(9) Confidence intervals

Study questions

1. Define the δ-confidence interval.
2. Give two interpretations of δ-confidence intervals.
3. Define the notions of exact and approximate pivots and δ-confidence intervals.
4. Write down the definition of the Z statistic and state its distribution.
5. Write down the definition of the U statistic and state its distribution.
6. Write down the definition of the T statistic and state its distribution.
7. Write down the definition of the Wald statistic and state its distribution.
8. State the steps involved in the typical construction of confidence intervals.
9. Write down the formula of the 95%-confidence interval for the expectation parameter of a univariate Gaussian distribution with known variance.
10. Write down the formula of the 95%-confidence interval for the expectation parameter of a univariate Gaussian distribution with unknown variance.
11. Write down the formula of the 95%-confidence interval for the variance parameter of a univariate Gaussian distribution.
12. Write down the formula for approximate 95%-confidence intervals for parameters based on maximum likelihood estimators.

Exercises (Theory)

1. Develop a 95%-confidence interval for the paramter of an exponential distribution (Held and Sabanés Bové, 2014, Example 3.74).
2. By means of example, show that a confidence interval is not a probability statement about a true, but unknown, parameter (Wasserman, 2004, Example 6.14).
3. Show that the T statistic has a Student's t distribution with $n - 1$ degrees of freedom (Casella and Berger, 2002, Definition 5.3.4 and p.223 - 224).

Exercises (Programming)

1. Write a simulation that verifies that the T statistic is distributed according to a t-distribution with $n - 1$ degrees of freedom.
2. Write a simulation that verifies that the 95%-confidence interval for the expectation parameter of a Gaussian distribution with unknown variance comprises the true, but unknown, expectation parameter in $\approx 95\%$ of its realizations.
3. Write a simulation that verifies that the approximate 95%-confidence interval for the expectation parameter of a Bernoulli distribution comprises the true, but unknown, expectation parameter in $\approx 95\%$ of its realizations.