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MES 6 Conference Logo

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TENSIONS BETWEEN CONTEXT AND CONTENT IN A QUANTITATIVE LITERACY COURSE AT UNIVERSITY

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University of Cape Town

We present our views on the purposes and nature of quantitative literacy and our position on the relationship between mathematical and statistical content and disciplinary contexts in a quantitative literacy course for university students in humanities and law. We locate these views in the existing literature. We discuss some of the tensions we experience in our teaching, which challenge our assumptions about the appropriate balance between contexts and content in our 'context-driven' curriculum. We use preliminary observations to highlight some of these challenges and use these to suggest questions for further study.

INTRODUCTION

The important role of quantitative literacy [1] in higher education curricula is increasingly being recognized internationally (for example, Yasukawa, 2007; Steen, 2004). Yet the quantitative demands of academic disciplines in higher education, for example in law and humanities, are often very different from those that are the focus of traditional mathematics courses. Completion of mathematics courses does not guarantee quantitative literacy (Hughes-Hallet, 2001).

Many students in South Africa are poorly prepared to meet the quantitative literacy (QL) requirements in university curricula (Frith and Prince, 2009). There is a lack of articulation between the demands of the university curriculum and students' experiences of school mathematics, as well as enduring inequalities in the education system. The aim of the Academic Development Programme, of which we in the Numeracy Centre are a part, is to address this articulation gap and work towards equity of access to (and outcomes in) university study at the University of Cape Town.

Though our primary aim is to equip students to deal with the complex quantitative demands of their studies in higher education, our location at a higher education institution in the new democracy of South Africa, as well as our personal histories, leads to our work also being driven by a concern for social justice and a firm belief that our students should be functioning, yet critical, citizens.

In this paper we are specifically referring to our experiences in 'stand-alone' courses designed for first-year humanities and law students [2]. In 2009 we began a reform process in these courses, with the aim of shifting the curriculum from what Graven and Venkat (2007, p.74) would call "mainly content-driven" to "context-driven". We begin by locating our views on the purposes of QL in the existing literature. This provides a frame for our position on the relationship between mathematics content

and disciplinary contexts, and the related decisions about curriculum. We then draw on our initial research into our curriculum reform process to discuss the tensions that arose in our teaching due to the assumptions we have made about the role of contexts in our QL curriculum.

WHAT DOES IT MEAN TO BE QUANTITATIVELY LITERATE?

We conceptualise quantitative literacy as practices imbedded in particular social (and academic disciplinary) contexts. We also subscribe to Johnston's (2007, p.54) view that numeracy is "a critical awareness that builds bridges between mathematics and the real world". Quantitative literacy can be described in terms of the contexts, the mathematical and statistical content and the behaviours and reasoning that are involved in quantitatively literate practice (Frith & Prince, 2009). Accordingly, we have adopted the following definition:

"Quantitative literacy is the ability to manage situations or solve problems in practice, and involves responding to quantitative (mathematical and statistical) information that may be presented verbally, visually, in tabular or symbolic form. It requires the activation of enabling knowledge and behaviours and can be observed when it is expressed in the form of a communication, in written, oral or visual mode." [3]

The development of this definition was most strongly influenced by the definition of numerate behaviour underlying the assessment of numeracy in the ALL survey (Gal et al, 2005) and the view of literacy as social practice (for example, Street, 2005).

PURPOSES OF QUANTITATIVE LITERACY IN HIGHER EDUCATION

In Table 1 we summarise the relationships between our purposes and some different classifications from the literature (the various authors' own choices of terms are used). The Numeracy Centre has an established view on the purpose of developing our students' QL, which can be explained by our location in a higher education institution in South Africa in the first decades after the abolition of apartheid.

Our work has been driven by three main purposes. First, the centre exists at the institution to assist students with the quantitative demands of their chosen discipline with a view to their future careers (in Table 1, categories 'tertiary study' and 'professional life and work'). Secondly, our work is driven by a social justice agenda in which we strive to sensitize students to the extensive social problems in our country (in Table 1, 'social justice' category). Lastly, we aim to equip students to become functioning citizens of the country. This goal has two elements, which we describe as 'everyday life' and 'citizenship' in Table 1 and which differ in the degree to which they have to do with the individual's engagement with society in their personal capacity.

Numeracy Centre, UCT	Jablonka, 2003	Gal et al, 2005 (ALL framework). <i>Purposes and goals for Numeracy</i>	Steen, 2001 <i>Expressions of quantitative literacy</i>	Yasukawa et al, 2008 (quoting Schuller) <i>Benefits of learning</i>	S A school subject "Mathematical Literacy", 2003. <i>Purpose of the subject</i>
Tertiary study	Developing human capital	Further learning	Education	Human capital	
Professional life and work		Work-related	Professions		Workplace
Everyday life			Personal finance		
		Personal Health	Everyday life		Everyday life
Citizenship	Societal or community	Citizenship	Societal capital	Citizenship	
Social justice	Social change				
	Environmental awareness				
	Cultural identity			Identity capital	
	Evaluating mathematics				
			Management		
			Culture		

Table 1: Some classification of the purposes for quantitative literacy

CONTENT AND DISCIPLINARY CONTEXT IN A QUANTITATIVE LITERACY CURRICULUM

The most important characteristic of quantitative literacy in the higher education curriculum is that it is situated in the contexts and disciplines where it is practiced (Steen, 2004). For a competent practitioner in a discipline the mathematical content is often dealt with so fluently that it is relatively “transparent” (Lave and Wenger, 1991, p.102), in the sense that the mathematical content becomes invisible and does not present any obstacles to the engagement with the context. The practitioner of QL in a particular discipline must simultaneously deal with mathematics concepts and the concepts of the context in which they are operating.

Consider a very simple example: A report in a South African newspaper in July 2009 said that striking construction workers (building 2010 Football World Cup infrastructure) settled for a 12% pay rise. To understand what this really means in the social context, you need to know that the average daily wage of a construction worker is R140 (context), to be able to accurately calculate 12% of R140 and to realise that this calculation would be useful (mathematical content). To assess the meaning of calculated quantity in the social context of the workers, you need to understand something about prices, the inflation rate, the average number of dependants a worker has and so forth (the context). Now you might feel that to derive greater insight you need to recalculate the daily wage figures as annual income so that you can make a comparison with salaries of workers in other sectors more easily, in which case the context will be dictating what mathematics to do. So even the process of interpreting a simple quantitative statement can lead to fairly complex trains of thought involving mathematical competencies applied in an integrated way within one’s thinking about a particular real context.

Our role as QL lecturers is to ensure that students know when and how to do the necessary mathematics within a given disciplinary context, which is only possible if students have an understanding of the context itself. This presents challenges for our course design.

The first challenge relates to the choice of contexts. Zevenbergen, Sullivan and Mousley (2002) point out how contextualisation of mathematics in unfamiliar contexts can create barriers to students’ learning. However if we use contexts familiar to students, this could compromise our goal of sensitising students to social issues in society. If we use contexts that students are not familiar with, we may be overloading them with having to learn about the mathematics and the features of the context at the same time, at the risk of letting the mathematics be submerged. Given our experience that our students have significant difficulties with the required mathematical and statistical content, they cannot afford to spend much of the course time on learning about the features of specific contexts. Furthermore, the choice of contextual material is also determined by the availability of suitable authentic material in which the necessary mathematical and statistical content topics are

embedded. Jablonka (2003) highlights the difficulty of finding contexts that are exemplary in terms of social and political relevance but also exemplary in terms of the practices of mathematics.

The second challenge relates to achieving an appropriate balance between context and content in the curriculum. Graven and Venkat (2007) present the possibilities as a spectrum of approaches to the balance between content and context, as well as some of the associated tensions, for example aiming for authenticity of the context as well as mathematical understanding. In our practice at the most general level, it entails a choice between a ‘content-driven’ curriculum and a ‘context-driven’ one. In both cases the development of the mathematical and statistical knowledge and techniques and their appropriate use is our major goal. For many years (for pragmatic reasons) the underlying organisation principle of our course has been the mathematics and statistics content, which we always presented using relevant contexts. This would correspond to what Graven and Venkat (2007, p.74) characterise as “mainly content-driven”.

Our belief has always been that ideally we should strive for a higher degree of contextualisation, where students would engage with substantial real contexts, and the necessary mathematics and statistics would arise and be developed as needed. In Graven and Venkat’s (2007, p.74) terms this would be a “context driven” curriculum. The motivation for this is that it more closely mimics the reality of the practice of QL in the disciplines; our assumption being that similarity between the features of the context selected for the stand-alone QL course and of the disciplinary context will enhance transfer of what students learn in our courses to their practice in the disciplines. We believe that it is a similar assumption that underlies all arguments in favour of teaching QL in context (for example Steen, 2001, 2004; Department of Education, 2003).

TENSIONS ARISING ON IMPLEMENTING THE CURRICULUM

In accordance with our belief that the curriculum should be context-driven, in 2009 we structured the first part of our curriculum for humanities and law students around three contexts: a module on children’s rights, a module on xenophobia and one on prison overcrowding (rather than having modules built around different categories of mathematical content, which is the structure we retained for the second half of the course covering financial mathematics and statistics). We present students with edited extracts from research reports of the kind which we believe they will encounter in their disciplinary studies. The extracts are accompanied by comprehension-type questions which are used as a basis for workshops in which students are assisted in making sense of the quantitative information presented in the contextual material.

In order to study the curriculum reform process, we collected data on lecturers’ and students’ experiences. This data is in the form of field notes from classroom

observations, annotated lecture notes, audio-recordings of the weekly staff meeting where the courses were discussed, three focus group interviews with students and copies of students' work.

A number of tensions experienced by lecturers and students when implementing this kind of curriculum in 2009 have led us to question our assumptions about the degree to which the curriculum should be driven by the contexts studied. We discuss these below, and draw on preliminary data from our study as illustration.

Achieving a balance between content and context

Our decision to emphasise contexts presented challenges in terms of classroom practice. Firstly students were less familiar with some of the concepts and terminology associated with the contexts than we expected they would be. The additional activities designed to improve their understanding of the contexts affected the pace at which we were able to work through the tasks, something that the students identified in interviews, for example: "I felt like ... we drew it out for a very long period of time". Classroom discussion of the contexts also highlighted for us differences between our own understandings of the country's history and those of our students who have grown up in post-apartheid South Africa. Reflecting on discussions in her class about the need for an act of law to resolve backlogs in social services, one lecturer commented, "you forget ... they don't realise that there was this momentous thing that happened in 1994 and things were completely different".

The second challenge relates to the depth at which students engaged with the contexts. Those students we interviewed clearly appreciated the fact that the material was current, felt that it had relevance to their lives, as opposed to what they had experienced before in school: "Like in school they just give you like fabricated stuff, like made up things, but a lot of the stuff was real and relevant to South Africa like with xenophobia". Yet we were disappointed that many students did not engage with the contexts in the critical manner that we had envisaged. For example we aimed to highlight the denial of human rights of prisoners (especially the considerable numbers of awaiting trial prisoners) in overcrowded prisons, but many students voiced unconsidered views that anyone in jail deserved bad treatment.

Our original teaching plan included a weekly 'summary lecture' in which we would review the mathematical content and link this to its use in context. However, these fell away as we spent more time dealing with the contexts. As lecturers we became increasingly concerned that the mathematical content was being eclipsed in favour of the context, for example the following was voiced in a staff meeting:

"I'm feeling concerned about the discomfort that students are feeling ... I think we need to give them some structured framework they can slot the content into. ... I'm kind of wanting to stop and say, "Hey now this is what we've done. All of these questions let's go and look at them. This is percentage increase, or percentage change, these are percentage points."

Students' responses to the materials suggest, too, that they were not engaging with the content in sufficient depth. For example, a student expanded on his view that the materials were tedious by identifying the content quite generally as "percentage change": "But this was almost like percentage change, then percentage change within prisons and then with xenophobia". However, formal assessments suggested to us that the students were not distinguishing between different types of percentage change calculations or identifying when appropriate kinds of calculations were needed.

Students' confusion about the nature of what they were learning and what was valued in the course

As we attempted to balance the content and contexts in our practice, we sensed discomfort on the part of our students. Students had expectations of the course, often based on their experience of school mathematics, for example, "Please can you tell us what formulas we've done". But they were also confused about what they were learning; when preparing for the first assessment which occurred when the context was children's rights, a student queried whether it was necessary to learn the details of the relevant act of law. This kind of confusion, particularly for disadvantaged students, is similar to that identified by Zevenbergen et al. (2002).

We also recognised that we were expecting students to engage with the contexts in different ways in different circumstances. During class discussions of the social contexts, we encouraged students to draw on their personal experiences. Yet our assessments valued arguments that were substantiated with data and calculations. Although it was not rewarded, some students continued to ground their arguments in their personal experiences, often not recognising subtle textual clues as to what was required.

Concerns about transfer

We do not at present have solid evidence that the knowledge and ways of thinking we are trying to promote in the context-driven curriculum will be transferred to the disciplinary practices elsewhere in the students' curriculum or in a profession. This lack of evidence is due both to our concern about selecting an appropriate methodology for researching the concept of transfer, and to the practical limitations on our ability to collect evidence. It is a matter of concern, therefore, that we are basing our curriculum choices on a fundamentally untested assumption about transfer, and responses of past and present students to our courses serve to highlight our concerns. For example, a psychology graduate, who experienced the previous content-driven curriculum, expressed the opinion that she was able to use her learning in the QL course in subsequent years, "I attribute my current knowledge and ability to grasp statistics to this course". It is possible that the structuring of knowledge in our course around specific statistical concepts facilitated the transfer of this knowledge in subsequent courses, which also highlighted the same content

topics. This observation does not support our assumption that a context-driven curriculum is the most effective in promoting transfer.

Furthermore, our observations of and discussions with students who studied the new context-driven curriculum suggest that many still view the course as a “mathematics” course. These perceptions serve as a reminder that the context selected for curriculum material cannot be viewed in isolation from the wider social setting, that is, a course that is coded by the institution as a mathematics course. Students will use their perceptions of the broader setting of the course to make a “negotiated judgement” of the context (Evans, 2000, p.85), and this judgement may have implications for the successful transfer of practices to other settings.

Impact of using language-based texts

Students characterised the course as having particular “writing” and “explaining” requirements. Those students interviewed identified that the course valued a particular way of writing; for example: “... there were times we would write a paragraph and I’d write what I thought ... and all the information I gave was information I’d gotten from the data, but then they’d just be looking for something different”.

The interviewed students could recognise and articulate the differences between school mathematics and the QL presented in the course. Yet our observations suggest that students still approached the context-based material rather like school mathematical word problems. They identified cues in the text for how to proceed rather than using the text as a resource to identify an appropriate strategy.

Many students attributed their difficulties to their view that the questions were not “clear”, yet we identified the problems as deeper, and related to the complex demands of balancing content and context in QL practice. For example, presented with a table of data on prisoners for three different years, students were asked to, “Draw up a table that shows the total (absolute) number of unsentenced prisoners and the number of unsentenced prisoners as a percentage of the total (relative number) for each of the three years”. In order to establish what is required by this task students must identify meaningful parts of this sentence, for example, “for each of the three years”, but also hold the whole sentence in view. In addition, to complete this task students need to work with and across tabular and textual representations.

In this section we have identified some of the difficulties students encountered when working with language-based texts. These highlight the need to find a way to provide the necessary language support, particularly for students for whom English is not the home language. We have established an ongoing collaboration with an academic literacies specialist to assist us in this regard.

CONCLUDING REMARKS

In changing from a content-driven to a context-driven curriculum in a QL course for students at university, we believed we would achieve our purposes and serve our

students' needs more appropriately. The experience of implementation has caused us to question some of the assumptions we made about the balance between context and content in such a curriculum. This discussion of the tensions we have experienced is based on our preliminary observations during our first experience of our newly structured curriculum. It raises the questions that concern us, rather than attempting to present arguments for changes that will resolve these questions. We regard these questions as indicating the areas for further research which will point the way to achieving a more effective quantitative literacy curriculum.

Specifically, the questions can be summarised as:

What interventions will be effective in ensuring that students have a clear idea of the objectives of the course, in terms of the relative importance of content and context?

What balance between context and content is optimal (and how should these be related in the course) to maximise students' ability to transfer what they have learned to new situations?

What interventions will be effective in supporting students' reading and writing in a language-based quantitative literacy curriculum?

NOTES

1. The terms quantitative literacy, numeracy and mathematical literacy are often used interchangeably when naming a wide range of practices related to dealing with quantitative information. We prefer the term 'quantitative literacy', since in South Africa, 'Mathematical Literacy' is the name of a school subject and 'numeracy' is most often associated with learning in the early stages of a child's schooling. However the term 'numeracy' means the same as 'quantitative literacy' (often referred to as QL) in this paper.
2. Given the context-bound nature of QL practice, we do not advocate separate QL courses as the optimal kind of intervention, although for structural reasons they are sometimes the best choice. The fact that our teaching of QL is separated from the disciplinary practice in this way leads to many of the tensions we experience.
3. A detailed summary of our view on what a student needs to be able to do in order to practice QL is given in Frith and Prince (2009).

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OUR ISSUES, OUR PEOPLE: MATHEMATICS AS OUR WEAPON

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This paper is based on a critical action-research project in a Chicago public school, in a low-income community of color. In the 2008-09 school year, I taught a 12-grade math class in which students used and learned math to study social reality, in particular, aspects of injustice. Math content areas included algebra, discrete math, pre-calculus, probability/data analysis, and quantitative reasoning; the overarching theme was mathematical modelling of reality. Real-world contexts we investigated were whether the 2004 US presidential election was “stolen,” neighborhood displacement (gentrification/immigration/deportation/foreclosures), HIV/AIDS, criminalization of youth/people of color, and sexism. Students used this article’s title to name their end-of-year presentations to their communities about our work in class.

INTRODUCTION

Paulo Freire (1970/1998), among others, posed the question: What should be the purpose of education? He might have answered using words from Amílcar Cabral, one of his mentors. Education can serve “the difficult but inspiring struggle for the liberation of peoples and humankind and against oppression of all kinds in the interest of a better life in a world of peace, security, and progress” (Cabral, 1973, p. 15). How, then, could or should a mathematics class contribute to this struggle? How can one address this question in a mathematics class in an urban neighborhood public school, a district wedded to high-stakes accountability measures, punitive disciplinary policies, what youth call the “school to prison pipeline,” and within a stratified education/social system designed to have education for servitude rather than for emancipation and humanization? Answering this question is a far larger task than one paper can undertake, obviously, but I argue that we can respond to Freire’s question from the perspective of critical mathematics (Frankenstein, 1987; Gutstein, 2006; Skovsmose, 1994) and from a math class in a Chicago public school—a space which we sometimes refer to as “the belly of the belly of the beast”. In this short paper, I provide an example showing what this *can* look like and briefly discuss some concomitant complexities.

In the 2008-09 school year, I taught a 12th-grade mathematics class at the Greater Lawndale/Little Village School for Social Justice (aka “Sojo”). Sojo is 70% Latino/a (mainly Mexican), 30% Black (African American), and 98% low-income. Any student from the neighbourhood may attend. The school, which opened in Fall 2005, grew out of a struggle to build a new high school in an overcrowded Mexican immigrant community, culminating in a 19-day hunger strike in 2001 by neighborhood activists (Russo, 2003). I was part of the design team that founded the school and have been working with administrators, mathematics teachers, and

students there since December 2003, developing and co-teaching critical curricula, and supporting teachers in learning to teach it.

READING AND WRITING THE WORLD WITH MATHEMATICS

Freire's perspectives provide the overarching conceptual framework informing this work. His words guided our class: "Problem-posing education does not and cannot serve the interests of the oppressor. No oppressive order could permit the oppressed to begin to question: Why?" (1970/1998, p. xx). The purpose of our class was that students learned to question: Why? —but specifically, to do so using mathematics, while simultaneously learning the mathematics they needed to both get access to college and economic survival for themselves, family, and community, but also to be able to understand social phenomenon at a deep level. This latter idea is what Freire (1994) referred to as *reading the world*. He was concerned with people learning to read, not as a mechanical exercise, but rather as a way to make meaning out of, and to change, their reality:

From the beginning, we rejected the hypothesis of a purely mechanistic literacy program and considered the problem of teaching adults how to read in relation to the awakening of their consciousness....We wanted a literacy program which would be an introduction to the democratization of culture, a program with men [sic] as its Subjects rather than as patient recipients... (Freire, 1973, p. 43).

Though he rarely discussed math, I have taken his concept of reading the world and built on Frankenstein's (1987) framework to extend it to mathematics. My evolving understanding of *reading the world with mathematics* is:

to use mathematics to understand relations of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation and to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (Gutstein, 2003, p. 45)

Of course, Freire was not satisfied that we learn to read the world because he subscribed to Marx's (1845/1969) observation that the "philosophers have only interpreted the world, in various ways; the point is to change it" (p. 15). Freire referred to this changing reality as *writing the world*:

Reading the world always precedes reading the word, and reading the word implies continually reading the world....In a way, however, we can go further and say that reading the word is not preceded merely by reading the world, but by a certain form of *writing* it, or *rewriting* [emphases original] it, that is, of transforming it by means of conscious, practical work. (Freire & Macedo, 1987, p. 35)

In our class, both reading and writing the world—with mathematics—were very much the agenda. I asked George, a student in our class, what this meant to him, and why he thought we did it. His impromptu written response:

Reading and writing the world with mathematics means a lot. It means that you look at any issue happening anywhere in the world. When you read the world, you are getting background information and seeing why whatever problem you see is occurring. You then find a way to resolve it. This then brings in writing the world with mathematics. When writing the world, you are ready to use mathematics to prove your point. Also, every point you have will not be a solution. It will sometimes just be a way for you to bring light to a situation that no one knows about. So to me this is what reading and writing the world with mathematics means.

We do this for a reason. There are big corporations trying to take advantage of people. There are also plain old injustices that happen everyday. We do this to educate ourselves on global or local problems that can be solved with mathematics. We also do this to learn more advanced mathematics. Lastly, we do this so that we can take our knowledge back to our friends and family to educate them. Once we educate the ones that are closest to us, we then go out and educate our community on how to prevent things from happening to them and how to catch things before they are taken advantage of.

George's insight is clear. Not only did he link the local situation (friends, family, and community) to the globe ("issues happening anywhere in the world"), he understood that math meant both understanding and "resolving" problems, and using math to "prove your point." His justification for why we do this included, again, both the macro ("big corporations trying to take advantage") as well as the micro ("plain old injustices that happen everyday"). And his motivation is clear—to "educate our community" and "prevent things from happening to them."

Was the 2004 (US) Presidential Election Stolen?

To help readers understand how students in urban public schools develop such dispositions toward knowledge, I share some of what George and his classmates actually did in class. We started the year with a 10-week unit titled, "Was the 2004 Presidential Election Stolen?" The rationale was that we were in Chicago (President Obama's town), many of my students were voting for the first time (having turned 18), and several were poll watchers or involved in voter registration campaigns. Chicago went overwhelmingly for Obama—one student was a poll watcher at a voting precinct in her all-Black neighborhood and reported that it went 292-0 for Obama. Given that all Sojo students are of color, the historical significance of the Obama campaign—and the possibility that the 2008 election might be "stolen"—it is not at all surprising that students were so engaged in the unit.

We studied the 2004 election to understand that certain phenomena could *not* have happened by chance, in preparation to monitor and alert the public, in whatever small ways we could, about the 2008 possibilities. Using data from a book, *Was the 2004 Presidential Election Stolen?* (Freeman & Bleifuss, 2004), we investigated what we

called the *poll differences* (PD) between reported votes and exit poll results. Exit polls are anonymous, highly accurate polls conducted immediately *after* individuals vote and have been used internationally to certify the accuracy of elections (e.g., in Ukraine). We expect exit polls to differ from recorded votes, due to sample variation, but the difference in any given precinct should arbitrarily favor one candidate or the other (assuming just two); over a large number of precincts, these disparities should break roughly 50-50 in favor of each candidate (ibid.). However, the 2004 exit poll disparities, in many places, were extremely unlikely. In the 10 so-called “battleground” states (those whose outcomes were highly contested and important), the PDs *all* favored then-President Bush against candidate John Kerry, and in the 50 state polls, the PDs split 44-6 in favor of Bush. We also studied the actual discrepancies within the exit polls in key states. As Maria, a student wrote:

As I showed before, the probability [in the battleground states] was $.9766E-4$, that is about 1 in 1000. Then on the 50 PDs, 44 favored Bush and only 6 favored Kerry. That probability is $1.411E-8$ or about 1 in 100,000,000, almost zero. In Ohio, Kerry won the exit poll with 54.2% but his recorded vote was 48.7%. That chance is about 1 in 1,000,000,000. Does it sound reasonable? At least not by chance.

As Channing, another student, wrote, the math he learned was “a bigger piece of evidence that these events couldn’t, shouldn’t, and wouldn’t happen by chance.”

The unit culminated the week after the election (November 2008) and students collectively wrote an op-ed piece that was published October 31 in the popular on-line news magazine, *Huffington Post* (the website received 4.5 million unique hits in October 2008; see http://www.huffingtonpost.com/robert-koehler/students-ask-are-our-elec_b_139883.html). The op-ed concluded:

Our class is writing this to inform everyone about previous problems in the elections and to warn people to watch for similar troubles. We want to ensure that in this election, the same problems do not occur....In this election, it is up to all of us to question the results and to hold officials accountable for fairness. If the vote changes on the electronic machine, call for assistance. Let your vote be counted for the candidate of your choice. Let your voice be heard, and don't settle for less!

Remember--it didn't happen by chance!

We see this as an instance of *writing the world with mathematics*, that is, using mathematical arguments to advocate a position and fight for what students believe is just. Although I do not include specifics here, and students decided not to put their mathematics in the op-ed piece (knowing most readers would not understand), students learned about binomial and normal probability distributions and confidence intervals, and used their mathematical analyses to conclude that things could not have happened by chance unless one accepts a one-in-1-billion possibility. This was their way of trying to ensure that the 2008 election was honest. As Antoinette wrote: “writing the world with mathematics means being able to use mathematics to address

a real social issue and being able to make a change. Being able to address a point and being able to back it up with mathematics.”

