Mathematisation: social process & didactic principle

> Because mathematics is recognizable but not easily defined, we replaced it by a process or processes which can be made more tangible and that we named "mathematization". (Gattegno 1988, p. 1)

Introduction

The intention of CIEAEM 69 is to interrogate the concept of mathematisation which is commonly and undoubtedly accepted as a desirable outcome of formal mathematics education. One of the aims of the 69th CIEAEM conference is to make the mathematisation of social, economic, ecologic, etc. conditions explicit. The second aim of the 69th CIEAEM conference is to reflect on experience with curricular conceptions that pay particular attention to the relation of mathematical and everyday knowledge.

In this call for papers, mathematisation is used in its broadest sense. It may then include people's active use of some kind of mathematics, for example by interpreting notions (including mathematical objects) in the world mathematically, or by expressing one's ideas in a mathematical way. It may also include the way that people encounter mathematics as being used "on" them and their context, for example mathematics as being at the core of how a certain activity is described, or how decisions are made on a mathematically informed basis.

Mathematisation ––in its broad range–– is a concept that has received CIEAEM's attention for more than half a century. We can trace the occupation of CIEAEM and its members back to 1954, when Servais describes the global changes of society that he expects in the following words:

> Our time marks the beginning of the mathematical era. [...] This fact, whatever the reactions, the opinions and the judgments it may provoke, increases the responsibility of every teacher, who, no matter on which level, teaches mathematics. [...] If it befits to be worthy of a mathematical tradition, it is also important to allow the mathematization [of the world] to come. As much as it is true that he [sic] who devotes his life to teaching, accepts a mission of a world gone-by to build a world being born. The responsibility towards the future is greater than loyalty towards the past. (Servais 1954, p. 89; quoted in Vanpaemel, De Bock, & Verschaffel 2011)

This statement is informed by the prevailing optimism that by basing social and technological development on a mathematical tradition the future would be more prosperous than the past. Indeed, as Davis and Hersh show thoroughly 30 years later, "the social and physical worlds are being mathematized at an increasing rate" (1986, p. xv). The extent of the ongoing mathematisation makes Davis and Hersh warn us that "we'd better watch it, because too much of it may not be good for us" (ibid.). Keitel, Kotzmann and Skovsmose substantiate this warning by describing a circular process:

> On the one side society becomes formalized and mathematized by the influence of the self-produced technological environment and economic structures respectively; on the other, mathematics is "naturally" a magnificent help in dealing with technological and quantified surroundings. Society, therefore, needs more and more techno-mathematical help. In this process, many structures of human activity are recognized as having formal character. Hence, one can use mathematics to control or change these structures. It is a characteristic of modern technology and science that not only the purpose determines the means but also the other way round: the means determine or create the ends. (1993, p. 249)

The mathematisation of social, economical and technological relations in the form of formal structures is a double-edged sword. On the one hand, it has proven effective and efficient in terms of developing more and more complex structures. As Fischer points out, "[t]he more mathematics is used to construct a reality, the better it can be applied to describe and handle exactly that reality" (1993, p. 118). On the other hand, once established as the standard (or only) way of describing, predicting and prescribing social, economic, ecologic, etc. processes, it severely reduces the possibilities of finding non-formal, non-quantifiable, non-mathematical solutions to the problems we face (Straehler-Pohl 2017).

Moreover, the mathematisation of social, economical and technological relations cannot be fully understood without taking into account a process occurring in parallel (Gellert & Jablonka 2007) -- the demathematisation of social practices, for instance, the fact that taxes are nowadays deducted automatically from salaries and no longer calculated in the historical form of labour or grain to be given to the authorities:

> The greatest achievement of mathematics, one which is immediately geared to their intrinsic progress, can paradoxically be seen in the never-ending, twofold process of (explicit) demathematising of social practices and (implicit) mathematizing of socially produced objects and techniques. (Chevallard 1989, p. 52)

For Keitel, mathematics-based technology as a form of implicit mathematics "makes mathematics disappear from ordinary social practice" (1989, p. 10). As a consequence, the (explicit) demathematisation of social practices leads to a devaluation of the mathematical knowledge involved in these practices. What kind of mathematical knowledge, then, is helpful so that citizens can do more than simply "obey" the structures which seem so "inseparably connected with our social organization" (Fischer 1993, p. 114)? A threat to the democratic character of our political fundament is thus posed, which Skovsmose translates into the relation between technological and reflective knowledge:

> Technological knowledge itself is insufficient for predicting and analysing the results and consequences of its own production; reflections building upon different competencies are needed. The competence in constructing a car is not adequate for the evaluation of the social consequences of car production. (1994, p. 99)

From a pedagogic point of view, in which democracy and critical citizenship are taken into consideration as the overarching aim of education, the mathematisation/demathematisation of social relations, of economic and technological development can count as a starting point for curricular reflection and imagination. However, what do we really know about the structures and effects of mathematisation and demathematisation? Taken to an extreme, might it even be necessary to actively work toward preserving the capacity and confidence to reject, at least some of the time, the "solv[ing of] problems of social significance by means of mathematics" (Straehler-Pohl 2017, p. 49)?

