1 Introduction

1. Give definitions of the terms model formulation, model estimation, and model evaluation.
2. What is the difference between static and dynamic models?
3. Provide a brief overview of differences and commonalities between Frequentist and Bayesian statistics.
4. Define the terms independent experimental variable, dependent experimental variable, categorical variable, and continuous variable.
5. Explain the difference between within- and between-participant experimental designs.
6. Consider the GLM equation $y = X\beta + \varepsilon$. Which of the symbols y, X, β, ε represents independent experimental variables, which of the symbols represents dependent experimental variables?
7. Consider the GLM equation $y = X\beta + \varepsilon$. In an experimental context, which of the components of this equation are known before performing and experiment, which of the components are known to after the experiment before estimating the model?
8. The design matrix X is of dimensionality $n \times p$. What do $n \in \mathbb{N}$ and $p \in \mathbb{N}$ represent, respectively?
9. Express the GLM (matrix) equation $y = X\beta + \varepsilon$ as a set of (simultaneous non-matrix) equations for $n := 4$ and $p := 3$.
10. Name and explain the components and their properties of the GLM equation $y = X\beta + \varepsilon$.

2 Sets, sums, and functions

1. Give brief explanations of the symbols $\mathbb{N}, \mathbb{N}_n, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{R}^n$.
2. Provide a numerical example for $x \in \mathbb{R}^5$.
3. Consider the sets $A := \{1, 2, 3\}$ and $B := \{3, 4, 5\}$. Write down the sets $C := A \cup B$ and $D := A \cap B$.
4. Write down the definition of the interval $[0, 1] \subset \mathbb{R}$. Is 0 an element of this interval?
5. Evaluate the sum $y := \sum_{i=1}^{4} a_i x_i$ for $a_1 = -1, a_2 = 0, a_3 = 2, a_4 = -2$ and $x_1 = 3, x_2 = 2, x_3 = 5, x_4 = -2$.
6. Explain the meaning of $f : D \to R, x \mapsto f(x)$ and its components f, D, R, x, and $f(x)$ using an example of your choice.
7. Is the function
$$f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := 2x + 2 \tag{1}$$
a linear function? Justify your answer.
8. Sketch the identity function.
9. Sketch the exponential function.
10. Sketch the natural logarithm.

3 Calculus

1. Give a brief explanation of the notion of a derivative of a univariate function f in a point x.
2. Provide brief explanations of the symbols $\frac{d}{dx}$, $\frac{d^2}{dx^2}$, $\frac{\partial}{\partial x}$, and $\frac{\partial^2}{\partial x^2}$.
3. Compute the first derivative of $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := 3\exp(-x^2)$.
4. Determine the minimum of the function $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := x^2 + 3x - 2$.
5. Compute the partial derivatives of $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto f(x, y) := \ln(x) + \sum_{i=1}^{n}(y - 3)^2$.
6. Write down the definition of the gradient of a multivariate real-valued function.
7. Write down the definition of the Hessian of a multivariate real-valued function.
8. Evaluate gradient and Hessian of $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto f(x, y) := 2\exp(x^2 - 3y)$.
9. State the intuitions for the definite integral $\int_a^b f(x)dx$ and the indefinite integral $\int f(x)dx$ of a univariate real-valued function f.
10. Evaluate the definite integral $I := \int_1^3 5x^2 + 2x\, dx$.

4 Matrices

1. Write down the definition of matrix addition, subtraction, and scalar multiplication.
2. Write down the definition of the matrix product.
3. If X is an $n \times p$ matrix and β is an $p \times 1$ vector, what is the size of $X\beta$?
4. Let
$$A := \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \text{ and } B := \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}. \tag{2}$$
Evaluate the following matrices: $C := A + B^T, D := A - B, E := AB, F := BA$.
5. Let
$$A := \begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}. \tag{3}$$
Evaluate
$$x = Ab, B = bb^T A^T, \text{ and } C = b^T A^T A. \tag{4}$$
6. Let $X \in \mathbb{R}^{10\times 3}$ and $y \in \mathbb{R}^{10}$. What are the sizes of $X^T X$, $(X^T X)^{-1}$, $X^T y$, and of $(X^T X)^{-1} X^T y$?
7. Explain the concept of a matrix inverse.
8. Evaluate the inverse A^{-1} for $A := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.
9. Evaluate the determinants of the matrices
$$M := \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \text{ and } N := \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}. \tag{5}$$
10. Write down the definition of a positive-definite matrix.