Studying Neighborhood Displacement

After completing the elections unit, we studied neighborhood displacement. Sojo serves two distinct and physically separated low-income neighbourhoods: North Lawndale, almost all Black, and Little Village, primarily Mexican. Displacement has specific meanings in various contexts. In our situation, it first means gentrification, when more well-off people move into low-income communities, as rents, house prices, and property taxes rise, forcing people out. This is particularly the case in North Lawndale. Second, the mortgage and broader global economic crises have severe repercussions for both Sojo communities. Family members have lost jobs due to layoffs, and foreclosures have skyrocketed, including affecting Sojo families. In 2008, almost 10% of all mortgageable units went into foreclosure in North Lawndale. Third, estimates are that thousands of Little Village residents are unauthorized in the US. Displacement, for them, means the possibility of being swept up by US immigration authorities and summarily deported to Mexico. The spectre hangs over the community. In April 2007, in one such raid, “federal agents in full gear, some holding machine guns, surrounded the parking lot of the Discount Mall in Little Village” (Garcia, 2007). And fourth, displacement refers to the impact free trade agreements (e.g., NAFTA) have had on both communities—industrial jobs have been lost in North Lawndale and relocated to parts of the global South, and Mexican farmers have been forced off the land due to the influx of highly subsidized US corn (Oxfam, 2003). Many displaced Mexicans eventually find their way to Little Village, where they exist in shadows, vulnerable and fearful of the other displacement of deportation. Thus displacement was *very* real to my students.

The displacement unit was far-reaching politically and long—it took three months. One key theme was the meaning of *affordability*. Developers were building (or rehabbing) homes in North Lawndale, and prices were rising even in Little Village (at least until the housing crash), so students investigated whether “ordinary” residents could afford to still live in the communities. Students learned that a North Lawndale family with the annual median income (around \$20,250, very low for the US) could afford to buy a house without “hardship” (defined by the US government to be no more than 30% of gross income) of only about \$84,000 at the prevailing interest rate at the time. New condos in North Lawndale were selling for triple that.

A second key idea was for students to understand an aspect of the mathematics of capitalism: how home mortgages work. Given that home foreclosures in both communities have soared (the number in each neighborhood more than tripled from 2005 to 2008), and that subprime, predatory loans hit Black and Latino communities the hardest in the US (Bajas & Fessenden, 2007), this was particularly relevant. There were students in class whose families were either in foreclosure or had lost a

home recently (in both communities). We started the unit by me telling the story of one family who had given me permission to interview them and share their tale.

To understand the detailed mathematics of subprime mortgages, students learned to use *discrete dynamical systems* (DDS) (essentially, discrete versions of differential equations; Sandefur, 1993). A DDS has a starting and a recursive equation, and one can iterate them to define a sequence, such as a mortgage amortization schedule. For example, if one has a 30-year (360 month), fixed-rate mortgage with a 6% interest rate (a common scenario with which we started the investigation) and buys a house with a mortgage of \$150,000, the monthly payment (not including escrow items, such as property tax and insurance) is \$899.33. The DDS looks like the following, where U_n represents the balance of the mortgage due at the start of month n :

$$U_1 = 150,000.00$$

$$U_n = U_{n-1} + .005U_{n-1} - 899.33 \text{ [for clarity, we did not initially combine like terms]}$$

Using graphing calculators, students graphed curves of time (X-axis) vs. the unpaid principal balance (Y-axis), traced the values, saw how the curve changed shape (and why) over the 30 years, and examined the values. We analyzed various subprime mortgages, including adjustable-rate, interest-only, balloon, and pay-option loans.

Enacting Critical Mathematics in the Classroom

Reading and writing the world with mathematics entails, among other things, using mathematical ideas to develop sociopolitical consciousness. In this unit, this occurred in several places, including as students examined how much of a monthly payment goes to interest and how much to principal. In Little Village, a family with the median income (\$32,320) can afford, without hardship, a 6%, 30-year fixed-rate mortgage of just under \$135,000, and would pay about \$808 per month. However, over 80% of the first payment goes to the bank for interest, not to pay off principal. And it does not get much better, from the borrower's perspective, until well into the mortgage (e.g., after 15 years, almost 60% of the payment still goes to interest). In fact, after 30 years, the borrower will pay about 291,000, over twice the original amount. This led us to the question of what happens if a Little Village family with the median income wanted a \$150,000 mortgage, somewhat more than the roughly \$135,000 such a family can afford without hardship. Given that their monthly payment would be (maximum) \$808, after 30 years, the family would still owe the bank almost \$92,000 after paying \$291,000 on a \$150,000 mortgage. This astonished and dismayed students, even though I explained, from capital's perspective, the rationale for collecting interest. As we examined this, we had the following dialogue in class.

Mr. Rico: [This is my classroom name] Think about that, after thirty years, you still owe them ninety two thousand, you only borrowed one hundred and fifty. Do some quick math. How much have you paid in thirty years?

Darnisha: Too much money.

[We work through on the overhead calculator that the family has paid about \$291,000.]

Mr. Rico: [slowly] You've paid two hundred and ninety one thousand dollars, on a hundred and fifty thousand dollar mortgage...

Alex: [interrupting] Nearly double.

Mr. Rico: And you still owe ninety two thousand dollars. [pause] Check that math out. That's good math. Let's look at that math. [I write on the board: $150,000 - 291,000 = 92,000$ while saying:] One hundred and fifty thousand minus two hundred and ninety one thousand equals ninety two thousand dollars. [pause] Look at that math. [pause] Think about that. You started with a hundred and fifty—you paid two hundred and ninety one—and you still owe ninety two thousand dollars. Good math, huh? [students are astonished] What's going on here?

Alex: They're taking your money!

Darnisha: [in a matter-of-fact tone] The bank is taking advantage of you.

Mr. Rico: This is legal. This is how banks loan money and make money. [pause] [slowly] This is legal. [pause] [slowly] This is how banks loan money and make money.

Rut: Wouldn't a family have to pay more than eight hundred and eight a month to be able to afford the one hundred and fifty thousand loan? [she is right]

Rut: [I then asked students what were their questions.] Why is it legal?

Mr. Rico: Why is it legal is a really good question.

Darnisha: Why don't more people look into it? Why don't they have people who look into it, to make sure that their finances are favourable, where they could actually pay it off instead of waiting till after 30 years? [class ended]

This discussion continued over the next several weeks as students carefully analyzed various types of subprime mortgages and discovered, through their own calculations, that these are higher-cost (to the borrower) loans. We also examined data showing that families of color disproportionately received these loans, even when they qualified for prime loans, and thus linked the question of racism in banking practices to the displacement affecting students' communities. As a key way of writing the world with mathematics, the class had agreed to do two public presentations involving all students (one in each neighbourhood) at the end of the school year to inform their communities about issues that students felt were critical to know. They felt it particularly important that they tell neighbourhood adults about displacement and the traps of predatory lending.

Complexities and Risks in Learning to Read and Write the World

This dialogue and our work in this unit (indeed, in the class) raise central questions about critical pedagogy, some of which I briefly discuss here (see Gutstein, 2006 for

more). Can education ever be neutral? What is the teacher's ethical and pedagogical responsibility with respect to multiple perspectives? Her own perspective? Affirming himself while simultaneously not disaffirming students (Freire's terms)? Concretely, how might one teach about capital's math, from within a capitalist country, while providing students space to develop their own views?

To address these issues, I turn to Freire (1994), who wrote:

There neither is, nor has even been, an educational practice in zero space-time—neutral in the sense of being committed only to preponderantly abstract, intangible ideas. To try to get people to believe that there is such a thing as this, and to convince or try to convince the incautious that this is the truth, is indisputably a political practice, whereby an effort is made to soften any possible rebelliousness on the part of those to whom injustice is being done. It is as political as the other practice, which does not conceal—in fact, which proclaims—its political character. (pp. 77-78)

Drawing from Freire, I argue that to *not* analyze the mathematics behind the “injustice being done” to Sojo students (and others) *is* a political practice. People throughout Chicago are being forced out and losing their homes, and communities are being destroyed and “rebirthed” as upscale gentrified spaces, for the “new” people with means. Obviously, fully understanding this process is complicated, as it involves transnational capital, the drive for cities to compete on the global market, and the workings of neoliberal urbanism (Lipman & Haines, 2007). But these young people's lives are affected—by these “political practices.” My goal, as a teacher/researcher/ learner is not to “soften any possible rebelliousness” of my students, but to *strengthen* it, through providing them opportunities to use mathematics as a weapon in the struggle for social justice. But does this mean to provide only one view of reality? In contrast, I also build on Freire to address this:

Respecting them [students] means, on the one hand, testifying to them of my choice, and defending it; on the other, it means showing them other options, whenever I teach—no matter what it is that I teach! (p. 78)

What is altogether impermissible, in democratic practice, is for teachers, surreptitiously or otherwise, to impose on their pupils their own “reading of the world,” in whose framework, therefore, they will now situate the teaching of content....The role of the progressive educator, which neither can nor ought to be omitted, in offering her or his “reading the world,” is to bring out the fact that there are other “readings of the world,” different from the one being offered as the educator's own, and at times, antagonistic to it. (pp. 111-112)

Thus, my conscious effort to have students understand that capital, too, has its values and goals, but based on its position in the world. That is why I explained that “this is legal, this is how banks loan money and make money.” Whether students think that this is just or not must be their decision; I did not hide my own views (though I did not always put them out initially), but I challenged and encouraged students to develop their own. Students need to *begin* to understand that the conditions affecting

their lives are complex, both global and local, and that they can develop analytical ways of thinking with respect to both mathematics and sociopolitical realities.

Given the above, was I at all concerned that students might take my views without sufficiently questioning and critiquing them? Yes, and at times it happened. That is why, from the first day of class, I told students to question me as much as they question any other view, person, or text. Freire commented on this also: “Is there risk of influencing the students? It is impossible to live, let alone exist, without risks. The important thing is to prepare ourselves to be able to run them well” (p. 79). The goal, regarding this issue, is for students to begin to interrogate their lives and the reasons behind what they experience. This questioning is precisely what a critical mathematics curriculum seeks to engender in students, and is exactly why it threatens. The earlier quote from Freire (1970/1998) speaks clearly to this, “No oppressive order could permit the oppressed to begin to question: Why?” Evidence that Sojo students learned to read and write their worlds with (and without) mathematics come from Vero, who said:

I’ve learned to question how and why...Mr. Rico told me that I was just giving people the mathematical answers...I went from questioning things in math to questioning things in life. Now I question everything and everyone....[I asked her: Why?] Because we’re taking [pause] regular math and implementing it, we use our knowledge to address other issues that affect others, people of color, low-income people, etc.

She continued, turning from mathematics to aspects of her lived reality:

The reason why some people act so aggressive is not because that’s how we are, but because that’s how we are meant to be because of what’s happening to us. So like all the police and stuff, all these North Lawndale shootings, Little Village shootings, another shooting, another kid dead, or something like that, it’s just that that was led by something else. It’s just not, people don’t just pop out with a gun and start shooting. It’s because something is going on that is leading people to do certain things....it’s not a way of excusing it, but it’s a way of addressing the question: Why?

As an end-of-year present, I had t-shirts made for all my students, depending on whether they were Latina/o or Black. The t-shirts read, on the back, *SOJO-2009; Math for Social Justice*, and on the front, *Danger—Educated Black Woman* (or Man) or *Danger—Educated Latina* (or Latino). Vero, as an educated Latina, poses the danger to the status quo because she is educated, and through her critical education, is beginning to ask the question: Why?

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STUDYING THE EFFECTS OF A HYBRID CURRICULUM AND APPARENT WEAK FRAMING: GLIMPSES FROM AN ONGOING INVESTIGATION OF TWO SWEDISH CLASSROOMS

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We report from an ongoing study of two Swedish mathematics classrooms, in which the students and their teachers are together for the first time at the very beginning of their upper secondary education. The students are enrolled in two different programmes, in which they study the same mathematics course. The investigation of these two classes is part of a larger comparative project that studies the emergence of disparities from a theoretical perspective that examines their social construction in the context of the practices of the mathematics classroom.

THE RESEARCH PROJECT

The data to which we refer are collected in an ongoing comparative research project involving classrooms from Canada, Germany and Sweden [1]. The researchers collaborating in the project are concerned with the emergence of disparity in achievement in mathematics classrooms. The project investigates the emergence of disparities from a theoretical perspective that examines their social construction in the context of the practices of the mathematics classroom while taking into account “outer” factors that might lead to the systematic exclusion of some students and to the success of others. That is, to identify discursive and interactional mechanisms that can explain if and how structural elements can be found in classroom interactions. A comparative approach aims at identifying the characteristics of these mechanisms in relatively homogeneous and in heterogeneous groups, in socially advantaged and disadvantaged groups. We start with the observation that teachers and students in mathematics classrooms after a short time come to know which students perform well in mathematics and which do not. This occurs even within different streams in which students are supposed to be starting at a comparable level.

The Swedish data comprise video-footage of the first eight respectively nine mathematics lessons in the two classrooms, recordings of interviews with all students and one of the teachers, copies of a test from one of the classrooms, textbooks and other material used as well as information about the students’ social, cultural and economic backgrounds obtained from questionnaires. The two classrooms to which we refer are both situated in upper-secondary school. The students are studying the same mathematics course, but are registered in two different programmes. One focus of our study is to identify the ways in which the difference between the settings affects the students’ access to academic mathematics. Some of the emerging issues might be of relevance in other educational contexts that have undergone similar curriculum transformations, from a more specialised version to an integrative.

THE SWEDISH EDUCATIONAL CONTEXT

In Sweden, the classic model of progressive welfare state education comprised the introduction of the 9-year comprehensive school (*grundskolan*) in the 1960's and lead to an expansion of secondary education. The education system has undergone considerable changes since then. Strong central management and detailed regulation gave way to a system with increased autonomy for schools, which can decide about the organisation of work, choice of methods and class sizes. Important features of Swedish education can be seen in the high participation of children in public child-care and pre-school education, little formal streaming and tracking in the comprehensive school as well as the provision of both vocational and academic programmes in the same organisation in secondary education (*gymnasieskolan*, year 10 to 12). After completion of all programmes students are eligible for higher education, although there are different criteria for different university subjects as to the nature and amount of courses chosen by the students within the programmes. Consequently, the choice of the programme after nine years of comprehensive school and the selection of courses within the programme are important choices that affect students' identity, career opportunities and relationship with mathematics.

The number of compulsory mathematics courses varies across the programmes in the *gymnasieskolan*. The courses are named A, B, C, D and E. Course A is compulsory in all programmes. Theoretically, students in all programmes can choose more than the compulsory courses. The completion of each mathematics course comprises a national test, the results of which are meant to inform the teacher's evaluation of the students' attainment and are also reported to the school authority. The curriculum prescriptions are outcome based, formulated and tested for the end of year three, five and nine and for each of the courses in upper secondary education. The prescriptions include an outline of the evaluation criteria. The grading system comprises four levels: not pass, pass, pass with distinction, and pass with special distinction. The curriculum does not include recommendations for the order of topics, time allocation, pace and teaching methods.

Swedish classrooms very often show a high proportion of lessons devoted to individualised work with exercises from the textbook, where students work at their own pace occasionally scaffolded by the teacher. Harling, Hansen and Lindblad (2008) investigate changes in classification and framing in Swedish classrooms based on comparing recorded fragments of lessons from 1968 and 2003. In a mathematics classroom from 2003, they identify a pedagogy of weaker framing and classification, hence, a more invisible pedagogy. They also found more active students "that on one side challenge hierarchical power structures, expressing individuality and on the other side are much more visible and therefore more open for governing techniques" (p.16). The more "visible" students in 2003 are to a much greater degree responsible for their own successes and failures. There is indication of students' self-exclusion based on their participation, the students (dis)qualify themselves, both individually and as a collective as competent members.

THEORETICAL BACKGROUND

The theoretical framework has to allow one to describe the emergence of disparity in terms of the relationship between everyday knowledge derived from the larger socio-cultural context and the type of knowledge that is related to success in a mathematics classroom. In addition it has to allow one to describe the implicit rules to which not all students have equal access. An obvious choice is to elaborate the notions of classification and framing (Bernstein, 1996). Differences in the instructional and regulative classroom rules can be described as variations of the type of knowledge which can be accessed and of the ways this knowledge is made accessible. The underlying principles can be captured by the strength of external and internal classification and framing. Weak internal classification indicates that there are weak boundaries between the mathematical sub-areas; weak external classification means that many connections are made to other disciplines or everyday practices. The degree of specialisation of the discourse can be captured by these categories. Framing refers to the nature of the control over the selection of the communication, its sequencing, its pacing, the evaluation criteria for the performance, and the social base which makes access to knowledge possible (Bernstein, 1996, p. 27). The students can have more or less control over these dimensions independently. According to Bernstein's theoretical analysis, variations of classification and framing relate to differential access to institutionalized knowledge.

The curriculum of the course taken by the students in the two Swedish classrooms can be characterised by a weak external classification. Reference to local, particular and situated everyday knowledge is frequently made, most commonly through contextualised mathematical tasks, metaphorical expressions and images. Empirical evidence suggests that the institutionalisation of segments from everyday discourse within school mathematical discourse has a tendency to allocate the everyday insertions to marginalised groups (see, for example, Cooper and Harries, 2009; Dowling, 1998). Boaler (1994) shows how female students are more likely to engage with aspects of "reality" when addressing contextualised tasks. Dowling's (1998, 2007) distinction between content and expression of a text, both being weakly or strongly institutionalised, allows for a description of the weakly classified hybrid discourse often found in school mathematics. He describes a relational space of four domains of action. Much of what constitutes the hybrid operates in a public domain of recontextualized domestic practices (Dowling, 1998). Recontextualization has to be understood as the process of subordinating one practice under the evaluation principles of another, as for example school mathematics. The contents of these four domains change. School mathematical practices strive for the esoteric domain of strongly institutionalised content and forms of expression. This leaves the question open as to how to conceptualise different forms in which the esoteric domain appears and whether these have a differential effect on students' participation. Also, what constitutes the public domain, consists in a set of activities that differ with respect to

the evaluation principles. In engaging with the empirical material, we attempt to clarify and re-structure our language of description.

FINDINGS

In the following we present an initial analysis of the first five lessons in both classrooms with a focus on episodes, in which the ground rules are developed and the evaluation criteria are discussed. All episodes, in which the criteria (for selection, pacing and evaluation) are explicitly addressed, were identified in the video recordings and selectively transcribed. In our presentation and discussion we also rely on our first impressions from the student interviews as well as on an initial analysis of the textbooks used in the two classrooms.

The students in the two classrooms under study are at the very beginning of their upper secondary education. One class with 32 students, is from an Arts Programme (Estetiska programmet, short: ES), the other, with 10 students, is from a special track aiming at the International Baccalaureat (IB). In the latter, English is used as the language of instruction. Both classes are engaged in the course A. In the IB track both course A and course B are compulsory, in the ES only course A. The IB group uses a textbook produced for the U.S. American market, while the ES textbook is designed for the A mathematics course within the Arts Programme.

Not surprisingly, there are differences between the mathematics A course in the two programmes revealed already in the first lessons. For example, both teachers stress the importance of the use of technology. In the ES class the students are told that “the calculator does not have to be of any advanced type”, but they should when buying it make sure that it “includes sine, cosine and tangent.” The teacher suggests that “the Casio fx 82, for example, is a good calculator for a cheap price.” In contrast, in the IB group, the teacher reveals: “And then I will also give you your calculators...that is...are also for you to borrow and you may have them during three years cause it’s a requirement on the IB that you need to have a graphic calculator.”

Setting the rules for the work and commenting on the criteria

In the course of the first five lessons in the IB class, the criteria are explicitly addressed only in the first lesson. The teacher hands out the official course plan from the school authority. This 2-page document contains short outcome-based descriptions, phrased as the ability to use the named repertoire of mathematical procedures and concepts in different situations. Under the heading “evaluation criteria” the criteria for each of the three pass levels are described. After illuminating the topic descriptions, the teacher addresses the evaluation criteria. While some students are looking at the handout and some are listening, the others look disengaged.

IB: Lesson 1[2]

(00:22:15) What about the grades... have you looked at the grading criteria that you have... you have it at the back [of the page]... we have pass ...pass with distinction and pass with special distinction. If you are aiming for... pass... and I hope that is your least

goal to get a pass...hopefully a higher grade and at least a pass should be your goal... then you are...or you must use appropriate concepts... learning what about what different things are called what different methods to use and how you solve problems. And for pass it's required that you can solve problems in one step...at least... and some oral and written reasoning of course... that is important that you can show your work both orally and in writing... it's difficult to know how students are reasoning sometimes if you don't see it... and then use of course mathematical terms and symbols and so on... and understand and know what that is. And that you also can differentiate between guesses and assumptions...when you are given facts you don't think you can solve... and some proof.

(00:24:18) To get pass with distinction...the biggest difference between pass and pass with distinction is that you can solve more types of problems... you can use... maybe use several methods to solve one problem... and you can connect different knowledge when you do your reasoning...and that you have a more deeper knowledge so that you can interpret different kinds of situations and when you solve your mathematical problems.

This expansion of the evaluation criteria matches largely what is stated in the text about pass and pass with distinction. According to the official interpretation of the Swedish grades, the pass level should not be taken as the minimum threshold but as the outcome expected to be reached by all students. This might be the reason why the teacher concentrates on elaborating the pass criteria and does not illuminate the criteria for the pass with special distinction. Being able to “differentiate between guesses and assumptions” and “some proof” can be seen to constitute an important part of the esoteric mathematical discourse. The inclusion of these in the pass criteria reflects the intention to include all students in this discourse.

(00:25:07) What is your goal... is that a question one is allowed to ask...have you thought about that...do you think about that now when you have started to take the different courses...what level do I want to achieve with my studies... do you think about that sometimes...would you...that is a good thing to think about because sometimes you have to choose...and think hard about what you want to achieve...or maybe I will put a pass with special distinction for everyone in the class...that would be nice... that would be good...mmm...

“Maybe I will put a pass with special distinction for everyone in the class...” is to be taken as encouragement (and not as a form of sarcasm), as such an outcome is indeed an intended possibility within the framework of the grading system. Then, after handing out the book, the teacher advises the students to look at the sections with the worked examples and start solving the tasks in the category “on your own”.

(00:30:34): ...just the odd ones and you will do one...three...five... and so on...because that's what you have answers to...just the odd ones... and then that will be good enough.

In the first lesson of the ES class, the teacher hands out a working plan for the term. For each week, it contains the page numbers of the textbook where the tasks to be

dealt with can be found. The tasks in the book are labelled (by colour) as category A, B or C. In the chapters the group is dealing with within the first weeks, there are 25 A tasks, 12 B tasks and 2 C tasks in the chapter “tables and diagrams”, and 108 A tasks, 87 B tasks and 15 C tasks in the chapter “numbers in different forms”. In each chapter there is one task marked as an open task amongst the B tasks. On the first page in the textbook there is a short explanation:

After the theory exposition follows a solved example that illuminates the theory. There are tasks at three different levels and of different character. Open tasks do not have one given answer and often require a mathematical discussion. A-tasks are standard tasks that generally can be solved in one step, while B tasks often require a solution in several steps. C tasks are more complex in their character and for solving them you need to, amongst others, apply mathematical knowledge from several areas.

In a meeting about organisational issues before the first lesson, the teacher handed out the textbooks and explained:

“The book is grouped by levels and you will get a feeling which level suits you.”

In the first lesson, after a student calls the teacher and asks, the teacher expands on the principles of the task levels.

ES Lesson 1:

Teacher (00:07:33): So.

Anna: With these tasks... then... should one do A B or B C or only one of those

Teacher: Ehm...

Anna: So if we say that I have done type A tasks... will one then pass the test or does one need B tasks in order to get all tasks done [in the test]... because sometimes it is like this [?] tasks C... that is how it was in lower secondary...which come in the test... if I had done A I did not grasp what it was all about.

Teacher: Nope... these are grouped by level of difficulty and if you go in the first place for solving A tasks and it works very well on the A tasks... then you probably don't need to solve all A tasks... but then you go to a B task which is a bit harder and take up a challenge.

Anna: Yes/

Teacher: /And the C tasks are of course a bit more tricky.

Anna: Yes...

Teacher: But the minimal requirement is that you have done A tasks to an extent where you feel that it works well with the A tasks.

Another episode in which the criteria are discussed occurs in the second lesson. In this lesson, the students again work individually on the textbook tasks while the teacher walks around between the desks.

ES Lesson 2:

Thomas (00:33:52): You [name of the teacher] I'm wondering about something.

Teacher: Yes...

Valter: No we are wondering about something.

Teacher: Well we then will make a collective wondering/

Thomas: /Does one have to/

Valter: /Does one have to... it is A here and B and also C here does one have to do a task for all...

Teacher: Yes you should if you feel that you succeed very well with the A tasks then you will have to get up a level and do B tasks yes of course.

Thomas: But A counts as G [abbreviation for pass] B as VG [abbreviation for pass with distinction] and C as MVG [abbreviation for pass with special distinction]

Teacher: Yes roughly it can indicate that it is roughly that level of difficulty for the somewhat more difficult B tasks but one stretches oneself up a little extra when sorting out the B tasks...

Hannes: But [for the] B tasks it is unnatural.

Teacher: No of course you should sort them out.

Later in this lesson, the criteria for pass with special distinction are at issue:

Kerstin (01:03:57): Was one supposed do these...

Teacher: Not now we save these MVG [abbreviation for pass with special distinction] tasks and take them in a lump using an MVG lesson so to speak for those who really want to aim at MVG so that we can discuss them properly.

The teacher refers to a set of tasks given at the end of the chapter on a separate page. These tasks are all related to a non-mathematical topic. The page differs considerably in layout and design from the other pages in the book.

Finally, we found another incident when the criteria for the type of discourse that has to be produced for a pass with special distinction are touched upon:

ES Lesson 5:

Teacher (00:46:00): So what basic calculating operation is it that you have here.

Hannes: Yes it is minus then.

Teacher: Mm minus...so you end up at about pass and subtraction...so this is pass with distinction language...yes.

DISCUSSION

In the first five lessons, weak framing is apparent in both classrooms, although to a different degree. In both groups, the students can choose their own pace for working with the tasks. Framing over the communication is weaker in the ES lessons. When solving tasks, students can choose, for example, whether they talk with their peers,

get help from the teacher or work on their own. The option of openly discussing is usually not available in the IB lessons. In both groups, students have an apparent choice over the criteria as far as they can “choose” out of a given set of levels or grades to aim for, if they want to achieve more than pass.

