

(8) Asymptotic estimator properties

STUDY QUESTIONS

1. Write down the definition of an asymptotically unbiased estimator.
2. Write down the definition of a consistent estimator.
3. State the mean squared error criterion for estimator consistency.
4. State the bias and variance criterion for estimator consistency.
5. Write down the definition of an asymptotically normally distributed estimator.
6. Write down the definition of an asymptotically efficient estimator.
7. Name five properties of maximum likelihood estimators.

EXERCISES (THEORY)

1. Show the consistency of the sample mean without recourse to the mean squared error consistency criterion ([Casella and Berger, 2002](#), Example 10.1.2)
2. Show the consistency of the sample variance and sample standard deviation ([Casella and Berger, 2002](#), Examples 5.5.3 and 5.5.5)
3. Sketch the proof of the asymptotic efficiency of maximum likelihood estimators ([Casella and Berger, 2002](#), Theorem 10.1.12 and its proof).

EXERCISES (PROGRAMMING)

1. Write a simulation that verifies the asymptotic unbiasedness of the maximum likelihood estimator for the variance parameter of a univariate Gaussian distribution. Include a verification of the unbiasedness of the sample variance.
2. Write a simulation that verifies the asymptotic efficiency of the maximum likelihood estimator for the parameter of a Bernoulli distribution.
3. Write a simulation that verifies the asymptotic efficiency of the maximum likelihood estimator for the variance parameter of a univariate Gaussian distribution.