Turning from the discussion of making mathematisation explicit, we now consider the second aim.

The second aim of the 69th CIEAEM conference is related to a practice where, in most countries, school mathematics, particularly elementary school mathematics, is, and has historically been, constructed as a subject in which everyday knowledge and scientific knowledge are somehow brought together. In these practices it seems to be a commonplace assumption that mathematical knowledge may be useful in all kinds of professional and occupational contexts. See, for instance, an old German mathematics textbook for seventh-graders, on the cover of which mathematics is constructed as prevalent in manual work (Fig. 1). Examples like this abound. Keitel refers to a US textbook of 1937, in whose table of contents mathematics is overtly related to the supposed community needs, when arguing that "a trivial though dogmatic social-needs orientation" (1987, p. 398) is often the driving force for curriculum construction.

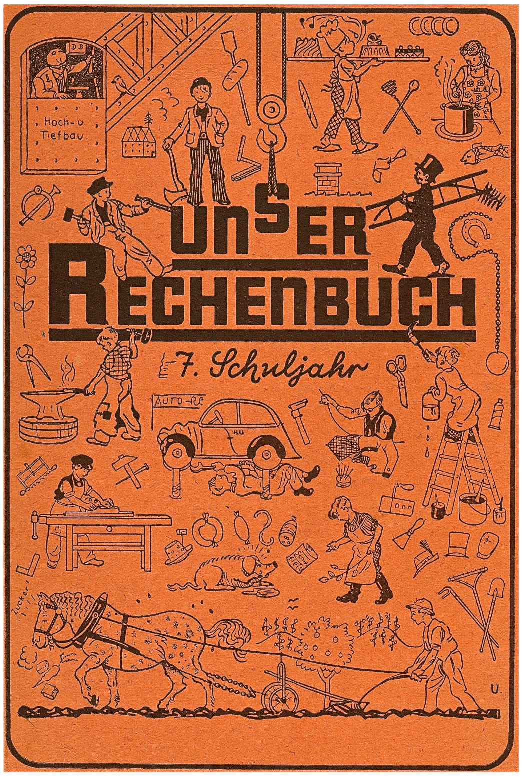

Fig. 1 Front cover of *unser Rechenbuch*, Baßler et al. (1949)

Non-trivial considerations on the relationship of mathematics and the everyday have served, and continue to serve, as the cornerstone of several curriculum conceptions in mathematics education (Jablonka 2003, Verschaffel, Greer, Van Dooren, & Mukhopadhyay 2009). In some of these conceptions, mathematisation is taken as a key didactic principle for the teaching and learning of mathematics.

An internationally influential example of a curriculum conception drawing explicitly on mathematisation(s) is Realistic Mathematics Education (e.g., Treffers 1987, de Lange 1996). RME distinguishes between a horizontal and a vertical mathematisation. A horizontal mathematisation denotes the students' activity of expressing mathematically a realistic everyday situation from which mathematical meaning can be developed. This can be interpreted as a sideways shift between discourses. However, the everyday situations are valued mostly for their didactic potential as a starting point for the mathematisation to occur. Their purpose is illustrative and motivational, and authenticity is not the main criterion for the design of the everyday situations. Once a mathematical formulation of the everyday situation has been arrived at, the next step is a vertical mathematisation, in which the organised structure of mathematical knowledge is the focus. The students get 'deeper' into the mathematics, or arrive at 'higher' levels of abstraction.

Mathematical Modelling (e.g., Blum, Galbraith, Henn, & Niss 2007, Stillman, Blum, & Salett Biembengut 2015) is another orientation for curriculum construction that attracts worldwide attention. Within Mathematical Modelling, the authenticity of everyday situations is of relevance. From these everyday situations a 'real world model' is generated and, further the 'real world model' is translated into a 'mathematical model', which can be used for calculation or other mathematical procedures. This translation is called mathematisation. In this curricular perspective, mathematics education is constructed as a didactically simplified version of applied mathematics.