According to Bernstein (1990), the relation between transmitters and acquirers is essentially (intrinsically) an asymmetric relation, but there are various strategies for disguising the asymmetry. Consequently, its realisation may be very complex in certain modalities (p.65). This is certainly the case for the realisation of the pedagogic practice as we observed it in the first five lessons in both classrooms. Although to a different degree, the instructional discourse is largely delegated to the textbooks. The most obvious difference between the textbooks is the levelled nature of the tasks in the book for the ES group and the absence of such levels in the one used in the IB class. It can be argued that in the ES class, the instructional discourse consists of two different discourses that operate in parallel and different students at the same time participate in those, based on their „choice“. According to the teacher’s advice, the sequencing rule entails that the different task levels have to be mastered one after the other. However, in the interviews, students reveal different strategies. Some choose to „jump“ between the levels, that is, after trying some of the A tasks, they decide to deal with the B or C tasks immediately. In the book, the A, B, C tasks are grouped and the groups appear in each chapter in the order of the levels, and all tasks are numbered consecutively, that is, such “jumping” is not suggested. The tasks in the textbook used in the IB group are also numbered consecutively, suggesting a strong framing of the sequencing. If they follow the teacher’s advice, all students will solve the same tasks, although at a different pace. The amount of tasks to be solved in each lesson is not specified in both classrooms. There is an apparent choice. Indeed, as we know from the interviews, some students from the ES group decide to solve mathematics tasks at home to make up for the time in the lessons, in which they talked to their classmates about issues note related to mathematics. In the IB group too, some students decide to work at home with the tasks. One student is much ahead.

In the ES class there is no teacher exposition, but the teacher assists the students while walking between the desks. As the textbook embodies the rules for evaluation and controls the selection and sequencing of the topics and tasks, it is not quite clear whether this is a pedagogic relation, in which the teacher is the “transmitter”. However, it is in the end the teacher who evaluates the competence of the students. Some weeks after the start of the school year, the teacher conducted a test in the ES class. Thirteen students did not pass this test, two students achieved pass with special distinction, and one pass with distinction. The episodes quoted reveal the conscious attempts of some students to get their teacher to make the criteria explicit. But as can be seen from these episodes, the teacher’s explanations remain vague. The responsibility for the decision about which tasks to solve is left to the students who have to evaluate their own performance by “getting a feeling” for the level of tasks

that suits them. The choice for solving tasks that suit their own level turns out to amount to self-exclusion for many.

In the IB class there is more public discourse and, due to the small size of the group, conversations between the teacher and individual students are more likely to be audible for the others. At the start of each of the subsequent four lessons, the IB teacher expands for about 10 minutes on a topic. However, this expansion is not necessarily related to the tasks that the students then continue to solve (in ascending order of odd numbers) in the remaining time of the lessons. We do not yet have the results of the test, but we expect a much more uniform achievement distribution in this group. The teacher's expansion on the evaluation criteria at the beginning indicates a more visible pedagogy. But even though the criteria are explicit, they are not specific and not stated in relation to the different topics to be dealt with. The work set up in the subsequent lessons does not yet appear to contain opportunities to acquire all of the modes mentioned in the criteria (such as explaining the reasoning orally or in written form). The notion of proof is not specified in the textbook and the tasks do not invite alternative strategies and/or solutions.

In the ES programme, the construction principle behind the levels of tasks, as we analysed them, is not consistent and does not match the short description in the book. A difference is visible between the A/B level and the C tasks, as there is a mystification of this level, which is indicated by "tricky" as the teacher describes it, or "more complex" as to the explanation in the textbook. The more "tricky" level does not move into the esoteric domain, neither do the special tasks mentioned in the short episode about the criteria for pass with special distinction. But in one of the episodes quoted we find indication that the use of strongly institutionalised language is an evaluation principle.

The IB textbook contains more esoteric domain text, that is, the external classification of the content seems to be stronger than in the other book. But we see a mismatch between the criteria for the grades and the evaluation principles manifested in the book, that is, in the expected solutions to the odd numbered tasks which the students solve. In both classrooms we are faced with a curriculum (through the textbooks in use) that operates mostly within the public domain of recontextualized non-mathematical practices with occasional insertions from the esoteric domain. Consequently, the question must be asked of how a hierarchy within such a discourse is established in the interaction between students, teachers and the texts in use, which translates into a hierarchy of students in terms of their achievement. If there is no progression towards the esoteric, the distribution of achievement would be arbitrary in relation to the knowledge domains. Only an analysis of the conversations between teacher and students when s/he is walking around between the desks will show whether and how access to the esoteric domain is facilitated and how different students are positioned in this process.

In the two classrooms, much of the responsibility for their own learning is delegated to the students. The teacher is there as a resource for the students, and it is their own decision how to make use of it. Especially in the ES lessons, there is no obvious sanctioning of a lack of participation. Success is likely to depend largely on study habits. The students are faced with choices at more than one level. They choose which program to study at age 16-19. This has implications for which mathematics courses they are required to take. They choose which optional (further) mathematics courses – if any – to take. In both classrooms of the study the teachers present the issue of choosing levels to work at and which grade to aim for as a choice for the students to make. On which basis can such a choice be made? Choices are a reflection of the possibilities a practice apparently offers. Negative choices might be made, that is, based on things to avoid of which one actually is relying on projected expectations. This seems to suggest that students with an advantaged background have more opportunities to make appropriate choices. Apparently free choice and self-selection can amount to self-exclusion.

NOTES

1. See <http://www.acadiau.ca/~cknippin/sd/index.html>
2. Transcription conventions:
 - ... indicates a pause of less than 3 seconds. At the end of a turn it indicates an offer for the interlocutor to get a turn.
 - Punctuation is introduced for improved readability.
 - / indicates a cut.
 - [?] indicates inaudible speech.

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PEDAGOGIC IDENTITIES IN THE REFORM OF SCHOOL MATHEMATICS

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This paper begins with an interpretation of Bernstein's concept of pedagogic identity, which Bernstein himself describes as "no more than a sketch, no more than an embryonic outline, rather than a completed painting ready to be signed and framed" (2000, p.65). In the second part of the paper, after a short description of the Swedish school system, the current reform and the efforts to improve mathematics teaching and students' achievement are analysed. The theoretical framework is based on the concept of pedagogic identity, showing different groups and institutions struggling to turn their bias and focus into state policy and practice. It is a first attempt to explore the diverse and manifold enterprise of the mathematics reform in Sweden in order to understand the positions and oppositions within the official pedagogic arena of reform.

INTRODUCTION

In a sense, educational change proposals resemble political parties. They represent a 'coalition' of interests and projects brought together under a common name at a particular point in time. The more harmonized these separate segments of projects and interests are, the more powerful the social movement behind the party or the educational change (Goodson, 2000, p. 3).

In the second half of the 1990s, the rules for admission into the national programs at the upper-secondary school changed in Sweden. 'Leaving certificates' in the subjects Swedish (or Swedish as a second language), English and Mathematics became necessary requirements. Ever since then the proportion of students who leave compulsory school with no marks in Mathematics has been steady between 6.0 and 7.4 percent. For this (cumulative) group of students, mathematics works as one of the gatekeepers to higher education. This is just one of the reasons behind the national wide initiative concerning mathematics education in school. The initiative, or rather the range of respective initiatives, comprises a mix of incentive, support, resources, accountability and pressure. Some of the initiatives are linked to the current school reform as a whole but some are especially directed to mathematics.

In this paper, I will analyse the current reform efforts regarding school mathematics in Sweden using Bernstein's concept of *pedagogic identity* (Bernstein, 2000). Under conditions of change, the positions and oppositions in the official pedagogic arena project different identities in the struggle for dominance. The concept of pedagogic identities can offer a useful way of looking at state education policies and educational change. This is a first attempt to do so.

THE OFFICIAL PEDAGOGIC ARENA

Official knowledge is “the educational knowledge which the state constructs and distributes in educational institutions” (Bernstein, 2000, p. 65). In different modalities of reform, the bias and focus of the official knowledge construct different pedagogic identities. The struggle between groups to turn their bias and focus into the State policy and practice can in this perspective be seen as the source of curriculum reform. The model of the official arena, in which the struggle takes place, shows four positions (Bernstein, 2000). The positions represent different approaches to regulating and managing change, approaches that are expected to shape the pedagogic identity of teachers and students. In the Bernstein’s model, two of these identities (*retrospective* and *prospective*) are managed by the state and two are generated from local resources. I use a modified model in which the identities can be generated from both centring and local resources.

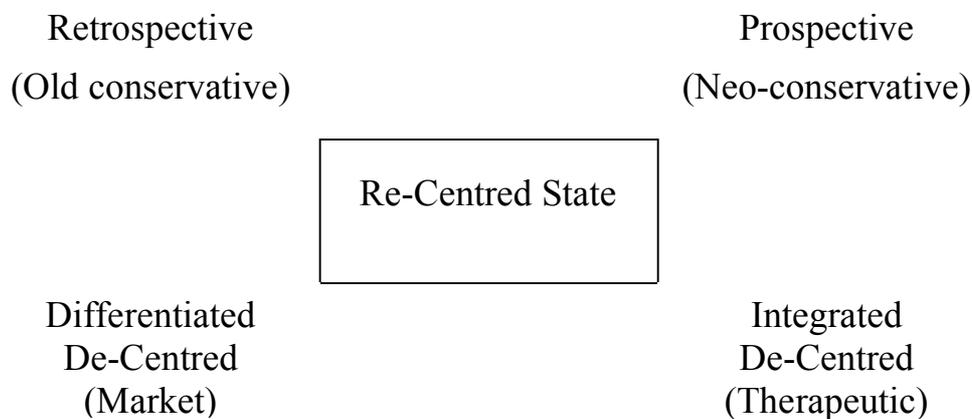


Fig 1: Modelling Pedagogic Identities Classification (Bernstein, 2000, p. 67)

Retrospective identities

The resources that construct retrospective identities are not related to the economy. Retrospective identities are shaped by “grand narratives of the past” (Bernstein, 2000, p. 66) that are suitably recontextualised to stabilise that past and project it into the future. The identities are “formed by hierarchically ordered, strongly bounded, explicitly stratified and sequenced discourses and practices” (p. 67). Revealed by the recontextualised grand narratives of the past, the collective social base is prioritised before individual careers. Retrospective identities are opposed to the de-centred identities since these modes reject past narratives (as the source for criteria, belonging and coherence for the present and future). There are two basic modes of retrospective identities: fundamentalist and elitist. The fundamentalist identity “consumes the self in all its manifestations” (p. 75). Fundamentalists make allowances for and often encourage change. It is entirely opposed to the other mode of retrospective identities, the elitist. Elitist identities can be constructed through education and social networks, apart from the intervention of background. Just like the fundamentalist identities, the position has strong classifications and internal

hierarchies, but unlike fundamentalists it refuses to engage in the market. Elitist identities are perhaps triggered and maintained by “narcissistic formations [...] whereas fundamentalist identities are maintained by strong super-ego formations and communalised selves” (p. 75-76).

Prospective identities

Similar to the retrospective identities, prospective identities are formed from the past. It is however not the same past. The discursive base has a different focus and bias since these identities are constructed to deal with cultural, economic and technological change. Prospective identities are shaped by selected features of the past which are recontextualised to defend or raise economic performance. The position in the official pedagogic arena is solid but the players can change. These identities are essentially future oriented. The narrative resources that they rest upon ground the identities in the future, not in the past as for retrospective identities. Prospective identities involve a re-centring since the narratives create a new basis for social relations, for solidarities and for opposition.

Differentiated de-centred identities (market)

The de-centred market is recognised as an identity projected from positions in the official arena. Local units, groups or departments are monitored by the management in terms of their effectiveness in satisfying and creating local markets. The market demands create a culture and context to facilitate the survival of the fittest. The messages that arise produce an identity whose products are valued by a market. “The transmission here views knowledge as money. And like money it should flow easily to where demand calls” (Bernstein, 2000, p. 69). One can say that these identities are reflections of external contingencies; they are likely to be formed through mechanisms of projection rather than introjection. The position “constructs an outwardly responsive identity rather than one driven by inner dedication” (ibid.).

The focus is in the short term rather than long term, on the extrinsic rather than the intrinsic, upon the exploration of vocational applications rather than upon exploration of knowledge (Bernstein, 2000, p. 69)

The capacity to project discursive practices themselves, which fit the external contingencies, is crucial for the maintenance of this identity.

Integrated de-centred identities (therapeutic)

The integrated de-centred identities are produced through introjection. Bernstein (2000) calls these identities ‘therapeutic’ because they are “produced by complex theories of personal, cognitive and social development, often labelled progressive” (p. 68). Autonomy is necessary for such an identity since it constructs its own characteristics: “an integrated modality of knowing and a participating, co-operative modality of social relation” (p. 68). Non-specialised, flexible thinking, team work and active participation are favoured by therapeutic identities. Their construction is internally regulated and relatively independent of external consumers’ signifiers.

Power is disguised by communication networks and interpersonal relations. The maintenance of this identity depends upon internal sense making procedures. If these fail then a shift to other resources is likely. Therapeutic identities may shift to prospective (Bernstein, 2000).

THE SCHOOL MATHEMATICS REFORM IN SWEDEN: A CASE STUDY

The school system and the curriculum

The public school in Sweden includes both compulsory and non-compulsory schooling. The non-compulsory preschool is for children aged 1-5 years. When children turn 6, they are offered a place in the preschool class and the schooling becomes free of charge (this remains throughout the whole public school system). For all children aged 7-16, attendance at school is compulsory. The upper secondary school, 'gymnasium', is not compulsory but almost all students continue their studies at upper secondary level.

In the middle of the 90s the school system in Sweden changed from rule-regulated to goal-based. The goals are defined by the state but the responsibility for achieving the goals lies on the municipalities (the local educational authority) and the local school (principal and teachers). Steering documents are drawn up at different levels within the school system. It is the Government and the Parliament that specify goals and guidelines for preschool and school. The overall national goals are described in *The Education Act*, the curriculum and course syllabi for each school subject. The central administrative authority, the National Agency for Education (Skolverket), steers, supports and evaluates the municipalities' management of the schools. The task of the Agency is to work actively for the achievement of the goals and to ensure that all children have access to equal education. When the authority for school development was decommissioned in 2008 (it had been established by the former Government in 2003), some of their tasks was passed on to the Agency. A new authority was simultaneously appointed, the Schools Inspectorate. This supervises local authorities to ensure that those who are responsible for the schools follow the laws and regulations. One of the first tasks of the Inspectorate was to carry out an evaluation of the teaching of mathematics.

When the school system changed and became goal-based, a new kind of curriculum for compulsory school was required. The current curriculum, Lpo 94, differs from its predecessors in many ways. One obvious difference is the number of pages. The current curriculum consists of 20 pages (not even 10 percent as much as the previous one). The focus is on principles and normative values that should permeate the schoolwork. There are no instructions concerning teaching methods; how to reach the goals is locally determined. The mode of the 'new' curriculum makes it hard to compare with its predecessors. Nevertheless, the syllabus of mathematics includes a 'new' perspective; students are supposed to learn not only mathematics but also

about mathematics (Johansson, 2006). In the syllabus for compulsory school, this aim is expressed in the following way:

Mathematics is an important part of our culture and the education should give pupils an insight into the subject's historical development, its importance and role in our society. The subject aims at developing the pupil's interest in mathematics, as well as creating opportunities for communicating in mathematical language and expressions. It should also give pupils the opportunity to discover aesthetic values in mathematical patterns, forms and relationships, as well as experience satisfaction and joy in understanding and solving problems (Swedish National Agency of Education, 2009, p. 23).

A rule that was implemented when the school system changed in the middle of the 1990s is that students need to have 'leaving certificates' in Mathematics, Swedish (or Swedish as a second language) and English in order to be qualified for the three-year national programs. An alternative for the students that fail in one or more of these subjects, if they would like to continue their studies, is to attend an individual program. The program offers a personalised study plan for the individual student. Each year since the reform, nearly ten percent of students have not qualified for the national programs when they leave compulsory school. About seven percent of students do not pass in mathematics. The situation is described as unacceptable in the political rhetoric. A new reform of the upper secondary school suggests that the current model of individual programs will be replaced by other alternatives.

Improving mathematics teaching and students' achievement

The election of a new Government in Sweden in 2006, a shift to the right, can be seen as the starting point for the current comprehensive school reform. The school was however already 'pathologised'. A lot of attention was given to, for example, students' results in mathematics on the national tests and final grading, the programs and the syllabuses at the upper secondary and the individual program. In 2003, the previous Government had decided to set up a Mathematics Delegation whose task was to propose measures to strengthen mathematics and its teaching. The current reform effort in mathematics education could (to some extent) be seen as a result of the proposal from that committee (Matematikdelegationen, 2004). Further 'triggers' are the results from national and international evaluations (e.g. Skolverket, 2003; 2004; 2007a; 2007b; 2008). So, this is the background for giving priority to mathematics and this is how the Government (the minister of education) motivates it:

National as well as international evaluations in recent years indicate that Swedish students have weakened their knowledge in mathematics and science. The international evaluation TIMSS 2007 shows clearly declined results compared with the evaluations from 1995 and 2003. The Government takes the situation seriously. The future of the competitiveness of Swedish industry depends on young peoples' interest in and good knowledge of mathematics, science and technology (excerpt from the commission from the Government to the National Agency of Education, (U2009/914/G), p. 5, author's translation)

Throughout the financial year 2009, the National Agency of Education is commissioned by the Government to perform the following tasks:

- through an application procedure, distribute 87 million (about 8 million Euro) Swedish kronor to mathematics development projects in schools,
- produce support material and disseminate information (about mathematics),
- analyse and suggest inputs that can facilitate the transition from gymnasium to higher education in the field of mathematics, science and technology,
- distribute funds to Science Centres.

Apart from the mixture of initiatives directed towards mathematics, the current school reform has additional consequences for mathematics as a school subject. Some of the changes already implemented are: a) goals and national assessments in school year 3 (previous only for year 5 and 9); b) written assessments in all subjects and every school year (marks were previous only given to students at grade 8 and 9); c) training in mathematics at the upper secondary school gives extra credits when students apply for higher education; d) an experimental work with elite upper secondary schools has started in three places in the country. The students are recruited from all over Sweden. Furthermore, a new curriculum and new syllabuses for compulsory school will be implemented in 2011.

The Read-Write-Count-Project (year 2008-2010) offers a directed State subsidy which the municipalities can apply for. It is a non-compulsory commitment for which the local authorities have to follow rules from the Government in order to get the subsidy. The financial support is distributed by the Agency. It can be used for increasing the workforce and the competence of teachers but it can also be used for curriculum material. The official aim of this project is to improve the work as regards students' basic skills. The focus is on early testing and diagnosis and the objective is to increase the students' goal-attainments.

Through an application procedure, schools can get financial support from the Agency for development projects in mathematics. Three types of project can be granted: projects that aim to develop teaching methods; pedagogical developmental work; and further studies in mathematics. The schools can decide which type of project (more than one type is allowed); the Agency divides the money between the three parts. The number of students involved defines the size of the subsidy. Selected projects are financed with 3 000 SEK (about 280 Euro) per student.

Mathematics is also prioritised within the state initiatives *Lärarlyftet* and *Förskolelyftet*. These programmes entail in-service training for qualified, in-service teachers and pre-school teachers. The teachers can study full-time with a 20 percent reduction in salary. They can choose from a range of courses at colleges and universities that are approved and procured by the Agency. Continuing education for pre-school teachers is mainly directed towards language and mathematical development of children. The official standpoint is that:

The continuing education that is commissioned by the Agency shall depart from the priorities and requests from the responsibly authority. Additionally, the Agency shall prioritise continuing education in areas that have proved to be especially disregarded (resolution by the Government (U2007/3168/S), author's translation).

The state pays the costs for the courses and compensates the schools for part of the payment to the teachers. During spring 2009 there was 2 262 places on courses approved by the Agency, 2 137 teachers applied and 585 requested education in mathematics (this was ten more applicants than the available places). Within the framework for *Lärarlyftet*, teachers can also suggest courses from the regular selections at the colleges/universities. However, these institutions are not allowed to advertise these courses under the same conditions as the approved courses. As a consequence, teachers must be more active (and skilful) when searching for suitable courses.

'Mathematics change agents' (matematikutvecklare) is another initiative. This was started in 2006 and involves financial support to regional and local development projects in mathematics. The National Centre for Mathematics Education, NCM, runs this programme together with the Agency. Regional conferences are arranged and the municipalities can have two participants attending each conference with no conference fee and selected literature for free. NCM reports that 93 percent of the municipalities have used this offer for the eight conferences that have been carried out in the years 2006-2009 (<http://matematikutvecklare.ncm.gu.se/> 2009-08-13).

School development projects are also supported through specially compiled support material, inspiring material and the dissemination of information and experience from previous projects. As an example, NCM is commissioned to produce research-based knowledge overviews. Funds are also given to other institutions, for example to a project for teaching mathematics for multilingual students (Webbmatte) and to the Educational Broadcasting Company, UR, (belonging to the Swedish public service). A further example is the support to develop inspiring material for Sámi School to help teachers to take a Sámi perspective in their teaching of mathematics. The state also gives financial support to a selected number of upper-secondary schools to support cooperation with colleges/universities.

The pedagogic arena: an analysis of the current reform in Sweden

In this part of the paper, I shall use Bernstein's concept of *pedagogic identity* as a framework for analysing the current reform in Sweden. The focus will be on the range of state initiatives concerning mathematics and the institutions that are involved. The assumption is that the institutions are projected into different pedagogic identities. The power or the empowerment of an institution, within the reform, is determined by the state or, indirectly, through other positions in the official pedagogic arena. A reform emerges and develops as a result of the struggle in the arena in which the different pedagogic identities try to turn their bias and focus into State policy and practice.

Retrospective identities could be mathematicians or educationalists who try to protect their view of the subject or their view of schooling. Discipline problems, a lack of interest in mathematics or school work in general could create the feeling that it was better in the ‘old time’ – when students were submissive, behaved properly and paid attention in class. For retrospective identities, a reform is positive because it opens up a possibility to claim their view. In the ongoing reform efforts, this position seems to have influence. Two examples are the Read-Write-Count-Project and the in-service training for pre-school teachers; both of which have a focus on basic skills. The development of school mathematics is neglected in favour of maintenance of well-known competences. Further examples are the initiative of elite upper secondary education in mathematics and the extra credits that mathematics gives when students enter higher education. The new syllabus in mathematics, to be implemented in 2011, could perhaps show if this position has a crucial role in the reform.

Prospective identities have to take responsibility for cultural, economical and technical changes. This position is grounded in the past but is supposed to mould the future, without jeopardising the economy. In the current reform, the Government, through the Agency and other institutions, tries to create a ‘new’ school. It is not about changing the view of what education and schooling is about, but rather an effort to reach anticipated results through a combination of pressure and support. The School Inspectorate supervises the schools so that they follow the rules determined by the Government. Students’ results on national tests and final grading are made public on the Agency’s website so that parents, students, journalists and others who are interested in a particular school have easy access to these results. Since the economy of the schools depends on the school voucher attached to each student, competition between schools (both independent and public) to attract students has become noticeable. Thus, local school authorities are projected to this position through this combination of pressure and support.

Differentiated de-centred identities (market) have considerable influence in the current reform. This position is projected through the expectations, needs and requests from other pedagogic identities. The most empowered institution in this position is the Agency. It is a central authority, so one should perhaps see it as a prospective identity. On the other hand, the Agency has some autonomy and works as a player in the market. It produces inspiring materials and research overviews, disseminates information on its website, and so on. The Agency chooses affiliates; the chosen institute (or person) will be empowered and influence the reform. In the current reform, the main ‘co-drivers’ and ‘performers’ are the colleges, the universities and the national resource centres (for example NCM). These institutions are ‘co-drivers’ and ‘performers’ in the sense that they could give support and advice to the authorities, which in the next phase could commission them to act in the suggested direction. In some sense, these institutions have to compete with each other to get their share in the official pedagogic arena.

The positions and oppositions are made visible through, for example, the choice of college/university courses within the programmes *Läraryftet* and *Förskolelyftet*. Without approval from the Agency, an institution will not be on the list of eligible courses and thus not get the same chance to influence teachers working in the schools or the pre-schools. In addition, the Agency creates a discourse about the teaching of mathematics through the selection of courses. ‘Outdoor mathematics’ and ‘laboratory mathematics’ seem to be adequate approaches as well as ‘the foundation of mathematics’. The criteria for approving the various kinds of courses are mostly of organisational type. A scientifically based approach is not explicitly asked for.

Integrated de-centred identities are recognised by their way of building communicative networks and social relations. Students’ associations, mathematics teachers’ associations and the network of ‘mathematics change agents’ could belong to this position. Their role in the current reform is vague but one can assume that these identities have a rather weak position. The school is pathologised; the ‘remedy’ is to improve teachers’ knowledge and to encourage students to work harder in (and enjoy) mathematics. The programmes of in-service teacher training and the extra credits for mathematics in upper-secondary school are incentives in two different directions. The freedom to choose courses could be seen as a possibility to empower this position. However, it is still the authorities who choose the range of courses and whom to permit to attend the college/university courses. One could also think that the position could be empowered through the subsidy from the state for developmental work. But even in this case, the final decision is made by the authorities.

DISCUSSION

Bernstein’s concept of Re-Centred State seems appropriate to apply to the Swedish school system and the current reform:

It refers to new forms of centralised regulation whereby the State decentralises and through (a) central setting of criteria and (b) the central assessment of the outputs of agencies, financially (and otherwise) rewards success and punishes failures: ‘choice’, selection, control and reproduction (2000, p. 78).

In the political rhetoric, teachers are sometimes ‘blamed’ for the failure of their students. Improving teachers’ knowledge and their teaching repertoire may be seen as a way to reform the school. This is also one of the focuses in the diverse and manifold enterprise of the mathematics reform in Sweden. The initiative was taken by the state but the responsibilities for the implementation are moved ‘down’ in the hierarchy of the school system. Autonomy is encouraged but the state controls through financial support, limits, criteria, inspections, annual reports and national testing. The *differentiated de-centred identities* (market) are empowered, for some institutions more than others. Parts of the reform, for example Read-Write-Count-Project, seem to be influenced by *retrospective identities*. The *integrated de-centred identities* are perhaps too silent in the current school political rhetoric.

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ANALYSING PISA'S REGIME OF RATIONALITY

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This paper explores the regime of rationality which PISA helps to reinforce. Bringing together certain approaches of Bernstein and Foucault, three levels of analysis, relating to social categories and communication, the self, and government are identified. A single PISA mathematics item is analysed, illustrating these levels of analysis and their interrelationships. We find evidence that Kenway's concept of the technopreneur as an agent of cultural and economic production might help in our analysis. The paper reports the first tentative steps towards a research agenda that brings together key contemporary theoretical resources in educational research.

