(3) Joint distributions

Study questions

1. Write down the definition of a joint PMF two discrete random variables X and Y with finite outcome set.
2. Write down the definition of the joint PDF of two continuous random variables X and Y each taking values in $\mathbb{R}$.
3. Write down the definition of a bivariate Gaussian PDF and comment on the interpretation of its parameters.
4. Write down the definition of the joint cumulative distribution function of two random variables.
5. Write down the definition of the marginal PMFs of a joint distribution of two random variables X and Y with joint PMF $p_{X,Y}$.
6. Write down the definition of the marginal PDFs of a joint distribution of two random variables X and Y with joint PDF $p_{X,Y}$.
7. Write down the definition of the independence of two random variables X and Y.
8. Write down the necessary and sufficient condition for the independence of two random variables X and Y with joint PMF/PDF $p_{X,Y}$.
9. Write down the definition of a conditional PMF.
10. Write down the definition of a conditional PDF.
11. Write down the definition of a multivariate Gaussian PDF and comment on the meaning of its paramters.
12. Write down the multivariate Law of Total Probability and Bayes theorem.
13. Write down the definition of the independence of n random variables X_i.
14. What does it mean for n random variables $X_1, ..., X_n$ to be i.i.d.?
15. What does it mean for $X_1, ..., X_n$ to be a random sample of size n from p?

Exercises (Theory)

1. Construct a mixed joint distribution comprising a marginal uniform distribution and a conditional binomial distribution DeGroot and Schervish (2012, Example 3.6.7).
2. Develop the formal definition of a marginal distribution based on DeGroot and Schervish (2012, Theorem 3.4.5, Definition 3.5.1).
3. Develop the theory of product measures and product probability spaces for two random variables (e.g., Fristedt and Gray (2013, Section 9.2), Billingsley (2012, Chapter 7), Schmidt (2011, Chapter 11.4)).

Exercises (Programming)

1. Write a simulation that demonstrates that the marginal distributions of a bivariate Gaussian distribution with expectation parameter $\mu = (1, 2)^T$ and covariance matrix parameter $\Sigma = \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.5 \end{pmatrix}$ are given by univariate Gaussian distributions with expectation parameters $\mu_1 = 1, \mu_2 = 2$ and variance parameters $\sigma^2 = 0.3$ and $\sigma^2 = 0.5$. For the simulation, make use of multivariate Gaussian probability density and random number generators. Visualize and document your results.

2. Write a simulation that verifies that obtaining samples from 5 independent univariate Gaussian distributions with parameters $\mu_i, \sigma_i^1 > 0, i = 1, ..., 5$ is equivalent to obtaining samples from a 5-dimensional multivariate Gaussian distribution with the appropriately specified parameters $\mu \in \mathbb{R}^5$ and $\Lambda \in \mathbb{R}^{n \times n}$. Visualize and document your results.

3. Write a simulation that verifies the analytical results on joint and conditional Gaussian distributions for the case of univariate X and Y, $A := 2$ and $b := 1$ by visually displaying the respective PDFs and their large sample normalized histograms.