(2) Random variables

Study questions

1. Write down the definition of a random variable.
2. Write down the definition of the cumulative distribution function of random variable.
3. Write down the definition of a probability mass function.
4. Express the value $P(x)$ of a cumulative distribution function of a discrete random variable X in terms of its probability mass function.
5. Write down the definition of a continuous random variable and its associated probability density function.
6. Express the cumulative distribution function of a continuous random variable in terms of its probability density function.
7. Write down the definition of the quantile function, and the first and third quartile of a random variable.

Exercises

1. Develop a probability space model of throwing two dice and the probability space model that results from defining a random variable that evaluates the sum of the pips (e.g., Moeschlin, 2000, Beispiel 3.1.1).
2. Let X denote a random variable with outcome space $\mathcal{X}$ and let P denote its cumulative distribution function. Show that for all $x \in \mathcal{X}$ it holds that $\mathbb{P}(X > x) = 1 - P(x)$ and that for all $x_1, x_2 \in \mathcal{X}$ it holds that $\mathbb{P}(x_1 < X \leq x_2) = P(x_2) - P(x_1)$ (DeGroot and Schervish, 2012, Theorems 3.3.1 and 3.3.2).

Exercises (Programming)

1. Visualize the probability mass function of a Bernoulli random variable and the probability density function of a Gaussian random variable. On top, visualize histograms of many samples from each random variable for the same parameter settings. For the Gaussian random variable, document how you normalize the histogram.
2. Visualize the probability density functions of a Beta random variable and a Gamma random variable (DeGroot and Schervish, 2012, Definitions 5.8.2 and 5.7.2). On top, visualize appropriately normalized histograms of many samples from each random variable for the same parameter settings.