INTRODUCTION

The official aim of the PISA is to “assess the extent to which students near the end of compulsory education have acquired the knowledge [of mathematics, science, literacy] and skills essential in everyday life”. Answers are sought to questions such as: “Are students well prepared for future challenges? Can they analyse, reason and communicate effectively? Do they have the capacity to continue learning throughout life?” To answer these, PISA says it needs to “develop indicators that show how effectively countries have prepared their 15-year-olds to become active, reflective and intelligent citizens from the perspective of their uses” of these subjects (OECD, 2006, p.114). Indicators are thus purposed to assess “students on their ability to *adapt* the knowledge they acquire at school to *real-life situations* as opposed to how they master a specific curriculum” (italics added). Within this context, PISA studies have amassed an almost iconic status in policy and discourse regarding mathematics educational futures – and doubtless do provide a large volume of useful and serious information. Nevertheless, we ask, does PISA do what it says it does?

In the assessment literature, this question is usually addressed in terms of the quality of the assessment regime, framed around the concepts of reliability and validity (Messick, 1989; Gipps, 1999; Broadfoot and Black, 2007; Black and Wiliam, 2007). However, we note that in these accounts analysis often narrows quickly to be construed in technical terms; quality is rendered against functional criteria relating to “fitness for purpose” - despite a rich parallel literature problematising exactly such criteria. Perhaps this tendency is explained by Patricia Broadfoot's remark that the “power to define what counts as quality in education is the single most influential source of educational control” (2007, p. 64). In other words, restricting the scope of analyses to the effectiveness of assessment methods *qua* techniques, self-controls the debate and creates a semblance of consent and rational commitment to PISA's aims. Nevertheless, all may not be as unproblematic as it seems. Indeed, as we argue

below, it seems unclear whether PISA does or even can do what it says it does. This suggests the need to ask a perhaps more fundamental question: What *can* PISA do?

In the next section we outline three separate but interrelated levels of analysis, which are key to our approach. The first level of analysis, taking up Bernsteinian resources, essentially argues that the validity of PISA's assessment instruments is undermined by the recontextualisation of categories such as school mathematics and pedagogic discourse and by the differential effects of this across student populations. We visit the seminal studies of Cooper and Dunne (2000) and open a discussion about the problematics of recontextualisation complicating PISA's work. In the second and third levels of analysis, we adopt Foucauldian approaches to try to better understand, more positively, what PISA *can* do. Here we are convinced of the importance of making visible the regime of rationality that makes PISA assessment possible. However, in order to do this we think it necessary to go beyond Bernstein's theory of power and control – a theory foregrounding interrelationships among social categories and communications (Bernstein, 2000, p. 5). Decisive in this is to understand how PISA performs the labour of governing. Using Foucauldian terms we want to explore how technologies of self are co-opted by PISA to work simultaneously as technologies of government. We will argue that these constitute the second and third levels of analyses needed in our enquiry. A fuller understanding of the interrelationships among these analyses is also required; our offering here, however, can only hint at these.

In order to illustrate how these key levels of analysis might be conducted we outline an investigation of a single PISA item, concluding the paper with a brief discussion. This paper represents a report in progress on an emerging large-scale agenda for research we have begun to design and implement. We are only too well aware of short cuts we have taken in presenting the flavour of our thinking within the space available. Our intentions here are to ask questions and provoke debate.

THREE ANALYSES

As indicated above, our efforts to recuperate the rationality of PISA mobilise analysis at the levels of categories (social unities) and communications, the self, and government. These levels of analysis are not arbitrarily chosen: they arise in remarks by Foucault on his methodological choices in unpacking problems concerning how forms of madness, punishment and sexuality have been produced and circulated within various regimes of truth. Foucault's analytic is driven by four interrelating "major types" of technology: of the production of things; the production of semiosis, semantics and signification; the production of the self; and the production of power (Foucault, 1988, p. 18). Though we find Foucault's refocusing of thinking around the concept of technology in keeping with his tacit foregrounding of production, this move begs an important question. If technologies are defined by the ends or *telos* to which they refer, what is the *telos* of technologies of power? Simply put, we believe the answer must be the deferred ends of the production of things and communication,

of the production of self, of government. In considering these, it is interesting that Foucault himself construed his work as unpacking technologies of power in understanding the interrelationships of the self and government. For us, a treatment of educational assessment, adopting this Foucauldian approach, would be illuminating. While Foucault avowed that his research was much less concerned with the production of things and communications, this area is a strong aspect of Bernstein's work. Thus we see prospects here for a useful complementarity and supplementarity between these substantial bodies of work. As a tentative exploration of this possibility, we set out three levels of analysis drawing on these resources. We seek an account of the circulation of rationality among the projects of production, communication, the self, and government.

The level of categories and communications

A dominant feature of PISA's discourse of mathematics is its emphasis on application. Indeed, mathematics is restyled as "mathematical literacy" and defined as dealing "with the extent to which 15-year-old students can be regarded as informed, reflective citizens and intelligent consumers" (OECD, 2006, p.72) rather than directly with the knowledge and skills of school mathematics. This constitutes both a particular form of *recontextualisation* of mathematical knowledge and a representation of the world of the 'everyday' (Dowling, 1998; Moore, 2007), constructing new meanings both for mathematics and for the 'real world'. In coming to this view, we feel supported by the seminal work of Cooper and Dunne (2000) who have demonstrated how the discourses of UK tests, which also combine school mathematics and the 'real world', compromise the validity of attempts to assess mathematical knowledge. They draw upon Bernstein's notion of code orientation (and Bourdieu's notion of habitus) to explain differences in patterns of response, and hence patterns of achievement, across and within social classes and other social divisions. Cooper and Dunne's focus is primarily on the ways that, depending on their social backgrounds, students with different styles of response may differentially negotiate the weakened boundaries implied in the emphasis on knowledge and skills essential in everyday life, arguing that this leads to the invalidity of certain mathematical assessments. Likewise, though the nature, variety and complexity of boundaries in the case of the PISA programme/system requires analysis of its own (one of our future tasks), we assume that such analyses will show that the PISA programme/system cannot in general validly assess what it claims to assess.

Thinking with Bernstein's concepts we can conceive of the PISA assessment system as an evaluative dimension of a *pedagogic discourse*, which OECD attempts to construct and disseminate. Pedagogic discourse, Bernstein suggests, has no content of its own, but is a set of principles for *recontextualising* knowledge for purposes of transmission, acquisition and evaluation (Bernstein, 1990). To construct school mathematics, educational agents in OECD operate by selecting, reassembling and relocating elements of discourse from the discipline of mathematics, incorporating within the discourse aspects of the social world as its field of reference (Dowling,

1998). Furthermore, they draw on other scientific domains related to education, crucially methodologies for pedagogical instruction (Bernstein, 1990). Two consequences might result from such a form of pedagogic discourse. One concerns its *orientation*, which positions the pedagogic subject in an activity whose criteria for appropriate action seem to remain implicit, suggesting that an *invisible* form of pedagogic practice may be at work (Bernstein, 1977). The other possible consequence is that in positioning the subject as an ‘actor’ within a ‘practical field’ a normative framework may be imposed, i.e., appearing to sanction particular ways for doing things, rather than indicating ways for students to seek mathematics specific resources in acting upon this field. As Moore (2007) puts it, pedagogic discourses differ in the degree to which they impose particular normative frameworks or create possibilities for critical questioning and interpretation. In Bernstein’s terminology, we must seek to describe the regulative characteristics of the discourse in which the instructional elements are embedded.

These analyses are potentially able to problematise the validity of PISA assessments, as they expose them to possible criticisms/charges of social class bias, as well as of symbolic control; still they do not of themselves explain in any elaborate way the symbolic control function and how this might relate, in this case, to PISA’s work of reinforcing particular versions/forms of rationality.

The level of self

Following Foucault, *technologies of self* are practices which “permit individuals to effect by their own means or with the help of others a certain number of operations on their own bodies and souls, thoughts, conduct, and way of being, so as to transform themselves in order to attain a certain state of happiness, purity, wisdom, perfection, or immortality” (Foucault, 1988, p. 18). Elsewhere (Foucault, 1997) he describes these as giving an ongoing commentary on the self by the self with respect to behaviours, desires and dispositions. These technologies open up for analysis the manner of individual commitment to social designations of identity. They go beyond identity in itself, to address how the self makes the self in relation to a particular identity (anticipating that the self will be encountered by others and by itself as a social type precisely in virtue of this identity).

Analyses at the level of self supplement the analyses of social categories and communication, which generally leave aside the manner of the inhabitation of these categories by the self. In particular, technologies of the self serve as self-regulative micro-level devices, thus contributing to the rationality of PISA, whether concerning the ‘scholastic identity’ of students or the ‘professional identity’ of the teacher (Ball, 1990; Meadmore, 1995; Evetts, 2003). Yet, without a third level of analysis into how the domains of categories and communication and of self are interrelated, we would have an incomplete picture of the rationality of the actions and behaviours of actors. This third level of analysis is an investigation into the mediating agency of government.

The level of government

For Foucault, government is not an entity, “developed above individuals, ignoring what they are and even their very existence”, but a “structure, in which individuals can be integrated, under one condition: that this individuality [is] shaped in a new form, and submitted to a set of very specific patterns” (Foucault, 1982, p. 214). The work of government is thus integrating work and paradigmatically performed by certain *technologies of government*. On the one hand, government secures the reproducibility of resources that are the grounds of power and symbolic control (as revealed in our first level of analyses). On the other hand, government also stabilises and makes predictable the formats required by individuals (and groups) to build dispositions and commitments to ongoing and future actions (the technologies of self, the object of our second level of analysis). Combining these, we follow the Foucauldian insight that work at the mediating level of government characteristically co-opts interactions accounted for at the other levels of analysis. In particular, for Foucault, *subjectivation* is the process whereby government co-opts technologies of self, and *governmentality* is the mode of working of such government. As Klein (1996) notes, governmentality for Foucault

applies itself to immediate everyday life which categorises the individual, marks him (sic) by his own individuality, attaches him to his own identity, imposes a law of truth on him which he must recognise and which others have to recognise in him. It is a form of power which makes individuals subjects. (Foucault, 1982, p. 781, cited in Klein, 1996, p.376)

It would therefore seem that unlocking the “laws of truth” might give us a way to understand the regime of rationality of PISA. But which are the salient laws? Here enquiry must go to the prevailing cultural realities of politics and economics. Lemke argues that individuals in a neo-liberal state regulate themselves in accordance with the “imperatives of flexibility, mobility and risk taking” (Lemke, 2002, p.6). Meanwhile, Kenway *et al.* (2006) draw attention to an emerging class of cultural and economic agents – *technopreneurs*: those who must be conditioned by Lemke’s neo-liberal imperatives, yet simultaneously work within bureaucratically controlled and managerialist settings. As a model for the kind of student who performs well in PISA assessments, Kenway’s identity designation seems thought provoking.

ILLUSTRATIVE ANALYSES

Here, using a single item taken from the set of example items published on the PISA website, we illustrate the three levels of analysis identified in the previous section. Clearly a single item cannot provide us with definitive conclusions. Nevertheless, we hope to suggest an approach to analysis that may prove illuminating and to raise issues and questions for further investigation. The item (see appendix) has a structure common to PISA items: a stem presenting the student with information about a “real-life situation”; a set of questions related to this context; and a scoring rubric for each

question indicating for the assessor how they should allocate marks to student answers. We use Question 44.1 to illustrate the first and second levels of analysis and Question 44.2 to illustrate and contrast all three levels of analysis.

Analyses of Question 44.1: Although the stem of this question might best be described as an instance of popular science discourse, the question quickly moves away from the scientific and real world reference, first instructing the student in how to read the diagram and then engaging them with a ‘pure’ mathematical calculation. Even here, however, the student’s task is not simply to calculate but to “show the calculation to demonstrate ...”. The statement of this task suggests that the activities demanded of the student are communication (*show, demonstrate*), while the nominalisation *calculation* presents the process as an agent-less object. The 11% result of this calculation *is obtained* using the passive voice, again obscuring agency. Thus, the student is distanced from the mathematical performance (“the calculation” is presented as having an unproblematic existence) and privilege is nominally allocated to communication. Despite this, as the rubric shows, in order to earn full credit the student must actually perform the necessary calculations correctly. Moreover, marks are deducted if the student offers correct arithmetic expressions but calculates incorrectly – thus contradicting the distancing from calculation created by the question itself. Most tellingly, marks are allocated for correct calculations, even where incorrect arithmetic formulations are proposed. Because credit is allowed where there is an invalid correspondence to the real world, a concerted preference for performance over communication is revealed. But this preference precisely contradicts that constructed, as we showed above, in the posing of the question and is thus concealed from the student confronting this mathematical problem.

The question begins: “In the diagram you can read ...”. Here the student is addressed directly as an apparently successful student who is able to interpret the diagram as required. This particular “you” may be read as the abstracted “one” of more formal speech. Yet simultaneously, the heightened modality of “you *can* read” and the fact that the correct interpretation is explicitly provided, opens up the possibility that a particular “you” actually is not able to “read” as stated, and therefore needs the help implicitly offered in order to answer the question. This “you”, a second “you”, brings to the question a real “you”, a personal “you”, one that might be taken by the student as “*me*”. The student then faces the instruction “Show ...”. This, together with the use of the definite article “the calculation” suggesting a single possible correct answer, is a common formulation within traditional forms of pedagogy and as such begins to demonstrate how the individual student is intended to negotiate their identity in the face of ambiguity among these possible “you”s. The assessment rubric is consistent with this traditional pedagogy: criteria for full and partial credit are explicit and draw on exclusively mathematical resources. All of this evidence powerfully indicates the recontextualisation of a pedagogical discourse; by virtue of different modes of access to this discourse, the student’s progress in the item is either

enhanced or depleted (as indicated by Cooper and Dunne's exemplary instance of such first level analysis).

The second level of analysis, exploring issues of self, occupies an alternative space. Here the concern switches from the structuring domain of the student's experience in encountering this question, to the student's way of encountering herself. The problem is to illustrate and understand the kind of self-monitoring that provides ways of being, exercised by choices between the two "you"s on offer in this example. Foucault refers to these as 'practices of the self'. Here the problematic is not accessibility, but the question of how a student may encounter this item: on what basis, with what manner of commitment, with what degree of mobility and preparedness to manage the risk of misconstruing one "you" for another? This second level of analysis goes beyond the actual accomplishment (or non-accomplishment) of the task to exploring the kinds of commitment to future ways of being belonging to the student's trajectory. The importance of this second level of analysis is that it asks about the ways students become the subject of their behaviours; and whether these behaviours, and therefore the subjects enacting them, do work also recognisable as the political work of government, and become committed to doing this work.

Analyses of Question 44.2: Here the question consists of a stem followed by an item posed as follows: "Do you agree with Mandy when she says this is not possible? Give an explanation to support your answer". It must be noted that the validity of the response cited in the marker's rubric depends on whether the lexical marker "possible" is taken to mean "possible in principle" (the 'of necessity' interpretation) or "possible in fact" (the 'contingent' interpretation). If the first, then Mandy is wrong precisely for the reason given in the rubric. But a student responding to this question may quite conceivably have had reasons (external to the data presented in the question) to believe that, for the specified period, it so happened that the net change of the EU's CO₂ emissions, excluding those of The Netherlands and Germany in this period, was negative, or at least less than a total increase of 4 million tons (this quantity being the salient difference between the decline of the EU total output of CO₂ in this period and the net decline in output of Germany and The Netherlands combined). In such a case, according to the rubric, the student would not receive any credit – despite their answer entirely corresponding to the "real-world" reality. Thus, allocation of the full score for this question critically depends on the student either only recognizing the 'of necessity' interpretation or recognizing this interpretation and actively dismissing the alternative. However to identify the 'of necessity' interpretation (let alone identify both and exclude the 'contingency' interpretation) is a matter of advantageous communication and depends on access to and recognition of the particular genre of pedagogic discourse in mathematics that biases toward necessity and away from contingency. It also depends on being able to decode texts in this genre appropriately. The issues at stake here are categorical and communicational, relating to the recontextualisation of everyday and school

mathematics discourses. However, these issues alone are not sufficient to fully understand the regime of rationality applying in this question.

We thus move to the second level of analysis which asks: What *kind* of self would act out of a commitment to a form of practical reasoning that entirely disregards indications of situatedness (with actual and specific dates, quantities, nations, and trans-national aggregations, etc.), no matter how detailed and elaborately set out (in numerical values and graphs and charts etc), in favour of a response that makes no reference to, or finds any utility for, these details? For us, such a self is one that is routinely oriented towards conceiving of specificity as a distraction. Such a self is habitually oriented towards locating abstracted relationships validly expressible in symbolic and always generalisable terms. As a corollary, such a self is reluctant to engage with concretely related entities expressible in terms where validity is contingent. Instead, this self disciplines itself in spying out necessary, albeit disembodied, relationships wherever the situation requires; and is practised at turning a blind eye to actual, factual, embodied, and contingent circumstances – even though these are overtly offered as a basis for practical reasoning. As a result, the disposition privileged here drifts towards greater flexibility and mobility, a socially acquired ability, related to the social class background of individuals and their differential relation to education, that presupposes recognition of the hierarchical relations structuring the abstract vs concrete distinction. What is emphasised here through the lenses of Foucault is, precisely, the ‘labour’ required to be able to work towards a rationality that allocates maximum value to higher levels of generality and minimum value to specificity.

In the third level of analysis we need to determine to what extent, if any, this kind of self-worked self does the work of government. That is, whether the technologies of self identified in the second level of analysis amount, in the third level of analysis, to an instance of governmentality. Given the scale of the data under examination, our approach to this investigation must be taken as extremely provisional. Nevertheless, we believe that the practical reasoning that encourages greater mobility and flexibility in producing assessment scores must at the same time lead to the most efficient production of scores (in the sense of maximizing symbolic benefits – the score; whilst minimising their cost – the amount of time on task). Thus, the effect of rewarding greater accumulations of positive scores is to reward efficiency. Yet ‘efficiency’ as such cannot be rewarded - only the *efficient self* can be rewarded. Thus, where the rationality of overall social production is dominated by the discourse of efficiency, as it is in under our current neo-liberal regimes of government, we can conclude that technologies of government and technologies of the self intersect. From this an outline of a possible regime of rationality in PISA emerges – a form of rationality we tentatively suggest resembles in some measure that of the *technopreneur*.

DISCUSSION

In this paper we started with the assumption that PISA's findings cannot validly support conclusions that go unproblematically to the official aims of the PISA programme. We believe it would be possible and useful to work out to what extent and within what boundaries of recontextualisation its data do offer information of the kind it claims. However, we believe that such an analysis would not be easy and would very likely introduce further difficult problems. Notwithstanding, we have reported our progress in outlining theoretical resources that have potential to help scrutinise PISA in a way that acknowledges its weight and complexity, while providing tools to critique and confront its character. We want to explore further probable links between PISA and the kinds of rationality we have alluded to, and to make this rationality available to critical and further rational inspection. In doing so, we have consciously referred our argument beyond analyses of power and control as rendered among categories of the social and semiotic production (Bernstein's legacy), to the way government governs through the micro actions of its subjects working on themselves (Foucault's legacy). We believe that the theoretical resources offered by these legacies complement and supplement each other, and that to better understand these relationships is also to better understand the rationality of PISA. Clearly this is the work of a large agenda, not of a single paper. Our aim here has been to introduce and advance this agenda, however tentatively. As is typically the case when a certain boldness is attempted, far more questions than answers immediately surface. We conclude by mentioning just two of these: How may the levels of analysis illustrated above be related and combined to recognise the regime of rationality characteristic of globalised education? By what methods of data collection and analysis may technologies of self, in particular, be made visible in the context of PISA productions?

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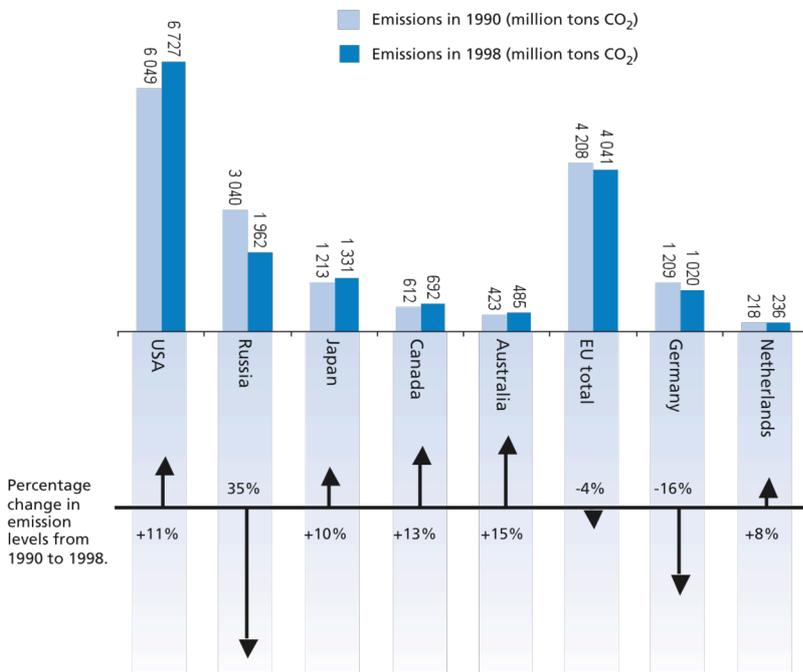
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APPENDIX: THE PISA TASK (OECD, 2006A)

MATHEMATICS UNIT 44: DECREASING CO₂ LEVELS

Many scientists fear that the increasing level of CO₂ gas in our atmosphere is causing climate change.

The diagram below shows the CO₂ emission levels in 1990 (the light bars) for several countries (or regions), the emission levels in 1998 (the dark bars), and the percentage change in emission levels between 1990 and 1998 (the arrows with percentages).



QUESTION 44.1

In the diagram you can read that in the USA, the increase in CO₂ emission level from 1990 to 1998 was 11%. Show the calculation to demonstrate how the 11% is obtained.

DECREASING CO₂ LEVELS SCORING 44.1

Full credit: Correct subtraction, and correct calculation of percentage.

$$\bullet 6727 - 6049 = 678, \frac{678}{6049} \times 100\% \approx 11\%$$

Partial credit: Subtraction error and percentage calculation correct, or subtraction correct but dividing by 6727.

$$\bullet \frac{6049}{6727} \times 100 = 89.9\%, \text{ and } 100 - 89.9 = 10.1\%$$

No credit: Other responses, including just 'Yes' or 'No', and missing.

To answer the question correctly students have to draw on skills from the connections competency cluster.

QUESTION 44.2

Mandy analysed the diagram and claimed she discovered a mistake in the percentage change in emission levels: "The percentage decrease in Germany (16%) is bigger than the percentage decrease in the whole European Union (EU total, 4%). This is not possible, since Germany is part of the EU."

Do you agree with Mandy when she says this is not possible? Give an explanation to support your answer.

DECREASING CO₂ LEVELS SCORING 44.2

Full credit: No, with correct argumentation.

- No, other countries from the EU can have increases e.g. the Netherlands so the total decrease in the EU can be smaller than the decrease in Germany.

No credit: Other responses and missing.

To answer the question correctly students have to draw on skills from the connections competency cluster.

MATHEMATICS EDUCATION, DIFFERENTIAL INCLUSION AND THE BRAZILIAN LANDLESS MOVEMENT

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The paper discusses an event involving the Brazilian Landless Movement that happened recently in the southernmost state of Brazil. Specifically, it describes the closing of the itinerant schools of the Landless Movement and the language games that constitute school mathematics and peasant mathematics of that form of life. Based on this description and the discussion about Hardt and Negri's notion of differential inclusion it is argued that school mathematics can be seen as working as a gear in the production of differential inclusion.

DIFFERENT FORMS OF LIFE, DIFFERENT MATHEMATICS

Postmodern times have been characterized by the proliferation of multiple interpretations of the social world, at the same time there has begun a “sort of suspicion of the place from which these interpretations are constructed, i.e., of the idea of reason itself” (Condé 2004:16). According to this author, from the second half of the 19th century and the beginning of the 20th, with the crisis in mathematics, the theory of evolution, the rise of human sciences, relativity theory in Physics and other movements rejecting the idea of a universal scientific rationality based on ultimate and true foundations were triggered. Using the ideas of Wittgenstein, Condé (2004:29) will say that “[...] *we need friction. Back to the rough ground* (PI §107)[1] of the social practices, and there to establish the criteria of our rationality”. Returning to the rough ground drives us to regard the Modern Project and, consequently Modern Science with suspicion. In particular, it allows us to problematize the existence of a unique and totalizing mathematics language, sustained by a specific rationality with its marks of asepsis, order and abstraction.

In his later work, Wittgenstein repudiates the notion of an ontological foundation for language. Language takes on a contingent, particular character, acquiring meaning through its different *uses*. “The meaning of a work is its use in language”, explains the philosopher (PI §43). In this way, since the meaning of a word is generated by its use, the possibility of essences or fixed guarantees for language is problematized, leading us to also question the existence of a single mathematics language with fixed meanings.

Highlighting the generation of diversified languages which gain meanings by their uses, Wittgenstein (1995) introduces the notion of *language games* as being the “whole, consisting of language and the actions into which it is woven” (PI §7). Hence, processes such as describing objects, reporting events, building hypotheses and analyzing them, telling stories, solving calculations, and others, are exemplified by Wittgenstein as language games. In aphorism 23 Wittgenstein states that language

games are part of a form of life, which leads Glock (1996:124) to highlight that the notion of form of life emphasizes the “intertwining of culture, world-view and language” or, as Condé (1998:104) writes: “The form of life is the last mooring place of language”, i.e., the meaning of the language games that institute the different mathematics and the rationality criteria embedded in them are constituted in the materiality of the forms of life in culture. Thus, academic mathematics, school mathematics, peasant mathematics, indigenous mathematics, in brief, the mathematics generated by specific cultural groups, can be understood as networks of language games engendered in different forms of life. However, these different games do not have an invariable essence which maintains them completely incommunicado from each other, nor a property common to all of them, but some analogies or relationships – what Wittgenstein (1995) call *family resemblances*.