5 Probability and random variables

1. Write down the definition of a probability space.
2. Write down the definition of the independence of two events A and B.
3. Write down the definition of a random variable.
4. Write down the definition of the cumulative distribution function of a random variable.
5. Write down the definitions of a PMF and a PDF.
6. Write down the definition of a random vector.
7. Write down the definition of the cumulative distribution function of a random vector.
8. Write down the definition of a multivariate PMF and a multivariate PDF.
9. Write down the definition of the independence of n random variables $\xi_i, i = 1, ..., n$.
10. What does it mean for n random variables $\xi_1, ..., \xi_n$ to be iid?

6 Expectation, covariance, and transformations

1. Write down the definition of the expectation of a random variable and discuss its intuition.
2. What does it mean for the expectation of a random variable to exist?
3. State the linearity and multiplication properties of expectations.
4. Write down the definition of the variance of a random variable and discuss its intuition.
5. Write down the definition of the standard deviation of a random variable and discuss its intuition.
6. Write down the expectation of the square of a random variable in terms of its variance and expectation.
7. For constant a random variable ξ and a constant a, what is $\mathbb{V}(a\xi)$?
8. Write down the definition of the covariance and correlation of two random variables ξ_1 and ξ_2.
9. Express the covariance of two random variables ξ_1 and ξ_2 in terms of expectations.
10. What is the variance of the sum of two random variables ξ_1 and ξ_2, if ξ_1 and ξ_2 are independent and in general?

7 Probability distributions

1. Write down the PDF of a univariate Gaussian distribution.
2. Write down the PDF of a standard normal distribution.
3. Write down the PDF of a multivariate Gaussian distribution and comment on its components.
4. State the theorem on linear-affine transformations of Gaussian random vectors.
5. State the theorem on independent Gaussian distributions.
6. State the theorem on squared standard normal random variables.
7. State the theorem on t distributions.
8. Write down the GLM in multivariate Gaussian form and comment on its components.
9. Write down the GLM formulation of independent and identically distributed Gaussian samples.
10. Write down the GLM formulation of a simple linear regression model.

8 Maximum likelihood estimation

1. Write down the general form of a likelihood function and name its components.
2. Write down the general form of a log likelihood function and name its components
3. Write down the general form of an ML estimator and explain it.
4. Discuss commonalities and differences between OLS and ML beta parameter estimators.
5. Write down the formula of the GLM ML beta parameter estimator and name its components.
6. Write down the formula of the GLM ML variance parameter estimator and name its components.
7. Write down the formula of the GLM ReML variance parameter estimator and name its components.
8. Define the sum of error squares (SES) and the residual sum of squares (RSS) and discuss their commonalities and differences.
9. Write down the GLM of incarnation independent and identical sampling from a univariate Gaussian distribution as well as the ensuing expectation and variance parameter estimators.

9 Frequentist distribution theory

1. Discuss the intuitive background of the Frequentist distribution theory.
2. State the Beta parameter estimate distribution theorem.
3. State the Variance parameter estimate distribution theorem.
4. Write down the definition of the T-statistic.
5. Discuss the intuitive meaning of the T-statistic.
6. Write down the definition of the centred T-statistic.
7. State the distribution of the centred T-statistic.
8. Write down the definition of the F-statistic.
9. Discuss the intuitive meaning of the F-statistic.
10. State the distribution of the centred F-statistic for $\beta_2 = 0_{p_2}$.