In relation to the second aim concerning curriculum, two things should not go unnoticed. First, from a psychological perspective on cognitive development mathematisation is strongly related to abstraction, or reflective abstraction, and decontextualisation. The issue has been substantially developed by Vergnaud, who describes the process of dissecting mathematical concepts from sets of problems via concepts such as operational invariants, theorems-in-action, and schemes. Students' symbolic representations and processes of instrumentation represent a major focus in this field (e.g., Vergnaud 1999). It is of interest that Piaget's work, as a central reference for Vergnaud's theoretical developments, has been a long-time influence on discussions in CIEAEM. See for instance Servais (1968), in which a shift from mathematisation-of-the-world to mathematisation-of-a-situation is visible.

> The true involvement of students in mathematical work can only be assured by an adequate motivation at their level: pleasure of playing or of competition, interest for application, satisfaction of the appetite for discovery, the affirmation of themselves, a taste for mathematics itself. In order to learn mathematics in an active manner, it is best to present to the students a situation to be mathematized. So today's didactic is based, as far as possible, on mathematical initiations to situations easy to approach at the basic level and sufficiently interesting and problematic to create and sustain investigations by the students. They learn by experience to schematicize, to untangle the structures, to define, to demonstrate, to apply themselves instead of listening to and memorizing ready-made results. (p. 798)

Second, much of the conceptual work that draws on mathematisation as a didactic principle refers explicitly to the writings of Freudenthal. In *Mathematics as an Educational Task*, his point of departure is an analysis of what mathematisation, or mathematizing, might mean on different mathematical levels:

> Today many would agree that the student should also learn mathematizing unmathematical (or insufficiently mathematical) matters, that is, to learn to organize it into a structure that is accessible to mathematical refinements. Grasping spatial *gestalts* as figures is mathematizing space. Arranging the properties of a parallelogram such that a particular one pops up to base the others on it in order to arrive at a definition of parallelogram, that is mathematizing the conceptual field of the parallelogram. Arranging the geometrical theorems to get all of them from a few, that is mathematizing (or axiomatizing) geometry. Organizing this system by linguistic means is again mathematizing of a subject, now called formalizing. (Freudenthal 1973, p. 133)

In this quote, the RME-concepts of horizontal mathematisation (as mathematizing the unmathematical) and vertical mathematisation (as axiomatizing and formalizing) are already elaborately preformed.

Subthemes and Questions

The theme of the conference *Mathematisation: social process & didactic principle* aims to attract contributions based on experience and analysis of a diverse nature and broad variety. Four subthemes, which represent possible thematic foci and will thus be used as a basis for the composition of the working groups, help to orientate and to categorize the contributions.

- Subtheme 1 is concerned with the issue of mathematisation as a didactic principle. It collects research on, and experience with, the teaching and learning of mathematics by mathematisations and in the classroom (or kindergarten, university, …) and also considers curriculum development in this field.
- Subtheme 2, in contrast to Subtheme 1, is not directly related to the learning of mathematics. It engages with the ways in which society is mathematised, and with the recent mathematisations by which the current local and global social, environmental, etc. situation are modelled.
- Subtheme 3 tries to bring the topics of the subthemes 1 and 2 into fertile interaction. The value of such an attempt has been described in the CIEAEM Manifesto 2000:

> Mathematics education has to provide understanding of the processes of "mathematisation" in society. […] How can mathematics teaching and learning be presented not only as an introduction to some powerful ideas of our culture, but also as a critique of ideas and their application? Do we teach about how mathematics is used in our society? Do we sufficiently understand in what ways, society is becoming increasingly "mathematised"? (CIEAEM 2000, pp. 8–9)

- Subtheme 4 is dedicated to analysis of, and self-reflection on, the effects of mathematisation on pedagogy. At stake are the ways in which the recent political emphasis on standards, assessment and evidence, influence, impact or impair the daily practices of mathematics teachers and researchers in mathematics education.

In the final part of the discussion document of CIEAEM 69, we further develop the four Subthemes. The descriptions as well as the exemplary questions that are posed are intended to stimulate contributions and discussions. They provide a tentative structure to the general topic, while explicitly encouraging the exploration of issues that are located in their intersection or in the space between them.

Subtheme 1 Mathematisation as a didactic principle

The focus of the Subtheme 1 is on teaching experience with, and research studies on, conceptions of mathematics education that interrelate mathematics and the everyday world. The contributions can be aligned to well-established conceptions such as RME or Mathematical Modelling, can question them or can explore new ways of connecting mathematics and the world. We encourage the contributors to Subtheme 1 to analyse the challenges and the potential of mathematisation as a didactic principle, as we invite critical reflections on historical developments and educational policy. A further issue is the implication of mathematisation as a didactic principle for students' learning and identity formation.