Later Wittgenstein’s ideas, briefly presented here, are the kernel of the discussion undertaken in this paper. Knijnik (2007a, 2007b) and Knijnik et al. (2005), taking support in empirical studies performed with the MST (the Landless Movement), have shown that the language games that shape Landless Peasant mathematics bear the marks of the orality of that peasant culture. Thus, for instance, to find how much could be made available monthly, during one year, with the 900 reais[2] obtained from the sale of thirty sacks of ecological rice, *Seu Otilio* – a 64-year old peasant who only had 4 years of schooling – explained:

Seu Otilio: We tried to know how much would be over to spend every month. For instance: nine hundred reais divided by twelve. Out of one thousand two hundred to get it to nine hundred reais, you had to take a quarter out of one hundred, which would become seventy-five reais. Because you take it out of ten, you have two and a half, making up the logic of ten. (...) So, as I reckon it, in this case there would be seventy-five reais a month to buy the other things. Any person who wants to use the machine or the pen will reach this value, I’m sure. (...) When I reckon it in my head I always have to look for the best path. I always have to round it out, to look for the large numbers. The closest, simplest way is to bring it to one thousand and two hundred reais. According to this logic it would be one hundred, but it could not be one thousand and two hundred because it is nine hundred. The twelve have numbers the same size as those that form nine. The nine can be formed by three times three, and twelve, four times three. So you have to take the total and see that twelve has a quarter more than the nine as a difference. This one quarter more is what I added, so I have to take it off the hundred (Knijnik and Wanderer 2008).

As shown by the excerpt above, the language game played by *Seu Otilio* considers the orders that are most relevant to find the final value. When he was asked about the ways in which he performed such language games *Seu Otilio* said:

Seu Otilio: I always tried to get to know and practice the three kinds of ways to do mathematical sums. I always used my memory, which I place first. I have also always used the pen. I use the pen a lot to contribute and check large sums, in which one becomes very tired and have to record it. And another thing I have also used is the little machine. What I learned today [one of the mathematics classes of the Course] was to operate those memories [of the calculator] that I had never managed to get explained, so I was treading water. One would buy the little machine and only use it to add and divide. And one has to know all of them, and realizes what does not fit (...). But, actually, I can reckon very well in my head, right. I can reckon very well in my head. And I even like to. But my logical reasoning about the numbers is always in my head. I always approach; I can't switch off the reasoning for reckoning the idea, with the machine sum or of the pen. I can reckon with a pen, but I always project so many bags will give, more or less so much. I have always practiced this and I think it is very good. Because one manages to see if the sum is wrong, one can realize that it is wrong. When you reckon it by pen, or even on the machine, I can see it immediately, OK. But this is not right. Because I have already projected it this way. So what I was trying to find out is how to theorize this (Ibidem).

In *Seu Otilio's* description, different language games can be identified: those of peasant mathematics – which use the “reasoning of sums through ideas”, the language games connected to school mathematics in which he had been socialized – in his words, “pen sums”, and the games involved in using the calculator – whose further learning had occurred during the pedagogical work that we were developing. However, despite the specificities of such games, *Seu Otilio* shows that he knows they bear a family resemblance, in the sense given to it by Wittgenstein.

In the next section, based on the above empirical data it will be discussed how school and, in particular, school mathematics works through what Hardt and Negri called *differential inclusion*.

DIFFERENTIAL INCLUSION AND LANDLESS MOVEMENT MATHEMATICS EDUCATION

In his class of March 17th in the year of 1976 at College de France, Foucault (2002) goes further in the discussions about biopower, showing its connections with the mechanisms of racism. He highlights first that it can be considered as a means to insert a cutoff in life, “the cutoff between the one who is to live and the one who is to die” (Ibidem:304). Secondly, racism allows maintaining a relationship of the kind “to make people live, you must massacre your enemy”, i.e, the “death[3] of the other, the death of the bad race, of the inferior race (or of the degenerate, or of the abnormal) is what will make life in general healthier; healthier and purer” (Ibidem:305). Thus, “taking life [...] tends not to victory over political adversaries, but to the elimination

of the biological danger and to the strengthening, directly linked to this elimination, of the species or race itself” (Ibidem:306).

The arguments presented by Foucault converge with the analysis undertaken by Hardt and Negri (2003) on imperial racism. For these authors, even with the end of slavery and of the apartheid laws, it cannot be said that racist practices have diminished. On the contrary, they continue as intense as ever, but now present themselves under different forms in our society. Étienne Balibar (apud Hardt and Negri 2003:192) considers these new forms of racism as “a racism without race, or more precisely a racism that does not rest on a biological concept of race”. As discussed by Hardt and Negri (Ibidem: 213), based on Deleuze and Gattari’s theorizations, the imperial racist practice is not sustained by a theory of racial superiority in which there would be a binary division between races and exclusion processes, but by mechanisms that act as differentiated inclusion. Thus, for the authors, the point of departure is not a difference among races that can generate antagonistic blocks to separate those “inside” and “outside”, but processes that act by inclusion and subordination. In their words:

White supremacy functions rather through first engaging alterity and then subordinating differences according to degrees of deviance from whiteness. This has nothing to do with the hatred and fear of the strange, unknown Other. It is a hatred born in proximity and elaborated through the degrees of difference of the neighbor (Hardt and Negri 2003:194).

In constructing their argument, Hardt and Negri also emphasize the impossibility of saying that there are no racial exclusions, but that it must be understood that this type of exclusion “arises generally as a result of differential inclusion” (Ibidem:194). For them, it would be a mistake to consider even the apartheid laws as “the paradigm of racial hierarchy” (Ibidem:194), since the racial differences would not be absolute or of nature, but differences in degree. “Imperial racism, or differential racism, integrates others with its order and then orchestrates those differences in a system of control” (Ibidem:195).

The theoretical tools briefly presented here will be useful to analyze how closing the itinerant schools of the Brazilian Landless Movement and incorporation of the children into the urban public schools – which we will discuss below – constitute a differential inclusion process in which school mathematics will be a gear in its production.

The schooling processes performed by the Landless Movement comprise specificities that have been studied by scholars of important international research centers (Kane 2000). Among these specificities it should be emphasized that their schools of Infant Education, Primary Education, Secondary School and, more recently, Higher Education, belong to the public system of education (at municipal, state or federal levels), i.e., they are subject to official guidelines and regulations. However, due to the relative autonomy given by the Brazilian Educational System to its institutions, the MST has organized the curriculum of its schools based on pedagogical and

philosophical principles (Knijnik, Alekseev & Barton 2006) which fulfill purposes of a schooling that will serve the interests of their struggle for land reform.

As explicitly stated by Caldart (2003:62), “under pressure from the mobilization of families and teachers, the movement decided to take on the task of *organizing and articulating, inside its organicity, this mobilization [for the schooling of its members], to produce a specific pedagogical proposal for the schools achieved and to educate people who are capable of working from this perspective*”. Thus, the MST considers it a key issue that their schools should not only be located in the camps and settlements, but, mainly, that they should develop pedagogical work closely connected with the Landless peasant culture (Knijnik 2007a), with its marks of the Brazilian rural culture in its interactions with the specificities of the practices of struggle developed by the movement.

This educational perspective is followed by the work developed by MST schools and its teacher training courses (Knijnik 2007a, Lucas de Oliveira 2004) in the sphere of teaching and learning mathematics. The pedagogical practice described below (Knijnik and Wanderer, 2008) very clearly exemplifies this approach.

It was centered on a report written by a woman student who belonged to the Landless Movement National Committee (the group of elected peasants who coordinate the movement at the national level). Her report was about a march that the Landless Movement was doing at the time in a specific region of the state of Rio Grande do Sul. The march involved hundreds of peasant families who were walking along the main roads of that region in order to press the state authorities to expropriate an unproductive large-holding whose owner had been in debt to the State for a long time, to the tune of about 32 million Reais. When the discussion about her report started, she interrupted what was going on in the class, stood up and, moving from a student subject-position to a leader subject-position, in a stentorian voice, as if she was in front of thousands of her comrades, explained:

Student: This is what is going on. We have eleven thousand and six hundred families settled in [the state of] Rio Grande do Sul. Following the data given by our Production Sector, the total amount of the State debts is seventy million reais, its total, counting everybody's debts. What is the point? The point is that Senhor Sotal, the landowner himself, has a debt of thirty-two million reais. In fact, it is not thirty-two, it is thirty-seven, but let's assume thirty-two million. Then, he alone has a debt of thirty-two million. And then I have a question because it is hard to debate about this in the schools, in the communities we are visiting during our march, in the media: What is the percentage that a single farmer took of public government money compared to our debts, to the “claims” that we are making? This question was asked on the first day of the march, already on the first day, when we sat down to prepare the people who were going to talk at the schools. This question came up and we looked at each other and couldn't [answer]. Then someone

said: it can't [the answer to the question] be more or less (...) we become insecure and afraid to speak. I never managed to explain this part, then (...) reckon what is the percentage that a single landowner took from the government. (...) To give you an idea we [in the report] only took economic data. So [if we were to take] the question, what is the social result [of this situation] certainly it would call much more attention even. But [what we wrote in the report] is an economic result. (...) Let us get into the economic issue, because if we get into the social issue, it can't even be compared (Knijnik and Wanderer 2008).

Her talk was interrupted by demonstrations from the other students, applauding her. The continuity of pedagogical work had as its center the analysis of her report. This analysis was performed using some of the Landless mathematics language games, which were briefly mentioned in the previous section. Initially the group was interested in discussing mainly the economic dimension of the situation, even if its social and political dimensions were always present. As one of the peasants justified: "With very concrete data, [one] strengthens our debate, our militancy". Another student completed this saying: "It is important to take this to the march". The group consensus was that it would be important to write a text with the results of the analysis of the situation we had carried out in our mathematics class. Thus, the next stage of pedagogical work involved writing a text, which not only showed the analysis results but also highlighted the reasoning developed by the group, marked by Landless mathematics language games associated to their form of life. From the following week onwards, the text was distributed in the communities through which the march moved.

As happens in all other marches, the Landless children who participated in that specific march with other member of their families did not stop their schooling thanks to the itinerant school[4] to which they belonged.

In her major research, Camini (2009) highlights that approximately four thousand six hundred students had already attended the itinerant schools of the MST camps in the State of Rio Grande do Sul, from the time they were instituted in 1996, to the end of 2008. Even following principles and guidelines that regulate the public schools in the state, such as the requirement of 200 days of school a year, the itinerant schools, ever since they were made official, had some unique characteristics: the students and teachers were Landless people living in the movement camps; teacher training was performed in Secondary School courses and courses of Higher Education belonging to the Movement; the students entered at any time during the school year; the general organization of the schools and of pedagogical work was implemented by the teachers and by the camp community; the school curriculum was structured by stages that were the equivalent of the initial grades of Basic Education and supported by the principles of the Landless Movement Pedagogy. Additionally, the teaching materials used by students were prepared by members of the MST Sector of Education,

comprising researchers and educators connected to the Movement, according to their interests, needs and purposes (Camini 2009).

Although Rio Grande do Sul was a pioneer in the organization and implementation of the itinerant schools in Brazil, it is in this state that, since March 2009, decrees of the State Government together with the State Attorney's Office have been issued for the purpose of interrupting this educational process. Closing the itinerant schools was preceded (during 2008) by the implementation of measures pointing to this, such as delays in paying the teachers' salaries and suspension of the delivery of teaching materials to students, as happens in all MST itinerant schools around the country. Thus, in March 2009, in the State of Rio Grande do Sul, the government agencies ordered that the children be transferred from the itinerant schools to the urban public schools of the municipalities where the camps are located. It was threatened that if this transfer were not made, the students would not receive their certificates at the end of the year and their parents would be held legally responsible for their "negligence". The debate about this issue has been widely disseminated by the national (and also international) media. In one of the most important newspapers, the State Attorney said:

The [Itinerant] schools perform brainwashing. We have to guide the children about the possibility of becoming part of the world that is there, of the productive world. [...] In a civil enquiry during which several things connected to MST were investigated, one of the proposals was an *Agreement on Conduct Adjustment* with the State Department of Education, for the public school system to absorb the students from these schools. This should be done so that they will have access to the knowledge imparted to all people[5].

Statements like the one above indicate that the *Agreement on Conduct Adjustment* involves sending the Landless children to urban public schools of the Brazilian educational system, enabling a more effective control of their presence and attendance at school. Thus, the members of the Landless Movement become a target of the technology of power Foucault called biopolitics. Such technology, exercised through biopower, "takes the population as its object, as a large living body, so as to manage to govern this population in the best way possible" (Veiga-Neto 2006:35). This government puts into action control mechanisms also on the knowledges that will be taught to the Landless children. As a government authority emphasized, "Mandatory public teaching must be the same everywhere. It simply aims at ensuring that these children will have a right which is to be in an equal situation to the others"[6].

In brief, the elements of pedagogical work presented here and the discussion performed in the previous section about different mathematics, in particular about language games that constitute the peasant mathematics and its family resemblances to school mathematics, point to the specificities of teaching and learning mathematics at the MST schools and in its teacher training courses. This is a schooling process that is strongly linked to the Landless form of peasant life.

The closing of the itinerant schools in Brazil's southernmost state was an event with unique characteristics. Hundreds of children were forced to move to regular schools, most of them situated in towns linked by precarious roads to the camps in which they live. The access to those towns is by using school buses in a state of disrepair. In these schools, they will be guided "*about the possibility of becoming part of the world that is there, of the productive world*" so that *they will have access to the knowledge imparted to all people*: a teaching that "*will ensure that the children will be in an equal situation with the others*". In other words, closing the itinerant schools would favor school inclusion and, consequently, social inclusion.

However, with the support of Hardt and Negri's theorizations, we are led to state that this inclusion will be above all a differential inclusion: when the Landless children are obliged to attend urban schools, they will in fact not be excluded from the official educational processes. However, this inclusion will be permanently marked by a differentiation that will produce hierarchies and subordination. The MST struggles, its history, the Landless peasant culture, the language games that constitute what we have called Landless peasant mathematics, all this will be distant (not only geographically) from the urban school. The teaching materials used at urban public schools, as well as the training of their teachers, is also very distant from the Landless peasant form of life. The State Attorney said that the school curriculum would also enable the Landless children "*to have access to the knowledge imparted to all people*". It can easily be deduced that this knowledge is not the one from the Landless peasant form of life.

In particular, it does not include the language games that constitute Landless peasant mathematics, that are possibly unknown by the teachers of urban public schools. Thus, the language games of their mathematics will be considered spurious and, therefore, absent from the school curriculum, "*repel[ed], out of its borders*" (Foucault 2001:33). So, school mathematics will work as a gear in the mechanism of differential inclusion: the Landless children will be in the official school, they will learn "*the knowledge offered to everybody*" and, at the same time, they will position their own mathematical knowledge at a lower level. This is how differential inclusion functions: it attracts alterity, but subordinates and hierarchizes the differences. This is how the post-modern racism discussed by Hardt and Negri functions.

NOTES

1. The author refers to aphorism 107 of Wittgenstein's book *Philosophical Investigations*. Following him, throughout the paper similar notation will be used to mention Wittgenstein's aphorisms in that book.
2. In August, 2009, 1 Real was the equivalent to approximately 0.3 Euro.
3. According to Foucault (2002:306), when using the word death he is not considering "simply direct murder, but also everything that may be indirect murder: the fact of exposing

to death, of multiplying the risk of death for some, or, pure and simply, political death, expulsion, rejection, etc.”

4. According to Camini (2009:135), official MST documents indicate that the *Itinerant School* received this name because it means “a school that follows the camp itinerary until the time when the families have achieved land ownership, the settlement. Then comes another stage of the process, obviously connected to the previous one. It is the time to take the legal measures to establish the Peasant School for the sons/daughters of those who, returning to the rural area wish to continue studying, working, living. The name Itinerant also means a pedagogical position of walking with the Landless, which is a great advance in the sense of affinity between the formal schooling processes and the educational experiences and practices of an organized social movement, such as MST.”

5. Source:

www.unisinos.br/ihu/index.php?option=com_noticias&Itemid=18&task=detalhe&id=2007

7. Accessed 5 April 2009.

6. Source: www.jusbrasil.com.br/noticias/884592/mp-manda-fechar-escolas-itinerantes-do-mst-no-rs-decisao-provoca-protestos. Accessed 5 April 2009.

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DEBATING FOR ‘ONE MEASURE FOR THE WORLD’: *SENSITIVE PENDULUM OR HEAVY EARTH?*

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INTRODUCTION

A central part of mathematics curricula, almost worldwide, is an emphasis on learning about methods of ‘measurement’ from primary to secondary levels and employing them for calculating perimeter, area or volume in geometry. Broadly speaking, mathematics curricula (and therefore textbooks and teachers) refer to the use of ‘meter’ as a conventional unit that is globally accepted, without ever mentioning how and why it has been established as a commonly used unit, let alone organizing along with children an investigation into this matter.

Our aim in this paper is to discuss how a group of adolescent students has been engaged in a cross-curriculum project about the processes of establishing the ‘meter’ as a commonly used unit during the French Revolution. The potential of this particular project can be recognised at various interrelated levels: a) increasing students’ awareness of mathematics as a socio-political product; b) cultivating a stronger sense of citizenship; c) developing students’ creative and critical thinking in the realm of their involvement in social problems within a specific economic and historical context.

Mathematics and Society: opening a closed relation

In recent decades, mathematics education has been characterized by researchers’ shift toward the so called ‘social turn’ (see Lerman, 2000: 23). This social shift consists of emerging theories that analyze meaning, thought and reasoning as products of social activity. This turn indicates simultaneously a research shift that results into the study and interpretation of phenomena related to mathematics teaching and learning taking into account social, cultural and political dimensions of the wider systemic context. Aspects of mathematics and mathematics education largely ignored in the past gain today a renewed interest within our scientific community. In the realm of these research directions Ernest’s call for an urgent need to consider new philosophies is pertinent.

Ernest (2008) speaks about the need for a broader view of the philosophy of mathematics to emerge, including the relevance of a range of traditionally excluded dimensions such as: culture, values and social responsibility; applications of mathematics and their effectiveness in science, technology and other realms of knowledge and social functioning; and the learning of mathematics, its role in the transmission of mathematical knowledge, and in the formation of individual mathematicians (Mellin-Olsen, 1987, Skovsmose, 1989, 1994, Restivo et al. 1993, Ernest, 2008). In an earlier article entitled “Mathematics and democracy” Ole

Skovsmose indicated that we cannot expect any development of democratic competence in school unless the teaching-learning situation is based on dialogue and unless the curriculum is not totally determined from outside the classroom. Such ideas seem equally relevant today. Since then a number of researchers have embraced such a standpoint (Valero and Zevenbergen, 2004).

Creativity, Critical thinking and Drama in Education: articulating a debate

The use of drama techniques in education has been proposed, among others, as a context that offers innovative ways for integrating different disciplinary curriculum areas and as a creative way for students to reflect on broader learning issues (emotional, cognitive, social, political). Drama in Education (DIE) can be a highly structured pedagogical procedure utilizing specific rituals and techniques of dramatic art aiming to focus participants' attention towards *the process* of participants' experience and not on the final product (Alkistis, 2000). It potentially constitutes a teaching approach that can embrace collaborative, active learning through experience, while giving participants the opportunity to develop acceptance, understanding, creativity, curiosity, self-consciousness, self-esteem (Alkistis, 1998; Wagner, 1999; Wooland, 1999).

The research results concerning the effects of Drama in teaching and learning Mathematics on students' understanding and retention of mathematical notions (Saab, 1987; Omniewski, 1999; Fleming et al., 2004; Duatepe, 2004) and in creating a positive impact on their attitudes towards mathematics (Kayhan, 2008, Duatepe, 2004) are very encouraging.

In an earlier study, Chronaki (1990) interviewed experienced maths and drama teachers who were involved in either long- or short-term cross-curricular projects aiming to teach mathematics and drama techniques. They all mentioned that, despite time and curricular constraints, drama techniques can provide the means to motivate children for active participation in mathematics problem solving. Being inspired by the work of Heathcote (1984), Way (1967) and Bousquet (1986) and confronted with the rigid structure of a maths curriculum reform in the early 80s, as well as specific needs for mathematical numeracy emphasised by the well known Cockcroft report, some maths and drama teachers in the UK context of the late 80s turned towards exploring innovative cross-curricular connections in specific cross-departmental co-operations. Their inner motivation was mainly geared towards battling children's demotivation and indifference for an abstract maths curriculum that stresses uniformity and fails to develop spaces for creative and critical thinking (see Chronaki, 1990 for more details).

Specific techniques such as the '*as-if*' create the context for teaching a concept, an idea or an event and offers opportunities for exploring mathematics in a variety of historical, social or cultural contexts. '*Teacher and students in Role*' is another basic drama technique -often supplemented with the use of open public '*debate*'. The use of '*debate*' in teaching can facilitate both knowledge acquisition and critical thinking

(Huber, 2005; Snider, 2006). Furthermore, participation in activities of this type familiarizes children with a holistic handling of issues, makes them able to justify their point of view, practices their mental flexibility and alertness, and enhances their ability of self-criticism and their conciliatoriness. Our project named '*Sensitive pendulum or heavy earth*' relies upon utilizing some of the above mentioned techniques with an emphasis on the dramatization of a public debate concerning the decision-making process of 'one measure for the world' that took place at the National Assembly during the French Revolution.

Historical events

By the end of the 18th century the diversity of weights and measures in France—about 2000 in total—was held responsible for great problems in economical dealings as well as the exploitation of people by feudal lords. This was because the majority was illiterate and thus incapable of making conversions amongst different measuring units. Due to the aforementioned reasons standardization of weights and measures was one of peoples' basic demands. The establishment of new units constituted a political decision. From the first year, the French National Assembly voted for uniformity in measurement units and sought new ones. In 1790, the French National Assembly accepted the definition of the meter as the length of a pendulum that has a period of 2 sec at latitude of 45°, and asked the Academy of Science to propose the base of the metric system. The Academy responded to this request and recommended a decimal system of measurement. In 1791 a committee of the French Academy of Science—Lagrange, Laplace, Condorcet, Borda and Monge—suggested that the new definition for a meter be equal to one 10 millionth part of the quadrant of the terrestrial meridian between Dunkirk and Barcelona and this was accepted by the National Assembly. The unit was given the name "meter" in 1793.

THE PROJECT: 'SENSITIVE PENDULUM OR HEAVY EARTH?'

The project was carried out in the 2nd State Lyceum of Ilion in Greece, in a low socio-economic area of Athens, during school year 2007-8. During this project, a role-playing debate was organized, which was a simulation of the confrontation in the French National Assembly. This particular debate entitled "The sensitive pendulum or the heavy earth?" was carried out twice with 11th grade students of two different classes. This confrontation concerned the choice of a length measurement unit, through the two aforementioned different approaches. The topic of the debate was selected to show students the confrontation, the protagonists and the historical context within which the unit 'meter' was introduced in order to help them comprehend that scientific theories are the result of both intellectual and social interaction.

Preparing the 'debate', seven teaching hours were required over a period of three weeks. In history-class, a powerpoint presentation concerning the establishment of the meter helped students to realize the problems stemming from the use of many

different units of measurement. Extracts from a documentary film about the French Revolution introduced them to the ambience existing before and during the first two years of the French Revolution. During the subsequent discussion, pupils started pondering why the choice of weights and measures by feudal lords was a privilege for them. They finally found out that the increase of size of the unit of volume resulted into an increase of taxes. The role of scientific unions concerning their decisions not only in scientific subjects but also in social and political ones was discussed.

During the language-class, the teacher helped students translate and understand texts concerning the origin and life, as well as the ethical and political role of scientists and other historical issues emerging in this revolutionary period.

The students were divided into 6 groups and had to locate their arguments for preparing a 'debate', reading extracts from Denis Guedj's book "Le metre du monde". Each group chose a representative to take part in the debate. The teachers in charge were present all the time to support the teams in their work. Students' ability to argue about the choice of the pendulum is directly correlated with their comprehension of its governing laws. Therefore, the students were taught the pendulum principles in physics- class and were also familiarized with the notion of the pendulum isochronism phenomenon. In the debate, in order to create a suitable atmosphere, simple settings and costumes were used. In order to find out the way the members of the Assembly were dressed and how they spoke, the children watched David's famous painting 'The Oath of the Tennis Court' and dramatized scenes from a documentary with Robespierre speaking in French Assembly.

In the debate all students adopted the role of a responsible citizen, member of the French Constituent Assembly, who had to take decisions about crucial matters, in this specific historical context. Certain students played the role of historical figures as Talleyrand, Bailly, Prieur, Condorcet, Borda. Prieur spoke in favor of establishing new units, Borda presented the proposition of the French Scientific Academy, while Talleyrand spoke in favor of the pendulum and Condorcet in favor of the meridian. Some students, as members of the Assembly, spoke either for the pendulum or for the meridian. During the debate students participated vividly either acclaiming or disapproving the speakers. Before reading the real decision of the French Assembly, all of them- in the roles of citizen members of the assembly- voted in favor of the meridian as the most appropriate unit of measurement, without knowing the real decision of the French Assembly.

PRELIMINARY ANALYSIS: QUESTIONNAIRES AND VIDEOTAPES

The project has been evaluated through the analysis of students' answers to a questionnaire and of particular episodes of the process as they were captured by means of videotaping the debate performance.

The reason for distributing a questionnaire was primarily to identify whether the project had any impact on students' knowledge concerning measuring units. The majority of students, even a month after the debate, answered correctly to questions involving the establishment of the meter and the laws of the pendulum. The students emphasized that this activity engendered the development of critical thought, familiarization with a more rounded approach, active participation and cooperation with each other. More importantly than what they learned, they emphasized the way in which they learned; they pointed out that they all took part, worked together, took the initiative and felt enthusiastic. Furthermore they liked the seriousness with which the debate was carried out; the good organization and the role play which made them believe that they were really taking part in the French National Assembly.

A preliminary analysis of the videotaped debate performance permitted us to identify a number of episodes related to processes of knowledge construction and values building such as: discussion of mathematics and physics; changing stereotypic images of mathematics; training skills in argument; and gaining a sense of becoming responsible citizens. Each of these is briefly analysed below:

Discussion about mathematics and Physics notions

Students discussed about arbitrary and conventional measures, connecting the conventional unit of length measurement with justice. From their arguments in the debate we concluded that students had understood the laws of the simple pendulum as they referred in their argumentation to the dependence of the period of a pendulum on π and on g , the acceleration due to gravity.

Changing the image of the nature of Mathematics

Instead of having a conventional lesson, in which the emphasis is in solving standardized and decontextualized problems, students faced a problem, not only mathematical but at the same time social and political. The social conditions were presented as decisive for taking decisions, while at the same time the ethical responsibility of scientists as an active member of society came to the fore.

Training in argumentation skills

Both sides had to prepare strong arguments to support the definition of the meter. The team in favor of the pendulum used the argument that the pendulum was a rational solution, simple, cheap in construction and easy and functional in use. The second team claimed that the definition of the meter as a part of a meridian was a global solution, not easy and cheap, yet accurate and reliable, as it was not dependent on approximate values of the numbers π and g .

