gamma model as introduced in the lecture. Visualize the likelihood, prior, approximate posterior, iterative variational distributions, as well as the ELBO.
3. Implement fixed-form algorithm for the nonlinear Gaussian model as introduced in the lecture. Visualize the likelihood, prior, approximate posterior, iterative variational distributions, as well as the ELBO approximation function.