10 Statistical testing

1. Define the notion of a test hypothesis.
2. Discuss the notions of simple and composite test hypotheses.
3. Define the notion of a statistical test.
4. Define the notions of a test statistic and a test decision rule.
5. Define the notion of a test rejection region.
6. Define Type I and Type II errors.
7. Define the size and the power of a test.
8. Explain the notion a test's significance level.
9. Define the notions of conservative, exact, and liberal tests.
10. Discuss the standard approach to test construction.

11 T-tests and simple linear regression

1. Discuss the extremes of the spectrum of GLM designs for the analysis of any data set.
2. Discuss commonalities and differences between continuous and categorical GLM designs.
3. Write down the GLM formulation of the one-sample t-test.
4. Write down the one-sample t-test beta estimator and variance parameter estimator.
5. Write down the one-sample t-test T-statistic for the contrast vector $c := 1$.
6. Write down the GLM formulation of the independent two-sample t-test.
7. Write down the independent two-sample t-test beta estimator and variance parameter estimator.
8. Write down the independent two-sample t-test T-statistic for the contrast vector $c := (1, -1)^T$.
9. Write down the GLM formulation of simple linear regression.
10. Write down the simple linear regression beta estimator.

12 Multiple linear regression

1. Write down the GLM formulation of a multiple linear regression with one offset variable and two predictor variables.
2. Define the notions of collinear, orthogonal, and correlated design matrix regressors.
3. Discuss, why for a design matrix $X \in \mathbb{R}^{n \times p}$ and contrast weight vector $c \in \mathbb{R}^p$ the function

$$\xi : \mathbb{R}^p \times \mathbb{R}^{n \times p} \to \mathbb{R}_{>0},\ (c, X) \mapsto \xi(c, X) := \left(c^T (X^T X)^{-1} c\right)^{-1} \tag{6}$$

   has some merit as a measure of the efficiency of the experimental design encoded in the design matrix.
4. What does it mean for the GLM to be used for fMRI data analysis in a *mass-univariate* fashion?
5. Describe the fMRI data organization after fMRI data preprocessing.
6. What is the difference between the haemodynamic response and a haemodynamic response function?
7. Which GLM design category is used for the analysis of fMRI time-series data?
8. What do the beta parameter estimates obtained in a GLM fMRI time-series data of a single voxel reflect?
9. What is a statistical parametric map?
10. How can fMRI designs be optimized?

13 One-way ANOVA

1. Provide a verbose account of the reference cell method reformulation for one-way ANOVA designs.
2. Write down the one-way ANOVA GLM in its reference cell, i.e., non-over-parameterized formulation.
3. Write down the beta parameter estimator of a one-way ANOVA GLM in its reference cell formulation.
4. Define the grand mean and the ith level means of a one-way ANOVA.
5. Define the total, between-level, and within-level sum of error squares of a one-way ANOVA.
6. Define the total, between-level, and within-level degrees of freedom of a one-way ANOVA.
7. Define the total, between-level, and within-level mean squares of a one-way ANOVA.
8. Define the F-statistic in terms of mean squares and discuss its intuition.
9. Write down the reduced and full model one-way ANOVA GLMs, such that the total sum of error squares corresponds to the residual sum of squares of the reduced model and the within-level sum of error squares corresponds to the residual sum of squares of the full model.
10. Write down the F-statistic in its mean squares and its residual sum of squares model form and explain their equivalence in verbose form.

14 Two-way ANOVA

1. Explain the terms *factorial design* and *2 × 2 factorial design*
2. Explain the terms *main effects* and *interaction* in factorial designs.
3. Write down the structural form of a purely additive two-way ANOVA design upon reference cell formulation.
4. Write down the GLM form of a purely additive 2×2 ANOVA design upon reference cell formulation.
5. Write down the structural form of a two-way ANOVA design with interaction upon reference cell formulation.
6. Write down the GLM form of a 2×2 ANOVA design with interaction upon reference cell formulation.