Fostering the sense of citizenship

Through the whole procedure, the value of public and responsible dialogue was brought out. During the introduction of the debate, the necessity of innovation 'in order for the people to stop being victims' was underlined. Also, the exploitation of

people by feudal lords through the use of arbitrary units of measurement was also emphasized by the students. The need for release from kings' units was emphasized, as these were local and also defined in an arbitrary way. Students associated the common meter with human rights while both teams argued that the 'meter' had to be defined in a way that could be understood by every citizen.

SOME CONCLUDING REMARKS

According to Ernest (2008) the adoption of mathematics as a cultural construction, as much from a historical perspective as from the perspective that examines knowledge in relationship to the context, can endow a human element to school mathematics once more. We believe that with all the aforementioned activities this aspect of mathematics as a socio-political 'tool' came to the fore. With this role-playing the central role of the historical and social context in the ways mathematics could be utilised was brought to light and students were able to experience this dimension of mathematics, not only mentally but emotionally and physically. We believe that such projects can help to achieve the major goals of education, *i.e.*, responsible creativity and an ethical [world] citizenship.

However, although maths teachers may start their involvement with dramatic techniques in the context of cross-curriculum co-operations with enthusiasm, such attempts soon seem to arrive at dead-ends given the time and space constraints created by a rigid demand for content coverage and national assessments. Teachers in Greece in the early 21st century, like teachers in the UK in the early 80s –and perhaps today- are faced with the same need to motivate students' engagement with mathematics and to support them towards 'changing' their image of a subject that suffers stereotyping through varied social terrains. As a maths teacher heavily involved with drama techniques in the UK claimed '*..children in drama are collaborators*' and '*..gradually trust themselves and their own abilities*'. However, the same teachers did not continue to utilise drama in their maths classes as they become prevented due to time, resource and energy requirements for cross-departmental co-operations (see Chronaki, 1990). This very fact denotes a paradox: *on the one hand*, drama techniques aid students' motivation and conceptual understanding of mathematics, and *on the other hand*, broader curriculum structures (such as guidelines and assessment) prevent teachers, even experienced ones, from utilising them as a legitimate resource on a continuous basis.

Chronaki (2000), based on the analysis of two teachers employing art-based activities in their maths classrooms, referred to such paradoxes as teachers having to deal with and bridge hegemonic and often conflicting discourses about what mathematics teaching should involve in a school classroom and about what might be the place and role of art related themes. It is obvious that such issues are rooted in broader debates concerning links amongst art and science related discursive practices. Since today there is renewed interest in revisiting this long-lasting bridging of art and science (e.g. Presmeg, 2009) there is a need to reconsider this line of research.

NOTES

D. Koutli, teacher of Physics, and teacher-librarian and N. Apostolopoulou the head-master of the school have also participated in the project.

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FABRICATION OF KNOWLEDGE: A FRAMEWORK FOR MATHEMATICAL EDUCATION FOR SOCIAL JUSTICE

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This essay is meant to spark discussion that seeks to pragmaticize the ideals of teaching mathematics for social justice. I wish to build a framework through which teachers can make decisions about the planning for content, pedagogy, and assessment of students' mathematics. The framework attends to three charges for a mathematics education for social justice: to attend to access, authority, and action. The constructivist embracing of knowledge as fabrication, rather than as truth, creates this sort of space for work toward mathematical education for social justice. The notion of curriculum is fundamentally altered, teacher's pedagogical decisions have great potential to be non-authoritarian, and assessment becomes a regular part of the ethical cycle of interaction.

Constructivism, as a theory for knowing and learning, has brought over the past three decades, a renewed wave of reflection and discussion about what it means to know mathematics, how does one teach mathematics, what are the goals of teaching mathematics, and even questions of what is mathematics. Constructivism's primary shift from the established behaviorist psychology of learning was to embrace the idea that the thinking mind could be considered, or at least modeled, while behaviorism restricted itself to considering only observable behaviour (Glaserfeld, 2007; 1995). The behaviorist orientation left the mind as a black box, and studied the inputs and outputs, while the constructivist set out to create models for what might be going on inside that black box. Constructivism embraced the learner as an active agent upon the world, rather than a passive recipient of the world. With slightly greater detail, the constructivist learner was imagined to either assimilate or accommodate the attended to perceptions of the experienced world.

Mathematics education embraced the constructivist view, as seen in promotion of the child as an active learner evident in policy documents of the early 1990's such as United States' National Council of Mathematics Teachers' *Curriculum and Evaluation Standards for School Mathematics* (1989) and those of other countries including Israel, Japan, China, Egypt, Canada, and South Africa (Malloy, 2002). Yet tensions remained about the status of knowledge the constructivist viewpoint suggested, that as a constructed way of knowing the experiential world, the truth of such constructed knowledge was in no way determinable (Glaserfeld, 2007; 1995).

The early 1970's marked a countercultural swing in Western cultures, during which constructivist ideas for education took seed (Papert, 1980; Piaget, 1970; Vygotsky, 1978; Wittrock, 1974), libratory and democratic movements in education found voice (Freire, 2002/1970; Illich, 1971; Kozol, 1972), and postmodern deconstructions of truth, power, and knowledge (Feyerabend 1975; Foucault, 1982/1972) emerged. That

similar goals for education emerged from each perspective, is unsurprising. Viewing children as authors of knowledge and to imbue the child with such authority, embraced the postmodern notions of power relations. Yet, save for the early levels of schooling, institutionalized education seems lost on how to proceed in a post-knowledge world. The unwillingness to relieve mathematics education from the encumbrance of a ontological existence to mathematics, the Platonic sense of truth, Erdős' "Book of Mathematics", has allowed for the unjust stratification of students that is at present the great challenge to cries for Mathematics for All, and other cries for equitable educational outcomes. The privileged knowing ascribed to certain people would not be possible if all learners were conceived as constructors of mathematics and/or mathematical ways of knowing the world.

This essay will go forth from this strong position that takes knowledge as constructed and thus embracing a new politics of truth, to create a 3-pronged orientation to teaching mathematics for social justice and then to consider the work of teaching in order to devise a pragmatic framework through which to enact a mathematical education for social justice.

MATHEMATICS EDUCATION FOR SOCIAL JUSTICE

Although there have been multiple definitions for what it means to teach mathematics for social justice (cf. Gutstein, 2006), here I suggest three cornerstones that help shape the enacting of teaching for social justice, in particular that each prong must be considered: *access*, *authority*, and *action*. For me, social justice in mathematics education does not end with greater *access* to mathematics or to education, or the larger culture. The notions of *authority* for knowing and the confidence and compulsion to *act* are of equal status when devising a notion of mathematics education for social justice.

To elaborate, I draw upon constructivist tradition to recognize a *children's mathematics* (Steffe, 2004), that I as a teacher assume a student to have constructed, the mathematical activity I attribute to the child. For the sake of the remainder of this paper, I will refer to such mathematics as lower case (m)athematics. Furthermore, *mathematics for children* are an adult's ways of knowing and operating, which are drawn upon in order to hypothesize a zone of potential construction for directing interaction with a child. Although still always a constructed knowledge, we as teachers treat this sort of mathematics, that which appears in textbooks and curriculum guides and standards documents, "the race-expression embodied in that thing we call curriculum" (Dewey, 1902, p. 31), as what is to be learned in the classroom. This particular mathematics, a mathematics for children, will be referred to with an upper case (M)athematics.

This distinction allows for further discussion of access, authority, and action. The notion of access is fully about (M)athematics. This privileged power/knowledge, an enlightenment era relic, retains a magnificent standing as a gateway to the cultural

capital that schools are directed to deliver. Gutstein (2006) noted that a teaching goal for mathematics must embrace this potential to read the (M)athematical *word*, quite similar to his teaching goal to succeed academically in the traditional sense. Gutstein extended this argument that mathematics education should embrace the goal to read and write the *world* with mathematics; however he did not note the constructivist distinction in mathematics as I have brought forth here.

To recognize the child both *writes the word* of (m)athematics and *writes the world* with (m)athematics is fully imbuing the learner as an author of their experiential reality, the second key notion for teaching mathematics for social justice. The child is an author of (m)athematics, and an actor upon the world with their (m)athematics. To both attribute this authority to the child, as well as foster the child's own awareness of this authority is the deference of power the constructivist epistemology allows for.

This shift in authority of knowledge justifies more simply the need to act upon society, the call for social action that underlies Gutstein's (2006) theory. That one does author knowledge, mathematical or otherwise, places the knower at the foreground of the world that unfurls in front of them. We know the world, the experiential world of constructivism, through our interactions with it. Inasmuch, we have a role in shaping that world. Through our (m)athematics, we act upon the world. To engage students in reflection, discussion, and decision on intentional acts and non-acts upon the world engages them in the ethics of determining and enacting what is fair, a fundamental activity of social justice. That children understand their role in authoring (writing) the world, and their decisions on how that authoring shapes the world, speaks to the third component of social justice education, action.

THE WORK OF TEACHING

In this first cut at creating a pragmatic framework for teaching mathematics for social justice, I simplify the work of teachers to defining curriculum, determining their ways of acting in the classroom—pedagogy, and planning for assessment activities. Constructivism helps distinguish there are two sorts of curricular goals in mathematics education. The first can be thought of in some ways as historical study, that there is a particular (M)athematics to be learned. Secondly, the constructive activity of the learner, that activity that we, as teacher-observers may deem mathematical, must also be developed. Here, one could say there is both a need to teach the child and to teach the discipline.

To intentionally raise awareness of mathematical authority and disperse authority among students (Cohen, 1994) are significant pedagogical moves of the teacher for social justice. And rather than assess to determine what the child cannot do—a orientation toward deficiency (Lee, 2003), assessment must have as its purpose the goal to build models of what a child knows and can do, the constructivist's *mathematics of children* (Steffe, 2004). Such practice allows the teacher to make

productive decisions “*to determine the environment of the child, and thus by indirection, to direct*” (Dewey, 1902, p. 31).

The teacher assesses in order to direct, even if by indirection, the child, that a consciousness of this mathematical interaction may make possible for the child to assert his present powers, exercise his present capacities, and realize his present attitudes (Dewey, 1902). So the mathematical development of the child—children’s mathematics—is never known before it “appears” in interaction, and then only emerges as mathematics of the child. Dewey’s concluding observation, “The case is of Child” (p. 31) is then to say; there is no getting around or free from the child. It is she who makes the mathematics she learns. I take this constructivist orientation to be my underlying premise for a socially just mathematics education.

A FRAMEWORK TO TEACH MATHEMATICS FOR SOCIAL JUSTICE

In sum, the postmodern, post-epistemological, post-knowledge framework for a mathematics education draws upon the demand for attention to access, authority, *and* action. The constructivist perspective redefines what access might be, repositions authority and authorship, and closely binds the embrace of social action as inherent in each of these first two cornerstones.

Students are learners who fabricate knowledge, where fabrication is taken to mean build, design, construct. Although the field of mathematics education seemingly has embraced the constructivist notions of the active learner and the constructing mind, it is most certainly a “softer” (Larochelle & Bednarz, 2000, p. 3) constructivism enacted in schools. The modernist truth agenda remains in place in schools and other educational structures. While student’s points of view may be increasingly valued in policy documents and elicited in the classroom, such elicitation only serves to determine what is “wrong” about a student’s point of view. Wrong, used in this manner, to mean from the perspective of a pre-existing knowledge, a truth-regime, something that is to be taught. In this soft version of constructivism, the fabrication of knowledge takes on a different meaning; it is a concoction, an invention, a forgery. In essence, the soft constructivism encourages a perspective toward the learner as to be one who constructs untruths, who fabricates lies.

The aforementioned political and social ramifications for a constructivist view on learning, and the related constructed view of knowledge, has yet to be enacted in the mathematics classroom, nor taken seriously when conceiving of the activity of or goals for mathematics education. Treating children as fabricators of knowledge, as little liars, may in fact be a greater injustice to the learner than teaching with the intent to deposit knowledge into the account of the knower, paraphrasing Freire’s (2002/1970) banking model for teaching and learning. In the present model for teaching our adolescent fabricators, we engage them in activity, engendering them with a momentary belief that we are truly interested in what they are thinking about their world. And then we tell them how it is, how it should be, how they should have

figured, how they should think. We not only continue to act in accordance with a belief that language may somehow transmit knowledge, of course an illusory notion (Glaserfeld, 1998), but we seem to enforce the modernist knowledge-as-truth agenda onto the adolescent learner.

When unquestioningly engaged in this epistemology of soft constructivism, we treat the learning activity as a process of discovery, holding tight to a knowledge that is to be discovered, listening *for* (Davis, 1997) cues to hear in the child our own ways of knowing this knowledge. The pedagogical practices of the teacher devolve to a guess-what-I'm-thinking state; the pressure of time and the testing of this pre-existing knowledge drive the maddening process of an education that began with a hopeful premise—that children make meaning through active engagement with their experiential world, that children are knowledge constructors, fabricators.

If the radical epistemology of constructivism is embraced and the fabrication of knowledge is recognized not as a construction of untruths but as other truths, a different mathematics education must be conceived. Such a mathematics education would mature from this postmodern epistemology of radical constructivism, and its concordant poststructural concept of power/knowledge (Foucault 1982). Such a mathematics education would be ripe to more powerfully embrace the socially just calls for access, authority, and action.

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“I WAS THINKING THE WRONG THING” / “I WAS LOOKING IN A PARTICULAR WAY”: IN SEARCH OF ANALYTIC TOOLS FOR STUDYING MATHEMATICAL ACTION FROM A SOCIO-POLITICAL PERSPECTIVE

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This paper focuses on the theoretical and methodological challenges of keeping the mathematical action of students in view when conducting research from a socio-political perspective. I present a theoretical perspective and associated analytic tools that are structured by the work of Fairclough in critical linguistics, but have been supplemented with the work of Morgan, Moschkovich, Sfard and Valero in mathematics education. I illustrate the use of the tools with data from a study investigating student action when solving problems with real-world contexts in an undergraduate mathematics course.

INTRODUCTION

Valero and Matos (2000) argue that the use of social, political or cultural approaches to mathematics education research, which often draw on theoretical perspectives in other fields of social science, enable us to engage with and understand aspects of mathematics education that are not necessarily offered by traditional psychological perspectives. Adler and Lerman (2003, p. 445) frame the choice of approach as an ethical one, and part of getting the description “right”. They argue that certain questions cannot be asked or answered when the “zoom of the lens is tightly on mathematical activity”.

Embarking on a research study related to my teaching at a higher education institution in South Africa, I was attracted by the possibilities suggested by adopting what I understood at the time to be a socio-political perspective of mathematics education and of research. In fact several features of my teaching and research practice suggested that I had no alternative. I was teaching (and at the same time wanted to research) an undergraduate mathematics course specifically designed for students identified as being disadvantaged by the enduring inequitable system of school education, with the aim of providing these students with access to and success in higher education. The unit of analysis was to be student action as they worked collaboratively to solve problems with “real-world” contexts [1]. My aim was not only to identify and describe the enabling and constraining discursive actions, but also to explain these in the light of the socio-political practices of the classroom and of the wider socio-political space.

Yet in advocating for perspectives that take into account the social, political and cultural aspects of mathematics education, Valero and Matos (2000, p. 398) acknowledge the “dilemma of mathematical specificity”; they note that such

perspectives can be regarded as “non-mathematical” in the sense that going “deeply” outside of mathematics results in the mathematics tending “to vanish or to be questioned”. Sierpinska (2005, p. 229) warns that such perspectives run the risk of “discouraging the mathematics away”. This dilemma is both a theoretical and methodological issue; since these perspectives draw on other fields, the appropriate analytic tools to study the mathematical content (or what I refer to as *the action on mathematical objects* in this paper) may not have been developed. For example, Sfard (2000, p. 298) notes that while discourse analysis has been used to study the “rules and norms constituting mathematical practices”, little attention has been given to using the method for the study of mathematical content and in particular for studying mathematical objects.

In planning my study I selected the work of Fairclough (1989; 2003; 2006) in critical linguistics on the strength of its potential for linking the micro socio-political activity of the classroom with wider socio-political practices. Fairclough’s method for critical discourse analysis proved productive in studying students’ positioning and the nature of their talk, yet as my study progressed I sensed that something about the mathematics itself was enabling and constraining the students’ work, and my tools were not allowing me to view this. I was “looking in a particular way”, and getting the description “right” in Adler and Lerman’s (2003, p. 445) terms required that I further develop my way of looking to allow me to bring the student action on mathematical objects into view, while not losing sight of the socio-political nature of this action.

In this paper I present the theoretical perspective and associated methodology that have emerged after an extended interactive process, working between my empirical data and my reading of the work of Fairclough (1989; 2003; 2006), Morgan (1998), Moschkovich (2007), Sfard (2000, 2007) and Valero (2007: 2008). I then illustrate the use of my tools on a selected piece of data from my study.

THE STUDY

The study is located in a first-year university access course in mathematics at a South African university. This course forms part of an extended curriculum programme designed for students identified as disadvantaged by the schooling system. After six weeks of the academic year, this group of students is joined by those students who are performing poorly in the mainstream first-year mathematics course.

The micro-level data for the study is in the form of transcripts of video-tapes and students’ written work; two groups of students were video-taped as they worked on selected real-world problems in the regular weekly afternoon workshop (see for example the “flu virus problem” in Figure 1). The students had access to a tutor and resources such as course notes. An extract from the worked solutions, provided to students a few days after the workshop is given in Figure 2. I transcribed the video-footage to represent both the verbal and non-verbal action of the students.

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

- Draw a rough sketch of the graph of P as a function of t , clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**
- What are the units of $P'(t)$?
- What does $P(4) = 1200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)
- What does $\frac{P(7) - P(4)}{7 - 4} = 350$ mean in practical terms? Give the correct units.
- What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.
- What is $\lim_{t \rightarrow \infty} P(t)$? Give a short reason for your answer.
- What is $\lim_{t \rightarrow \infty} P'(t)$? Give a reason for your answer.

Figure 1: “Flu virus problem”, Question 6, Workshop 10, 2007 Resource Book, p. 54

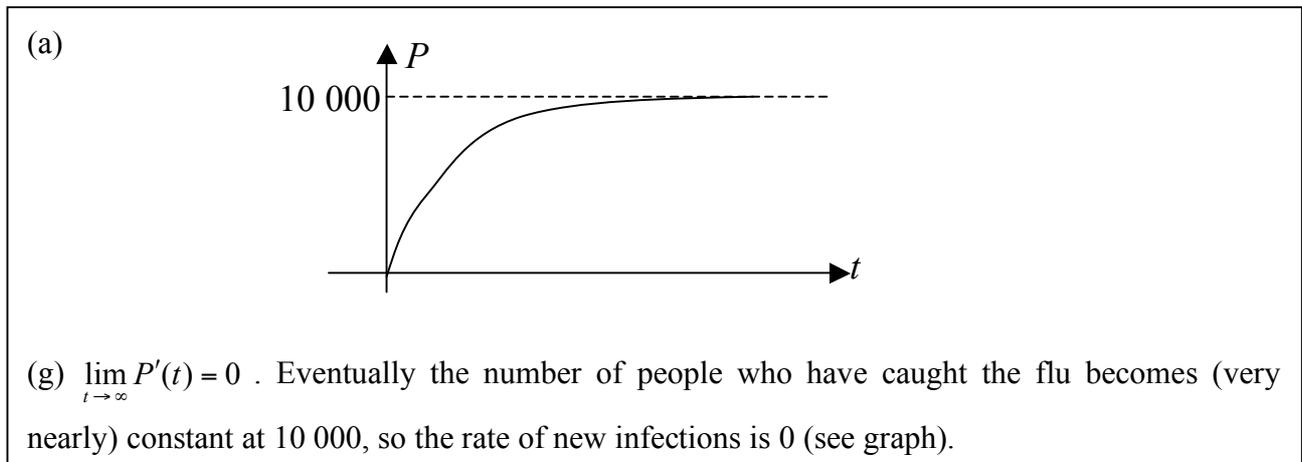


Figure 2: Selected solutions to the “flu virus problem”, Questions 6(a) and (g) [2]

THEORETICAL FRAMEWORK: A SOCIO-POLITICAL PERSPECTIVE OF MATHEMATICS EDUCATION

According to Fairclough (2003), a *social practice* is associated with certain activities, participants, social relations, objects, position in time and space, values and discourse. Any institution or organisation is characterised by a particular *network* of social practices, a network that is constantly shifting. Drawing on the work of Valero (2007) and Moschkovich (2007) I use the concept of *mathematical discourse practices* which differ “across communities, times, settings and purposes” (Moschkovich, 2007, p. 27) as a broad term for the network of social practices in

which mathematics teaching and learning is given meaning, for example, school or undergraduate classroom mathematical activity, teacher education, etc.

My reading of the work on mathematical discourse by Morgan (1998), Moschkovich (2007) and Sfard (2007), together with my empirical work, has led me to conceptualise mathematical discourse practices as characterised by certain *ways of acting mathematically*; ways of talking/writing/representing, ways of attending (of looking and listening), ways of making links and establishing relationships, ways of arguing, ways of evaluating, ways of interacting socially and discursively, and ways of identifying oneself and others.

This conceptualisation of particular ways of acting mathematically that may differ across mathematical discourse practices points to why these practices are not only social, but also political. Drawing on the work of Foucault, Valero (2007; 2008) argues that power is distributed when people participate in social practices, and she defines *power* as the capacity of people to position themselves in relation to what is valued in the practices. So power can be seen to manifest in mathematics classroom activity in two ways. Firstly, this activity is embedded in a network of socio-political practices, practices in which particular ways of acting are valued. Secondly, it is through the interaction of participants in the classroom that power is (re)produced.

The socio-political perspective of learning proposed in this paper draws on a socio-cultural perspective of learning which views learning mathematics as coming to participate in the discourse of a community that practises the mathematics (e.g. Sfard, 2007). Since mathematics education is inherently political, becoming a participant not only involves grappling with the content and skills of the community, but also determining what ways of acting mathematically are valued and negotiating one's identity and position in that community.

METHODOLOGY

In this section I explain the structure of and use of the analytic framework presented in Table 1. This framework derives its overall structure from the work of Fairclough, but is supplemented with work by Morgan (1998), Moschkovich (2007) and Sfard (2000; 2007). Fairclough (2003, p. 26-27) identifies three ways in which meaning is constructed by text and these are represented in column 2 of Table 1. Firstly, *representation* refers to how the text represents the classroom mathematical activity, for example, the ways of representing, making links etc. *Action* refers to how text enacts relations between participants and between other texts. Thirdly, *identification* refers to how text identifies people and their values. These three meanings are not distinct, but are separated for analytic purposes only.

Identifying these three meanings in the text involves a detailed line-by-line analysis of the transcript. The tools that I am using are summarised in column 3 of Table 1, and I explain their use with reference to line 482 of Transcript 1. Six students (Jane, Lulama, Darren, Hanah, Shae, and Jeff [3]) are solving question (g) of the flu virus

problem shown in Figure 1. By the time the Tutor joins the group the students have agreed that the limit does not exist (lines 472 and 473, Transcript 1). In line 479 the Tutor suggests that the students should be using their graph of the function P , constructed in question (a) (see Figure 2). Shae responds, “No but that’s of, that is not of the dash” (line 482).

Table 1: Analytic framework

Level of socio-political practice	“Meaning” of the text	Identifying features of the text
<i>discourse</i> as a relatively stable way of representing	<p><i>Representation</i>: What ways of acting mathematically are included / excluded / given significance?</p> <ol style="list-style-type: none"> 1. Ways of attending 2. Ways of making links 3. Ways of arguing 4. Ways of evaluating 5. Ways of talking and writing 6. Ways of representing 	<p>Focal analysis:</p> <ol style="list-style-type: none"> 1. <i>attended focus</i> 2. <i>pronounced focus</i> 3. <i>intended focus</i> <p>Critical discourse analysis: For</p>
<i>genre</i> is a relatively stable way of acting communicatively	<p><i>Action</i>: What action is the text performing in constituting relations (both social and textual)?</p> <ol style="list-style-type: none"> 1. Ways of interacting socially 2. Ways of making textual links 	<p>example, naming, pronouns, reference relations, mood,</p>
<i>style</i> is a relatively stable way of being	<p><i>Identification</i>: How does the text identify people, and their attitudes and values?</p> <ol style="list-style-type: none"> 1. Ways of identifying oneself and others 	<p>modality</p>

- 472 Tutor: Okay, I see two answers saying that ... P dash ... t as t tends to infinity is ... not defined or [does not exist] ((*He stretches across and points to Shae’s Resource Book*))
- 473 Shae: [Ja, it does not exist] ((*He looks up at the Tutor who is standing at his shoulder*))
- 474 Tutor: Okay, [[well]] do you all have that?
- 475 Jeff: [[Because it’ll]]
- 476 Lulama/
Hanah: Ja ((*Lulama, Darren and Hanah nod their heads*))
- 477 Tutor: What is your reasoning behind that?

- 478 Jeff: Because the graph ... it is s ... such a steep graph that it's tending more towards infinity ... than ... ((*All the others look at Jeff and then at Tutor, who has got down on his haunches next to the desk*))
- 479 Tutor: Okay ... well can I, where is the graph?
- 480 Jeff: Do we have to go and draw it? ((*He turns his page back*))
- 481 Tutor: No ... you have already drawn it
- 482 Shae: No but that's of, that is not of the dash ((*He looks at the graph for question (a) in his answer book*))

Transcript 1: Group 1, Question 6(g), lines 472 to 482 [4]

Sfard (2000; 2007) motivates for the use of *focal analysis* as a tool by arguing that since mathematical objects are abstract entities with no concrete referents, we use language and representations to talk and write about them. It is hard, therefore, to distinguish the object itself from the language and other forms of representation. Sfard provides three tools for studying the *discursive focus* of mathematical activity.

Firstly, the *pronounced focus* refers to the words the student uses when identifying “the object of her or his attention” (Sfard, 2000, p. 304). In line 482 Shae pronounces, “That’s of, that is not of the dash”. Having identified the pronounced focus I am able to do a critical discourse analysis of the words, for example, I note that Shae gives negative feedback (“No”) in the form of a description of the two graphs and he names the derivative using the symbol used to represent it (“the dash”). For this critical discourse analysis I use a list of textual features suggested by Fairclough (1989, 2003), but supplemented with work by Janks (2005), McCormick (2005) and Morgan (1998). Secondly, the *attended focus* is what the student is “looking at, listening to” when speaking (Sfard, 2000, p. 304). In line 482 Shae is attending to the Tutor’s reference to the graph in line 479, to the actual graph he has drawn for question (a), and possibly the visual image of the graph of the derivative that he and his classmates have been using. Lastly, the *intended focus* is what Sfard (2000, p. 304) describes as “the whole cluster of experiences” that are evoked by the pronounced focus and the attended focus. I use the pronounced and attended foci as clues to identify what meaning the speaker may be making; in line 482 it seems that Shae is arguing, “The graph for question (a) is the graph of the function P and not the graph of the derivative function P' .”