Some questions to start with:

- What qualifies a real-world context as a point of departure and/or point of arrival of a didactic arrangement that builds on mathematisation?
- How relevant is the authenticity of everyday contexts for the learning of mathematics?
- What are specific cognitive, social or discursive processes that occur in learning environments that have mathematisation as a pivot?
- Do all students benefit equally from these conceptions of mathematics education?
- Which material arrangements support students' learning of mathematics by mathematisation (e.g. artefacts, physical experiences, learning spaces, etc.).
- Which epistemologies of mathematics are built into particular didactical principles of mathematisation?

Subtheme 2 Mathematisation of society

Subtheme 2 studies the models, in which mathematics is partly or largely adopted, by which social, economical, ecological, etc. processes may be described, predicted and prescribed. These models often inform social and environmental policy on issues such as refugee migration, water, energy, climate change (Hauge & Barwell 2015), health (Hall & Barwell 2015); or they may be used for legitimizing political decisions. Subtheme 2 is concerned with the recent developments at the interface of mathematics, technology and globalisation: big data, security, internet of things, mathematisation of urban spaces, etc.; keeping in mind that mathematisation is not a naturally occurring phenomenon that we cannot avoid. It is done on purpose and it might be illuminative to ask whose intentions become realised (Davis 1989).

Some questions to start with:

- What do we know of and about the mathematical models in use? In what ways are they made public?
- Which experiences and practices are facilitated by mathematisation and would not have been possible without it? Are there experiences and practices that are made unlikely, or even impossible by such mathematisations?
- By comparing competing technologies that use different mathematical models/ algorithms for the same ends, what are or could be the unforeseen side effects?
- How is the mathematisation of society made an object of reflection in the media and popular culture (e.g. in advertisements, newspapers, novels, movies, documentaries)?
- How do mathematical models influence the fundamental conditions of life for particular social groups (e.g. by regulation of social welfare, supplies for refugees, or even transnational restrictions or sanctions for importing food or health supplies) (see, e.g., Alshwaikh and Straehler-Pohl 2017)?
- Considering the effects of mathematisation on mathematics education research: How does the increasing mathematisation affect the ways research is carried out? What counts as research? What are the "policy implications of developing mathematics education research" (Hoyles & Ferrini-Mundy 2013)?

Subtheme 3 Interconnecting mathematisation as a social process and as a didactic principle

It has been argued that we urgently need an "ethic of mathematics for life" (Renert 2011, p. 25) and that "the political and sociological dimensions of the relationship between mathematics, technology and society are fundamental" (Gellert 2011, p. 19). For such an ethic, it would be necessary to develop (classroom) activities that engage with this relationship, by not simply reducing mathematics to a remedy for and an answer to the problems we face, and by breaking with many myths about mathematics and its use.

Some questions to start with:

- "How are pupils to be enabled to criticise [and critique] models and modelling, including the formalised techniques that underpin so much the use or abuse of mathematics in society?" (CIEAEM 2000, p. 9)
- How can teacher education contribute to building up reflexive knowledge on mathematics necessary for pursuing this target?
- How do students and teachers balance the didactic fictionality and the reality of social, economical, environmental, etc. phenomena in mathematics education?
- What can we learn from examples of mathematics education practices that engage locally with social, environmental, etc. issues?
- How can we develop learning environments so that students learn to use mathematics as a tool of emancipation to question the social reality they live in?
- How can we develop learning environments so that students can emancipate themselves from mathematics, in order to assert agency over apparently mathematically validated necessities?

Subtheme 4 Mathematisation of pedagogy

Even when it is not intentionally used as a didactical principle or made an object of reflection, mathematisation does not remain out of school. It enters, for instance, in the form of standardised high-stakes testing and thus changes the "governing assessment dispositive" (Björklund Boistrup 2017). Sometimes directly, sometimes more indirectly, schools receive ‘support’, and teaching is ‘improved’, by evidence-based recommendations about what works in the classroom, and in education more generally (Biesta 2007). Randomised control experiments seem to be the gold standard for some policy makers and researchers in education (e.g., Slavin 2002). Once the impact of evidence-based recommendations is mathematised, interventions can be compared with each other, and moreover, measured against their monetary costs in terms of efficiency, promising policy-makers to find the "biggest bang for the buck", as Jablonka and Bergsten (2017, p. 115) critically capture. However, as Herzog (2011) asserts, “to expect that we would soon be able to control the education system more effectively and efficiently due to the politically motivated strengthening of experimental educational research, is naïve” (p. 134).

Some questions to start with:

- What are the effects of the mathematisation of research on mathematics pedagogic activity in school?
- What are officially stipulated strategies and instructions to implement evidence-based research results in mathematics education?
- How do teachers and students deal with the new regime as it affects mathematics education? How do they enact or resist it?
- What are the effects of the mathematisation of pedagogy on mathematics teacher education?

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