Column 1 of Table 1 is what allows me to link the micro-level classroom activity with the macro socio-political practices. Fairclough (2003, p. 28) states that *discourse, genre* and *style* (as given in column 1 of Table 1) are relatively stable ways of representing, acting, and identifying respectively, that operate on the level of socio-political practice. By asking which discourses, genres and styles are articulated in the text it is possible to make a link to these wider socio-political practices.

A SAMPLE ANALYSIS

In this section I present an analysis of the action of the group of six students (Jane, Lulama, Darren, Hanah, Shae, and Jeff) as they solve question (g) of the flu virus problem. I describe the action with reference to the socio-political practices of the classroom, and attempt to link this action to wider socio-political practices. I support my argument with quotes from Transcript 1 and also draw on my knowledge of the wider set of data.

As represented in much of the transcript, Shae and Jeff work ahead of the other four students. Shae looks at the derivative $P'(t)$ in the limit expression, links this expression to the task context, and identifies the limit with the maximum value of the derivative function; he pronounces that the answer is 10 000 “cause it could be 10000 people that catch it per day that would be the maximum amount” (lines 346 and 347). This link between the limit and the maximum value of the function is regarded as a common notion of the limit (e.g. Cornu, 1991), thus identifying this action with that of other undergraduate mathematics students.

Instead of making a link to the graph in question (a) as is valued in the worked solutions (see Figure 2), Shae and Jeff attend to the graph of the derivative. This is confirmed in lines 479 to 482 of Transcript 1 when there is confusion about which graph is being attended to and Shae explains the difference. Jeff proposes a vertical straight line, the “steep graph” referred to in line 478 of Transcript 1 (he demonstrates the graph in the air with his hand), a decision that may be cued by Shae’s argument in line 346 that 10 000 people can catch the disease in one day.

The graph proposed by Jeff becomes the graph that the students attend to from this point, and the other students do not interrogate his representation of the derivative. I have identified three possible explanations for the absence of any interrogation of this representation. Firstly, it is possible that the use of gesture to represent the graph as opposed to a physical drawing, a common feature of the data, may prevent the other students from focusing on the representation. Secondly, the easy acceptance of Jeff’s proposal may be linked to the promotion of co-operative group work as a genre in school mathematics and in the course itself. Adler (1997) suggests that a participatory-inquiry approach, in which students work together and are encouraged to value one another’s contributions, can inadvertently constrain rather than enable mathematical activity. Thirdly, it is possible to explain this acceptance of the vertical line graph with reference to the power relations in the group. Jeff and Shae, both first-language English speakers who joined the class from the mainstream mathematics class, identify themselves as the authorities in the group. They are the first to volunteer possible solutions (although often tentative solutions) and verbalise their evolving ideas publicly. In addition, the other students position Shae and Jeff as the authorities by consistently appealing to them for assistance and feedback.

Attention to the vertical line graph demonstrated by Jeff seems to set up links that are constraining. Darren makes a link to his lecture notes for that day by paging back in

his book; the lecture topic was the limit definition of the derivative and the lecturer presented cases where the derivative does not exist, for example, at the point where the tangent to the graph is vertical. It also emerges in a later discussion that Darren has possibly not yet looked at the required limit expression, $\lim_{t \rightarrow \infty} P'(t)$, in question

(g). Yet together, Darren and Jeff argue that the gradient of a vertical line “does not exist” and that vertical line graph is “not differentiable” / “non-diffable”, and hence the limit in question (g) does not exist. The link to Darren’s lecture notes and their language use suggests they are drawing on the Course discourse in their argument. Unlike Shae’s earlier attempt to ground his argument in the task context (as in lines 346 and 347), Darren and Jeff arrive at their answer without reference to the task context. This absence of a meaningful link between the mathematical content and the task context may be a result of their experience of the discourse of school mathematical word problems; for in such problems they need only “pretend that” the situation described in the task context exists (Gerofsky, 1996, p. 40).

An analysis of how the students talk also points to why they do not appear to critically interrogate one another’s reasoning. Although Jeff’s vertical line graph represents the derivative in the limit expression $\lim_{t \rightarrow \infty} P'(t)$, this is not pronounced in

the conversation and only emerges in line 482 (Transcript 1) in discussion with the Tutor. Yet the students’ argument suggests that their focus is on the gradient (the derivative) of this vertical line. The students’ tendency to use the pronoun “it” to reference different concepts rather than explicitly naming them may constrain them from evaluating their own arguments and from focusing appropriately. For example, in his explanation to the Tutor in line 478 (Transcript 1), Jeff uses “it” for the vertical line graph and possibly “it’s” to reference the tangent to this graph.

Furthermore, while the students look at the derivative function P' in the limit expression $\lim_{t \rightarrow \infty} P'(t)$, they do not appear to look at the symbols $t \rightarrow \infty$ in this

expression. It is possible that Hanah tries to draw the attention of Darren and Jeff to this when she argues, “But they are talking about the days ... time” (line 367). However, this pronouncement is not attended to by the other students, possibly because Hanah tends to position herself outside of the group by working as an individual and not entering into the group discussions. In an individual interview she indicated that she felt “intimidated” when working in the group.

When the Tutor approaches the group he reads the answers aloud from the students’ written work as in line 472. But the way that he reads the full answers by linking the mathematical symbols to their meaning in words and his insistence that the students explain their reasoning appear to be enabling. Jeff responds to the Tutor’s challenge to explain his graph, but then pauses, as he repeats the Tutor’s phrase “ t tends to infinity” (line 472, Transcript 1): “We thought that, okay, it will be... ° t tends to infinity° ((*looking up at the ceiling*)), okay wait, I’m thinking of the wrong thing” (line 485). From this point the change in time features in the students’ talk,

suggesting that they are attending to the symbols $t \rightarrow \infty$. For example, pointing to the problem text Shae says, “this is equal to the amount of people over time ... that is the increase ... °per day” (line 494). The link that the Tutor makes between the limit expression and the graph for question (a) also appears to be enabling, evidenced by the fact the Jeff rapidly comes up with a correct answer of “nought” (line 502) for (g) and indicates with his hands that he is visualizing an appropriate graph. Hanah’s naming of the expression $\lim_{t \rightarrow \infty} P'(t)$ as “the rate of change at infinity” (line 510a) suggests that she is now looking at the expression as an object with meaning, rather than looking at separate parts such as $t \rightarrow \infty$ and $P'(t)$. She also tries to explain her claim that the answer is zero by drawing on the task context.

CONCLUSION

Gómez (2008) argues that the methodological procedures involved in mathematics education research are usually only described in doctoral dissertations and tend to refer to methodologies that have already been developed. In this paper I have presented some of my personal journey in developing an analytic framework for my study. I have suggested that my original socio-political framework and associated tools did not mean, in Jeff’s words, that I was “thinking the wrong thing” (line 485), but rather that I was “looking in a particular way”. While my initial tools were useful in identifying important aspects of the student action, I needed to do some “mathematical work” in order to bring the student action on the mathematical objects into view.

The analysis presented here suggests that the students’ ways of talking, representing, making links, identifying themselves and one another, and interacting socially and discursively may constrain their attempts at solving question (g) of the flu virus problem. In contrast, the Tutor talks in such a way that he makes links between the mathematical symbols, the description of these symbols in words, and the appropriate graphical representations. His approach is enabling in that he allows the students space to talk and to work with the links he has set up. I argue that, with the Tutor’s support, Shae, Jeff and Hanah are able to position themselves appropriately in relation to the valued ways of acting mathematically for this problem. Yet there are absent voices in the data presented here; my analysis of the wider data set suggests that Lulama is positioned outside much of the group discussion, despite his ongoing attempts to participate.

By addressing the “mathematical specificity” (Valero & Matos, 2000, p. 398) of the student action from a socio-political perspective as described here, I argue that I am able to explain this action in terms of different ways of acting mathematically, rather than with reference to the cognitive mathematical ability of the individual students. So rather than viewing students’ action as “thinking the wrong way”, I can argue that they are “looking/linking/talking/etc. in a particular way”, and that these

mathematical ways may not be of value in the particular mathematical discourse practice in which they are engaged.

NOTES

1. I use the term “real-world” as a label for a group of problems in the Course. Developing a description of this group of problems is part of the wider study as reported in Le Roux (2008a, 2008b).
2. In the wider study I argue that in some cases the construction of the problem and/or the worked solution can be viewed as constraining.
3. The names of the students have been changed. The names Jane (gracious), Lulama (gentle and kind), Darren (great), Hanah (grace), Shae (gift) and Jeff (gift of peace) have been selected to acknowledge my admiration for these students and my gratitude for their willingness to take part in the study.
4. The transcription notation used in the study is based on Jefferson notation; three dots “...” indicates a short pause, square brackets around [text] or [[text]] indicates overlapping text, italicised text in double round brackets ((*text*)) presents the non-verbal action, underlined text indicates emphasis or stress, and °text° indicates speech said quieter than normal.

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QUESTIONING UNDERSTANDING!?

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In this article I examine the notion of understanding; how it is spoken into being and what work this does. I use post-structural analysis to examine primary school student-teacher interviews in relation to prominent socio-cultural research in mathematics education. From this I propose that understanding is tied to gendered and classed values which create hierarchical positions. I also argue that relying on psychological constructions of pupils is unworkable as student-teachers do not construct pupils as the rational automata presented by recent neo-liberal policies.

PART ONE: INTRODUCTION

In the field of teacher education the ‘quest for understanding’ is somewhat akin to the (re)search for the Holy Grail. Educational research is fixated with developing student-teachers’ understanding of mathematics and enticing student teachers to teach mathematics for understanding. In England, both of these issues are raised in recent influential reports, Ofsted (2008) note that pupils in school can be ‘successful’ in mathematics without ‘understanding’ the work and Williams (2008) argues that student-teachers often lack a ‘deep understanding’ of mathematics. In these documents and in the majority of research, ‘teaching for understanding’ is viewed as something everyone should aim towards. Although at first it may appear difficult to argue against this aspiration, any term can benefit from deconstruction to determine its ‘history of the present’ (Foucault 1977, p.31). To suggest we can ‘understand’ understanding’ is complicated. I do not deem that we can precisely determine what or how someone ‘understands’ but I do think we can discuss how understanding is spoken into being and the consequent work that this does. What happens when a teacher states that they desire understanding or to that they wish to teach for understanding? What is it they desire and what work does this do? What if our ‘quest for understanding’ (Boaler 1997, p. 111) is a masquerade for something else?

The research discussed in this article is post-structural. It is argued that this take upon mathematics education is particularly pertinent as many have the philosophical view that mathematics itself is rational and absolutist (Ernest 1991). If we continue to look at mathematics through a rational lens we will only see reason, by stepping outside dominant discourses, we not only challenge the nature of research itself but also the nature of mathematics (Walshaw, 2004a, Walkerdine, 1990). Primarily I use the work of Foucault (1972, 1977, 1980) and take meaning to be constituted through discourses and ‘truth’ to be a fiction. In this, language is ambiguous and meaning is created through discourses (MacLure 2003), where each discourse is specific to the current ‘regime of truth... that is, the types of discourse which it accepts and makes function as true’ (Foucault, 1980, p. 131). Furthermore power is ‘constituted through discourses’ (Walshaw, 2007, p. 20). However according to Foucault power can be

enabling and ‘needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repression’ (Foucault, 1980, p. 119). Thus people are ‘active agents with the capacity to fashion their own existences’ (Walshaw, 2007, p. 24). In addition it is worth noting that England is a broadly neoliberal society which relies upon the autonomous, psychological self (Rose 1999). Within this, surveillance is high as the public sector borrows management models of working from the private sector (Ball 2003). Governmentality, the principle that governments or social systems produce normalised subjects, is in the ascendance, whilst we live under apparent freedom.

To examine how understanding is created through discourses I will draw on ‘evidence’ from interviews with five prospective primary school teachers, where (after initial coding) it was found that understanding was a dominant theme. These student-teachers were interviewed throughout their three years on an undergraduate degree at a University situated in the North of England. The interviews presented are from the final year of their course as they would have had the most experience of university and school. Each interview was conducted between myself and the student-teacher and took the form of an open ended ‘discussion’. They lasted between twenty five minutes and one hour. The student-teachers were fully informed of the purpose of the interviews however I am also a lecturer on their degree course, though my role is minimal. It is important to note that I am deliberately using a small amount of data and participants to allow space for an intensive analysis; I am not seeking to generalise results or represent the population. To set the data within the current regime, the interviews are supplemented with extracts from recent influential policy documents for mathematics education; all of which the student-teachers will have had exposure to. Specifically I refer to the primary framework for literacy and mathematics DfES (Department for Education and Skills) (2006) and the primary framework for literacy and mathematics: core position papers underpinning the renewal of guidance for teaching literacy and mathematics primary (DfES 2006). These documents are written for the government by private companies and are fundamental in guiding teachers on the mathematical content to be covered, the pedagogy to be used and the nature of pupils’ learning.

PART TWO: UNDERSTANDING

Whilst there is not space for a full historical analysis of mathematics education and understanding, it is important to highlight some aspects of the current regime. There are those that analyse understanding in terms of cognition. Examples include Skemp (1976) and his oppositional (hierarchical) classification of instrumental and relational understanding and more recently Barmby et al (2009) who examine it as a process of connections and representation. Others have examined the notion of learning and understanding from a social-cultural perspective. Two particularly influential social studies in the UK were carried out by Walkerdine (1989) and Boaler (1997) both of whom examined the affect of the presence (or absence) of ‘understanding’ in the

classroom specifically in relation to gender. Throughout this article I use the terms boys and girls to fit in with the referenced sources, though I will attempt to critique these as indicators of enacted gender performativity (Butler 1999).

Twenty years ago Walkerdine (1989) highlighted an unnecessary gender division occurring in English mathematics classrooms. At this time, guidance on teaching mathematics emphasised the virtues of discovery teaching and of understanding and consequently the latter became a marker of proficiency with mathematics. Thus those who achieved high grades, but as a result of learning by rote, were positioned less favourably than those who achieved lower grades but could obtain true understanding. Moreover girls who worked hard were often constructed negatively whilst boys were often viewed as lazy or misguided but judged to have natural ability, with their superior and rational minds. In Walkerdine's view girls (or the feminine) were being punished for their achievements and science, rationality and reason had become the 'true' measures of mathematics and of success.

By the time Boaler (1997) was writing about girls 'quest for understanding' teaching for understanding would have been less popular in mathematics classrooms, as a result of the 'back to basics' agenda (of the Conservative and the Labour governments of the 1980's and 1990's). Boaler agreed with Walkerdine that 'girls' were alienated from mathematics; however Boaler differed by proposing that their exclusion was a result of the dominance of rote learning in the classroom. She argued that girls sought the understanding that was absent and thus excluded themselves. It would seem that girls were alienated in the 1980's and 1990's, but from very different classrooms, thus it is difficult to argue that pedagogy was the main reason for their isolation. In addition Boaler saw progressive classrooms (or classrooms which used 'open' approaches to mathematics) as offering freedom; though as Walkerdine points out – freedom is a fiction, the classroom is still governed but by covert rather than overt means. Being a pupil in Boaler's 'open' mathematics classrooms is not the free and liberating experience it is claimed; there is still normalisation and surveillance but it masquerades under a discursive construction of liberation. Whilst there is no disputing the influence of Boaler's work or the positives that have come from her conjectures you can of course offer alternative readings to some of her more essentialist claims. Although the multitude of policy (for example DCSF (2007)) that has accompanied these gendered assumptions may have raised some important issues about learning and may have helped improved attainment, this improvement could simply be because of the diversification of learning and teaching strategies, such that the pedagogy now meet the needs of more people regardless of their gender. However one point to contest is Boaler's treatment of gender. I argue that you cannot determine what boys and girls prefer because of their apparent gender (sex), instead you can only comment on their positioning and performance; to suggest anything more is too simplistic and can lead to heteronormative classrooms and classifications.

Since Walkerdine's study the guidance on the teaching of mathematics has largely been reactionary and cyclic, the movement has been from overt progressivism to traditionalism and now perhaps we are somewhere in-between. However many still adhere to the opinion that a significant amount of 'high attainment' in mathematics is the result of rote teaching and learning (Ofsted 2008). One thing that has changed over the last twenty years is the move towards specific policies that now govern the manner in which mathematics is taught in the classroom (see DfES 2006a). Furthermore, at present we live in a highly accountable society that is normalized and monitored by technologies of surveillance, such as league tables, performance management and the collection of pupil data. In 1999 (after Boaler's study) the National Numeracy Strategy (NNS) was introduced into primary schools. This is a piece of documentation which 'advises' teachers specifically how to teach; for example by advocating whole class teaching, three part lessons and mental/oral starters. In line with government advice the majority of classrooms took the NNS fully onboard. Initially its merits and content were questioned by academics in terms of the advice given and the overtly prescriptive nature in which it is written (Brown et al 2000). As a consequence of the strategy there was an improvement in attainment by some but it is argued that this does not translate to an improvement in the teaching of mathematics (Brown et al 2003); in addition mathematics classrooms have become clones into which pseudo-children are designed to fit. Very recently the NNS was revised in the form of the Primary Strategy for Literacy and Mathematics (DfES 2006a); in the current version it is meant to be less formulaic and in terms of mathematics there is more emphasis on the 'using and applying' strand of the curriculum (the aspect concerned with the development of thinking and reasoning skills). Thus student-teachers in the classroom today may be influenced and exposed to both versions of the Strategy.

PART THREE: THE INTERVIEWS

In the interviews below I wish to explore some of the points that arise from Boaler's (1997) and Walkerdine's (1989) work as well as drawing out other issues from my student-teacher interviews. In the first instance I discuss whether girls seek understanding (and consequently that boys do not). With an alternative reading you could argue that girls 'quest for understanding' (and boys lack of) may indeed be a quest for something else. For example Boaler discusses how boys replace the desire for understanding with a desire for speed. However if speed is seen as an indicator of 'natural ability', one can suggest that speed may be about taking up a discourse of naturally able, in an acceptable and masculine way (this would also position 'girls' as the other). Another point to contest is why are boys' 'quest for speed' and (not) their 'quest for understanding' in opposition and are these events mutually exclusive? Are we assuming that understanding cannot happen quickly, or that natural ability (which is being associated with speed) is distinct from understanding? This is confusing, especially when if consider that the government states that mathematically able pupils grasp new material quickly (DfES 2000). Whilst it is acknowledged that the

use of binaries is widespread within our language this does not mean that these oppositions are ‘natural’ (MacLure 2003). Moreover these constructions are not helpful as they serve to position events and people into conflicting boxes, for example, right or wrong; masculine and feminine. To suggest that all (or most) girls would choose understanding is too simplistic. I am not in disagreement with Boaler that people proclaim to desire understanding, but desire is not always about the obtaining of the object, the production of desire ‘involves a complex subject investment in ...subject-positions’ (Walkerdine 1990, p. 30). Indeed if we are to invest in psychoanalysis, Lacan’s version of desire is ‘about the quest for a secure identity (Walshaw 2004b. p. 130). Perhaps the girls’ proclamation of a quest for understanding was the socially acceptable response to give: the obedient, common sense response, the ‘trick of knowledge/power (Foucault 1972) which... leaves us unaware of the effects of our practices on ourselves and others’ (Hardy 2009). Or it may have been the only response left to give. Conceivably the quest for understanding is a mask for something else, for acceptance or for the other. In the extract below Jane does proclaim to desire understanding, however she develops her ‘understanding’ by ‘break(ing) them down’ into small manageable chunks. The understanding is not about developing connections across mathematical concepts or developing reasoned arguments, understanding is about ‘being able to do’ mathematics with competence and perhaps even confidence, which is quite different to the notions of understanding discussed earlier (for example Barmby et al. 2009). It is about the taking up of a subject-position, and belonging to an identity.

Jane: Now I’ll look to find, what I understand and from this course I’ve learnt that there’s things I do understand more than others. Like I taught myself the chunking method of division whereas before I didn’t understand long division at all, I just couldn’t do it, but I taught myself the chunking method by doing it with the children and that worked. I taught myself how to do the grid method with multiplication...But then because I’ve still got this back foot with maths, I think that’s helped because the things I do know and understand I’ll still break them down because I think well just because I understand them doesn’t mean that everybody does.

That student-teachers desire to be able to do is perhaps not that surprising if we look at an extract from policy (DfES 2006b); the influential government advice on teaching mathematics. Children must ‘have a secure knowledge of number facts and a good understanding of the four operations’ (DfES 2006b, p. 40, 57). Here I suggest that understanding is written as the ability to do – it is also preceded by ‘a secure knowledge of number facts’ hence keeping knowledge high on the agenda. Though this is just one example, the document contains many such deterministic statements.

‘As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.’ (DfES 2006, p. 40)

Here understanding is written as something which is unproblematic and universal. For example, the suggestion of ‘underlying ideas’ is not only overtly prescriptive but suggests a rational and linear view of mathematics. In addition, both of the phrases ‘use particular methods’ and ‘as children begin to understand’ imply that there is an expectation of certain cognitive and social behaviour – that of a normal child and thus anything that does not conform is othered. There is little space in these documents for children that fail to follow linear and rational views of learning. Which could suggest that what teachers are allowed to both read and say is similarly deterministic.

With Sophie the desire to be able to do is even more explicit. She expresses fear for the process (or the connections) or for what some (Barmby et al 2009) might term understanding. She expects mathematics to come to her naturally, which is a familiar story (Mendick 2006)¹.

Sophie: That’s my worst one! I hate the tables. 7s and 8s. 9s I know the trick with your hand so that’s not a problem. 5s and 10s and 3s are fine. 4s are fine. It’s just 7s and 8s.

Anna: Would you be happier, would you just like to know them and be able to recall them or work them out?

Sophie: I would love to be able to know them by rote and just be able to say ‘7 8s are...whatever it is’ and know it, off hand, but I just can’t. I haven’t got that in my head. Whereas some people are more- they can visualize numbers so easily and I just can’t do that. I need to sit down and go through them all one by one and get to the answer.

Anna: Right, so you know your fours?

Sophie: Yeah.

Anna: You could double your fours to get your eights.

Sophie: I can’t do that because that’s doing too much of a process so I could probably try that. I just think it’s going to complicate it more.

Anna: I do it.

Sophie: I’ll give it a try but...

Anna: Some people wouldn’t know them all but would calculate to get each individual one.

Sophie: Yeah.

Anna: Or if you know 8 8s is 64, use that to get back to 7.

Sophie: Mmm. But it’s all working out your long methods isn’t it?

Anna: Yeah.

Sophie: I'm not good at that sort of thing of working out how to do things and making it complicated. I'd rather it just came to me naturally. But I'm never going to be that one! (laughter) Unfortunately.

Sophie's assumption that true mathematics comes naturally succumbs to the hierarchical and rational view of mathematics which is in accordance with the view that understanding belongs to the most able. Furthermore this places mathematics as divisionary and positions those who achieve results by rote learning as non-mathematicians, as noted by Walkerdine (1998).

Nicola: My Highers can do fractions, equivalent fractions everything. The Lowers don't even understand basic fractions. The Highers are very good with their times tables. Or one boy, you can ask him anything in his zero to twelve times tables, he's like that [clicking of fingers], you get it back. He understands it perfectly. It's like using and applying is, I find more important for them.[the higher attaining]

Nicola views understanding as important, this is shown in part by the number of times she mentions it throughout her interviews. Her version of understanding is similarly aligned to the recall of knowledge and speed. This could be viewed as traits of cognitive ability or they could be viewed as social markers presented as understanding. In addition, her interview exemplifies Walkerdine's point that boys are more easily afforded the luxury of the appearance of understanding even when it is not apparent. Nicola also expresses the desire to work with the highers thus excluding the other - the lower attaining. This may imply that those who are able and can understand are worthy of praise, thus othering those who struggle to understand. The dangerous consequence is that these others are positioned as non-mathematicians and are no longer asked to understand (Bibby 2001). This is also shown where Nicola mentions the 'using and applying' strand of the curriculum, which can be tied to some cognitive notions of understanding. From this, I suggest that understanding is bound by hierarchical values that rely on rationality and reason and are tied to notions of gender and social class (Walkerdine 1989).

The next point I wish to build is the notion that understanding is produced as cognitive. Examining the interviews, I could not find one example of a student-teacher discussing a child in terms of cognition alone; more often their judgements had emotional contexts or the teachers discussed gender or confidence. Pupils were not positioned as the rational automata presented by government policy although there was tension between what the student-teachers 'rationally' expected (which aligns to over cognitive and simplistic policy) and the 'real' children that they worked with. Below are some extracts from each student-teachers' individual interview:

Kate: I know one of the girls is terrified of fractions, really hates them... My high fliers are all three boys. Girls tend to be middle of the road.

- Nicola: They can do stuff like that and it makes them feel good about Maths...When the shutters come down it's ... it is about convincing them they can do it.... The girls are higher but they're not as high as you could expect. With boys, they don't have the patience for it.
- Jane: Yeah, I see, I see children in two stances in maths whereas I don't in other subjects. I see the children that, that immediately think oh, oh no maths
- Sophie: I want them to learn in a way that they feel comfortable.
- Leah: But I would say that probably in Maths, boys are more confident on this table.

So what if understanding is positioned as the mathematical Holy Grail? What would this hierarchy contribute to the field of knowledge for student-teachers? To perceive something as innately good positions something else (rote-learning) as innately bad (MacLure 2003) however rote learning happens in mathematics classrooms and in universities. Indeed mathematics in society (which it is frequently argued school pupils should be exposed to) can be about the application of methods, for example within jobs such as accountancy or engineering. What version of understanding is relevant here – to be able to do or to make cognitive connections? In addition, if one of the purposes of school (or university) is to achieve qualifications (which it is in this high-stakes society) does the 'smart' person grapple with understanding or learn routines by rote, as the latter could help them achieve good grades? It is argued that in the interview data below Kate shows an understanding of systems and how to play the game. Tension is present around understanding where passing exams is presented as an unacceptable yet accepted version of learning and teaching.

- Kate: So I think it is important that they understand it, but then again it's not really - it is... a kind of learning a way to pass if that makes sense. A teacher knows what's coming up in the paper, like bar charts always come up, whereas line graphs don't. So I think they target what they teach in relation to SATS, especially in Year 6, which if SATS went out of the window maybe they'd get more free range of what they could teach.

PART FOUR: CONCLUSION

In this article I am not presenting data as a generalisation or as the 'truth', I am not even suggesting this is a finished argument. I am merely beginning to question some of the apparent truths that circulate within mathematics education. I do not wish to say that there is something wrong with teaching for understanding, but suggest that it should not be seen as a 'common sense' piece of truth, something which is beyond question. From the tentative analysis above, I suggest that understanding is produced as hierarchical, particularly in relation to gender, social class and ability. It belongs to the privileged few, to the 'naturally' able, which are often boys (another unnecessary and unhelpful classification). To suggest that girls have a 'quest for understanding' is over-simplistic and gendered and in the first instance we should unpack how each

version of understanding is constructed. Another danger is that such gendered assumptions produce and maintain hetero-normative classrooms and classifications. Finally, I suggest that students-teachers do not produce understanding as cognitive; the child is not an automaton who performs as the government text prescribes. Pupils and understanding are tied up with notions such as gender, confidence, and emotion.

There is no disputing the money and effort the current government has put into raising achievement in mathematics, however with greater prominence comes greater expectation. Targets are based upon exam grades that are publically dissected in league tables. Strategy documents (DfES 2006a; Department for Children Families and School (DCFS formerly DfES) 2009) promote the normalised expectation that pupils should be performing to certain levels. Within this, the perception is that learning is rational and linear and should happen when and how the documents advise. For this learners and teachers need to become autonomous psychological subjects; however this creates tensions within the student-teachers interviews as learners are produced as people with social and cultural identities. So should we look for different version of learners or teachers or should we look for different versions of mathematics? Perhaps we should question what else we do when we place teaching for understanding upon a pedestal? One version is not explicitly good or bad and every account has its technologies of surveillance and its regimes of power; though some are more covert than others. It is these less obvious forms of common sense masquerading as liberation that perhaps need closer inspection.

NOTES

¹ (another version of this story could be she is reacting against me, a lecturer, in a position of power)

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STRUCTURED OR STRUCTURING: SETTING UP A PROFESSIONAL DEVELOPMENT PROJECT

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Professional development is often seen as something that is provided pre-packaged to teachers who adopt or reject it depending upon their previous beliefs and knowledge. However, this does not take into account the influence of context and circumstances on the professional development providers. In this paper, we explore the constraints and opportunities on ourselves as the providers in setting up a mathematics professional development project in one school whose students came from a low socioeconomic area. Kemmis and Grootenboer's (2008) ideas on practice architectures were used to identify how the circumstances and context shaped what we were able to offer but also how we influenced the situation itself. Thus, we were better able to understand the complexity in which we worked.

PROVIDING PROFESSIONAL DEVELOPMENT

Mathematics and its alter ego numeracy have consistently functioned as gate-keeping subjects that regulate opportunities for students' future careers (Nasir & Cobb, 2007). The introduction in 2008 in Australia of the *National Assessment Programme – Literacy and Numeracy* emphasised again this perceived importance as well as identifying schools whose students are under-achieving in mathematics/numeracy. Consequently, there has been much discussion about the support that should be provided to the schools and students where underachievement has been identified (for example NSWPPA, 2008). Professional development has been considered as one way to “fix” teachers to improve student outcomes but this has not always resulted in success. For example, although the large scale numeracy professional development carried out in New Zealand did lead to increases in achievement for all students, the amount of increase differed according to ethnicity, socio-economic status and gender (Young-Loveridge, 2000; Young-Loveridge, 2003). Thus, the gap between the outcomes for different groups of students increased.

Recently, the complexity of factors that contribute to students' mathematics learning within a socio-political environment has been recognised (Kitchen, 2007; Nasir & Cobb, 2007). Although it is possible to separate the contributing factors that operate in mathematics classrooms, how they interact to produce particular outcomes differs according to the context. Case studies, such as those outlined by Kitchen (2007) and Nasir and Cobb (2007), show how mathematics learning is accessed by diverse learners in mathematics classrooms as a consequence of actions undertaken by teachers. However, there is not the same number of case studies about the impact of professional development on teachers of diverse students (Morton, 2005) and consequently teachers and schools can be blamed for the poor uptake of a professional development package. For example, in evaluating the relationship

between the implementation of a numeracy professional development project, *Count Me In Too* (CMIT), and results in a standardised numeracy test (BST) in Year 3 in New South Wales schools, Mitchelmore and White (2002) stated that:

There is a potential for schools with a poor history of BST performance to improve their results substantially. However, CMIT is no automatic guarantee of such improvement. The school must also provide the appropriate environment to support its effective implementation. (p. 22)

In 2009, we began a professional development project in a school with a diverse population of students who are underachieving in mathematics according to national testing. In this paper, we report on the constraints and opportunities that we faced in setting up the project. Joubert and Sutherland (2008) suggested that not only is the link between professional development and student outcomes unclear in the research but that:

There is very little in the literature that discusses the people who design, plan and deliver CPD [continuing professional development], but we think it is crucially important that we know about, and understand more about, this group of people because of their influential position on the teaching of mathematics (p. 29)

The requirements for effective professional development such as “build on what teachers already know, taking into account the voice of the teacher” (Joubert & Sutherland, 2008, p. 28) suggest that it needs to be adapted for teachers. The adaptation requires not just an understanding of the teachers’ background and needs but also the context in which they work. It was important for us to know how what the circumstances in which the teachers worked affected what we could offer them. We used Kemmis and Grootenboer’s (2008) ideas about practice architectures to better understand the process of setting up a project that we wanted to be effective.

PRACTICE ARCHITECTURES

Kemmis and Grootenboer (2008), using a scheme from Aristotle and adopted by Habermas, discussed how educators may come to perceive different actions as being available to them within certain situations. They saw educators as having dispositions of:

- *epistēmē* guided by the *telos* (aim) of attaining knowledge or truth
- *technē* guided by the *telos* of producing something
- *phronēsis* guided by the *telos* of wise and prudent action
- *critical* guided by the *telos* of overcoming irrationality, injustice, suffering and felt dissatisfactions by *emancipatory* action (p. 40)

Kemmis and Grootenboer (2008) described three extra-individual structures and processes - culturally-discursive, material-economic and social-political - that “shape dispositions and actions, both in the educator’s general response to a particular

professional development proposal, grant applications and ethics applications provided extra details. We analysed these data by looking for instances of different dispositions coming into play and then identifying how individual and extra-individual factors contributed to the enactment of the dispositions.

DISPOSITIONS

In the following section we describe an incident in which each of the first three dispositions – *epistēmē*, *technē* and *phronēsis* - are clearly visible in our actions. Other dispositions also are evident, but that they are not at the fore front of our understanding of the situation. The fourth disposition that of being critical, we see as being interwoven throughout each of the incidents and to some degree it is because we wanted to “overcome irrationality, injustice, suffering and felt dissatisfactions by *emancipatory* action” (p. 39) that the other dispositions became foregrounded. The incidents show how context influenced which disposition was brought into play and how this then affected what occurred.

Epistēmē

In the beginning stage of the project, there was a lot of knowledge gathering. A lucky chance meant that we met the principal just at the time when he had received substantial funding for the school. The following is his email following that chance meeting:

Sent: Friday, 28 November 2008 1:03 PM
To: Meaney, Tamsin
Subject:
Tamsin
Thank you for calling. Over the next 4 years we are going to focus on improving literacy and numeracy outcomes for students specifically from low SES backgrounds. We would be looking at appropriate teaching strategies and numeracy activities that could assist.
We are also investigating the application of the Quality Teaching Framework to the teaching of all areas of numeracy. We currently do CMIT and CMIT Indigenous employing the SENA for assessment purposes.
We are looking for assistance in the design and implementation of a successful program and would appreciate talking to you about these areas.
Kind Regards

The email provided details of the professional development programs that were already operating in the school, CMIT and CMIT Indigenous, and the other program Quality Teaching Framework that they were investigating. Our critical reflection at this point was to consider how we, as university-based mathematics educators, could

provide something that was different but in alignment with the programs offered to the school by the NSW Department of Education and Training (DET). In our first meeting with the school principal about a possible collaboration, we kept this need for difference in mind whilst finding out more about the school and its needs. Our ‘saying’, ‘doings’ and ‘relatings’ were focussed on understanding how we could integrate what we brought to the context into what was already happening. The fact that the school was involved already in a range of professional development activities did constrain what we could offer. However, the need for difference meant that we could contemplate the information provided to us, knowing that creative alternatives that met the principal’s criteria were likely to be well-received.

At the initial meeting, the principal clearly stated that he wanted a project that increased student engagement, community participation and teacher professional development. His belief was that once these were in place then there would be improvement in numeracy and literacy results in standardised tests. This approach to increasing students’ numeracy outcomes resonated with work that we had done previously (see Meaney, Fairhall, & Trinick, 2008; Lange, 2008). He also described how children often came to school without breakfast and that the teachers, although dedicated, were often exhausted and therefore could not be overloaded with more work. He saw 2009 as being a year where short, taster activities could be offered so that teachers would be better able to consider options for the following three years.

In these early stages, it was important that we enacted a disposition of *epistēmē* because, in order to put an appropriate proposal together for the school, we needed information. Although the DET provided a range of professional development programs to the school, they were ‘pre-packaged’ with set materials and tailoring them to the needs of the school or individual teachers was not a simple process. From respectful listening, we wanted to combine information about the school with our knowledge of successful professional development programs in other low socio-economic schools to produce an appropriate proposal through the disposition of *technē*.

Technē

After the initial meeting at the beginning of December, we produced a proposal that was sent to the principal in the middle of January, with the new school year starting later that month. In putting together the proposal, we relied on our own understandings, skills and values but were constrained by extra-individual conditions such as the discourse of professional development, the costings for different options and the logistics of having positions at the university which did not include providing professional development to a school. Nonetheless, our overriding concern was to ensure that we provided a project that had the greatest chance of supporting the school to improve students’ mathematical understandings and thus overcome inequity and injustice.

Proposals for schools are written in a certain genre that includes some information and excludes other information. For example, given that schools must keep to tight budgets, proposals need to include some indication of costs. A rationale for the proposed activities is also required but this needs to be kept concise. Our previous work with principals suggests that as busy people they only want the main points and will ask for further details when needed.

The proposal that we sent was ten pages long and outlined the different activities to be undertaken each school term. The professional development was the fourth activity and it was suggested that it should take place in Term 4 (October to December 2009). There were a number of reasons for choosing to do it so late in the year. Two earlier activities used our student teachers to engage in one-on-one sessions with primary school children, as part of their mathematics education assignments. The availability of student teachers meant that these activities had to be done in the first two terms. Another reason for leaving the professional development till later in the year was so that we could apply for funding to do research on what we were providing, so that the project could be seen as a legitimate part of our university work. Researching the constraints and opportunities that support or hinder teachers taking up professional development meant that we had to apply both to our university and to the DET's ethics committees. Applications to the DET ethics committee had a reputation for taking several months. We could not presume that permission to proceed would be granted until quite late in the year. Producing a proposal meant that we had to juggle these material-economic factors with our understanding of what was required by the school. Therefore, what we offered was structured by these considerations.

The genre of writing a proposal also required us to not only be attentive to the school needs but also to demonstrate that we were professional mathematics educators who had something to offer. We needed to show that we knew what we were talking about and were not suggesting an ivory-tower, non-realistic set of activities. The proposal needed to place us in relationship to the school where we had knowledge that they were interested in. However, at the same time, we did not want to present ourselves as all-knowing experts but show that we were respectful and valued the teachers as professionals. The way we presented the proposal was therefore constrained by the social relationships that we wanted to engage in, just as much as it was by the genre of the proposal and the material-economic commitments that we had to juggle.

Phronēsis

Our proposal was accepted by the school, although with some misgivings by some of the Assistant Principals as one activity involved giving the children disposable cameras to take photos of themselves doing mathematics at home. As the year progressed, our aim was to act rightly in regard to our relationship to the school staff and students. We saw this as contributing to being able to implement a professional

project later in the year that had the greatest likelihood of being successful. Acting rightly involved getting to know others with whom we would work on the project. We did this by carrying out the other activities and also by setting up the research component of the project.

The following extract from our field notes kept showed how we were seen by the one of the Assistant Principals (AP1) in August:

AP1 talked about how having the student teachers work with their children had been something that they had been sceptical about but which seemed to have turned out really well. I said that from our point of view the student teachers had gained a lot and AP1 also seemed to agree that the children had gained from being involved even though they were taken away from their normal programs. I think by showing how some of these off-beat ideas could work that she was more inclined to trust us with other suggestions. (11/08/09)

It may have been that our position as university mathematics educators gave us enough kudos for the school executive staff to allow the activities to go ahead. After the primary school children's learning increased, as well as the student teachers' learning, then the teachers were willing to admit that their misgivings were not justified. Consequently, the kudos of being from the university was enhanced by showing how we could support children to engage in mathematics. If the activities with the student teachers had been a dismal failure then our prestige as university lecturers who wanted to support the work done by a school may have been seriously undermined. Our 'doings' in the earlier activities had an impact on the relationships that we could develop with the teachers. Without having developed relationships with the teachers in which we were seen as having something to offer, it was unlikely that we would have any volunteers for the professional development project. As it was, the need for teachers to commit to being filmed each week so that they could analyse their own teaching was extremely daunting. We have four volunteers, with others watching carefully to see how it goes.

During the year, we applied for and received funding from our university to engage in research around the professional development. The funding enabled us to release Marianne Thurling, an Aboriginal teacher from another school to work as a co-researcher for 6 weeks. She would bring with her experience of working on other research projects and many years of experience of working in local schools with high Aboriginal populations. We felt that it would not be possible to improve outcomes for Indigenous children without the insights that Marianne could bring as an Indigenous researcher. We needed Marianne's expertise in order to ensure that we continued to act rightly. The university had also provided a semester off teaching and this meant that Tamsin could concentrate on the professional development project. The funding and other support provided by the university enabled us commit to running the professional development project. However, we were aware that we could find ourselves split in uncomfortable ways if the professional development

project and the research project did not run smoothly together. The need to do the research could restrict what we were able to do in the professional development project unless we continued to remain critically aware of what it was we wanted to achieve.

STRUCTURE OR STRUCTURING

Education of any kind, including the provision of professional development, is a highly complex set of interwoven practices. There has been ongoing concern about how marginalised groups of students have only restricted access to “learn significant ideas in mathematics and to develop an appreciation of mathematics” (Hodge, 2006, p. 378). However, this concern has not manifested itself to investigating how professional development for teachers of these students can be improved to ensure better outcomes for their students. Perhaps part of the difficulty is trying to understand what affects professional development and how this can be changed. Using Kemmis and Grootenboer’s (2008) ideas about practice architectures, we investigated how we structured a professional development project whilst simultaneously being structured by extra-individual features.

We explored the initial stages of setting up a professional development project because it seemed that if these stages were not done appropriately then it was unlikely that the actual implementation of the project would achieve what the school wanted. We drew on a critical disposition to try to overcome the “irrationality, injustice and suffering” (Kemmis & Grootenboer, 2008, p. 39) that children who underachieve in standardized tests are subjected to. By having this as our main focus, the constraints and opportunities provided by the extra-individual conditions were better understood. We could see opportunities for us to be creative in what we offered but also recognized the risk that our credibility would have if these activities failed.

As each disposition came into play during the different stages of setting up the professional development project, it was possible to see how our own understandings, skills and values were shaped by the extra-individual conditions such as the genre of a proposal, or the need to complete a research project based on the professional development. Although our understanding of the situation called forth different dispositions to guide our actions, the changing interactions with others and the circumstances in which we were operating meant that it was not necessarily possible to predict exactly what should or even would occur. For example, as our relationship with the teachers changed we were more mindful of acting in a way that the teachers as well as ourselves would consider as being right. We gained more from the level of discussions that we could have with the teachers, but were also more vulnerable if we acted stupidly because we were now seen as being expected to know more about how it was to act rightly in the teachers’ eyes. It could be some time before the relationship was strong enough to withstand the consequences of acts

of stupidity. As our understanding, skills and values grew and changed so did the extra-individual conditions change.

In the coming months, we will implement the professional development project in collaboration with the four teachers who have volunteered to work with us. The implementation will call forth the dispositions to act in similar ways as happened during the setting up of the project. It will be interesting to see how the teachers work within the constraints on the extra-individual conditions whilst we also work within a different but related set of constraints from a different set of extra-individual conditions. Knowing better how we have been affected by, but also affect, the extra-individual conditions, in which we operate, will provide us with a greater respect for understanding the teachers' negotiation of their own practices.

Practice architectures enable us to understand the structural framework in which we operate. We as professional development providers are not free agents who organically come up with appropriate programs to meet the needs of this school, or in fact any other school. We are constrained in what we can offer and how we do this. Kemmis and Grootenboer's (2008) ideas were useful in coming to grips with the complexity of how we operated without allowing that complexity to be so simplified that the meanings around what was occurring were reduced to superficial commentaries. However, the description of extra-individual conditions as a structure does not adequately represent the dynamic nature of this structure. It does not remain unchanged as we within it change, rather the structure is also changing because we are operating within it. The structure is also what is being structured.

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MATHEMATICS ASSESSMENT AND TEACHER TRAINING: A PERSPECTIVE OF CHANGE IN VENEZUELA

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This research raises the need to work with a new conceptualization of evaluation in mathematics in the context of teacher training in Venezuela. Curricular reforms in higher education have proposed assessment with an emphasis on ethical and social dimensions. This paper considers how to ensure that the evaluation is consistent with teacher education that promotes student learning of mathematics. We highlight the emergence of a student's awareness, the importance of collaborative work and the need to connect with everyday mathematical knowledge and to establish a link between their professional and social life.

APPROACHES TO THE PROBLEM

Currently we face profound changes in educational curricular design and a new conceptualization of evaluation has been developed in alignment with these transformations. The evaluation of learning is obliged to respond to a conception of the processes of teaching and learning in a cohesive and interactive way.

In such profound changes in curriculum design, assessment practice cannot be separated from teaching practice. In the area of mathematics education this also leads to a new conceptualization of what it means to evaluate (Kulm, 1990; Webb 1992; NCTM, 1995; Niss, 1993; Romberg, 1995; Moya, 2008).

Moreover, teacher training has been a recurring theme in educational research in Latin America countries. An alternative teacher education means understanding which are some of the mechanisms of power that it disguises (Becerra and Moya, 2008).

The curriculum reform in the Universidad Pedagógica Experimental Libertador (UPEL), the leading teacher-training center in Venezuela, prioritizes a qualitative approach to evaluation. It addresses ethical and social dimensions, stating that it must train teaching professionals who can develop assessment procedures relevant to the state of education in the classroom and beyond.

But, how can this new discourse on curriculum in Venezuela and other countries be developed to lead the practice? What new approaches and new ideas and conceptualizations regarding evaluation need to be made so that the theory and practice of tomorrow will not be as separate as they are today? In the fields of Mathematics Education and Teacher Education, how do we respond to proposed changes in curriculum design? How do we achieve a form of evaluation that is consistent with teacher education that promotes student learning?

Moreover, the changes cannot be enacted in isolation. They are determined by a number of factors that need to be supplemented in a consistent manner, so the need to consider new ways of evaluating may arise. Several of these factors have been part of the research conducted over the last twenty years, where there has been growing interest in the teaching and learning of mathematics at higher education level. One of those aspects concerns how the conception, implicit or explicit, the teacher has about teaching and learning mathematics will influence, to a certain extent, how he/she evaluates. Wilson (1994) argues that in the field of mathematics importance is given to what is evaluated and, therefore, the assessment gives a clue about what mathematical knowledge is important for the teacher. Smith and Wood (2000) state that the evaluation leads to what students should learn and that can lead to them adopting a surface approach or a deep approach to learning mathematics.

Additionally, students' preconceptions about what mathematics is have an impact on their perceptions about its teaching, learning and assessment (Berry and Sahlberg, 1996, Berry and Nyman, 2002). Crawford et al (1998) report that students enter college with different conceptions of what math is and different approaches to learning. Most of them conceive of mathematics as a fragmented body of knowledge and this is associated with an approach to learning consisting of a set of rules, algorithms and routine activities.

We face a picture of Curriculum Design, particularly in Teacher Education, that proposes changes, such as practice based on reflection, the transformation of teaching methods from transmitting knowledge to the process of generating it or transforming students into active agents in their own training. Within this context it is important that the evaluation makes sense, in line with teaching that promotes student learning. We share the position of Leder (1992), who argues that our approaches to teaching and assessment in mathematics cannot be separated.

Despite of that proposed context, very few UPEL students succeed with appropriate levels of achievement in the initial courses in mathematics. The question arises whether the assessment made in the university classrooms has an impact on the low levels of achievement of our students. Matched to this question would be one that forces us to inquire whether, indeed, the professional practice of the university professor of mathematics in the classroom is consistent with what is required by the curriculum design and, at the same time, wonder if the assessment is influenced by the teaching models and the students' own perceptions of what it means to learn mathematics. Likewise, we should analyze our assumption that if an assessment is correlated with a particular teaching model it may lead to greater learning achievement in mathematics for our students.

Consequently, the central problem considered in the research was to examine whether current models of assessment in mathematics, implicit or explicit, which are used in university classrooms in teacher education, are promoting mathematics learning for students, or are directed mainly to certify the mathematical knowledge

that teachers regarded as valid and that the student must exhibit as a sample of having achieved the goals. Depending on the results, it might be necessary to generate an alternative model of assessment in mathematics, to consider the specificity of the discipline and the many facets of what could be conceptualized as mathematical knowledge. However, this model should be in correspondence with a teaching model, so it is necessary to unravel the ways of organizing and managing the process of teaching mathematics in university classrooms.

METHODOLOGICAL DIMENSION

Every research involves knowing, wanting to know about something. Thus it is necessary to make explicit our considerations about what is meant by a deep understanding of the topic being addressed. In the first place, we assume that knowing is always a process that does not end with the completion of an investigation. It is a successive approximation that shapes truths that may be temporary and shared. This leads to a demystification of knowledge as something static and unchanging, something that is done. As researchers, we undertook the search for a truth, where the investigator himself was a subject of knowledge. We shared Freire's position (1990): "the object of knowledge is not the end of knowledge for the subject of knowledge, but a mediation of knowledge" (p. 113).

We understand knowledge as a dialectical process, where "my vision" does not prevail over the "vision of the other", where my beliefs are not more valid than the beliefs of others. Therefore, dialogue is an essential tool in this research, understood as something more than a simple conversation or a lively exchange of ideas. This dialogue involves the confrontation of different views around common interests, not with the intention to impose an idea that we consider less successful than another one, but with the goal to understand, to know and to advance in the search for truth that is shared with others.

In that search for understanding and knowing, we consider it essential to understand the rationalities (Giroux, 1997). It is necessary to approach the set of assumptions and practices that allow individuals to understand and shape their own experiences and those of others. On the other hand, one must decipher the interests that define and qualify the way each one is facing the challenges presented by their experience. This understanding may enable us to avoid merely causal explanations or oversimplification of the complex relations that exist inside and outside the classroom. We assume a strategic rationality (Heler, 2005), from the standpoint that we work with individuals, not with objects; we do not reify people. This rationality is aimed at trying to order the action between individuals pursuing interests that could diverge but still maintain interdependence relations among themselves. This creates the need to understand the viewpoints of others in order to decide on courses of action that can offer the group a certain "degree of security" in the realization of interests that may become shared.

We worked with teachers of mathematics, students majoring in Mathematics at the university and the investigator with their rationales, their theories and practices. There were two methodological moments. The first was a study of theoretical development aiming to propose elements for an evaluation model that would function as an explanatory and organizational principle, from critical analysis of empirical data and existing theories. This study was supplemented by documentary critical in-depth interviews conducted with teachers.

The second methodological point of the research came from the field work done with students in the Geometry course. This work allowed us to build reflections that nourish both teacher and student perspectives, depending on the search for missing links and the gathering of shared views.

We emphasize in this report the fact that our research is framed within an critical-interpretative perspective which used grounded theory.

SOME RESULTS

A conclusion regarding the teacher's perspective is that it is not possible to establish a direct correspondence between what teachers think and what they implement, between the desirable and feasible, between their epistemological and educational conceptions and the evaluative aspects. It is not possible to define a path-way, rather there is a complex framework that cannot be deciphered by a single approach or determined by a relation of transitivity. From that perspective, what we might consider contradictions are due to individual teachers' rationalities: their visions, assumptions about their practice and how they face it. That rationality is mediated by what Giroux (1997) calls *cultural capital* that is made, inter alia, by the forms of knowledge, linguistic practices, values and styles that make up the quality of each teacher.

The research raises a number of teacher's beliefs that are related to perceptions about students, learning and self-evaluation. As elements to emphasize, they perceive that mathematics' students value alternative forms of assessment lowly and have a "mechanistic conception" of the discipline that is a product of their experiences during the previous stage of their higher education that have developed a unique insight about solving problems. The problem of evaluation is seen as something external to the teacher's own practice, and responsibility for failure falls on the students themselves. They recognize assessment as an important and complex process but also recognize its limitations in this regard. A landmark opinion is that one of the teachers assumed: "I think the way we evaluate the student does not correspond with the things that I think about mathematics".

Although no one can say with absolute certainty that the prescriptive or normative conception (Ernest, 1989, 1991), represented mainly by the philosophical current of formalism, is what characterizes the group under study, we can see that for some of these teachers the values of objectivity, theoretical considerations of the discipline

and the “universality of the mathematical knowledge” are fundamental components of its design. Despite some suggestions that the importance of the applicability arises as a concern, possibly linked to their condition as mathematics teacher trainers, the weight of the formal and conceptual seems to mark their views.

The students gave importance to the variety of assessment activities that expanded the single scheme of examination papers. They achieved a shared vision of evaluation as an instance that promotes mathematical learning. This finding is at odds with the belief that some teachers have about the "low value" that students give to alternative forms of assessment. There seems, therefore, to be a "student's consciousness", which they develop by joining the word with the action and theory and practice. But as Carr (1999) argues, this "has little to do with the" hostile "attitude of teachers or their inability to understand or implement the theories" (p. 52). It has to do with the conceptual foundations on which to build an educational practice.

The students argued for the acquisition of what might be called added value to the proposed activities, such as when they state: "One feeds as [...] sometimes not what you were looking for but you begin to nourish with other things from other parts of mathematics, then you say this is interesting, let me see other work and you get to read, read. " Here we get an important clue about evaluation as an instance promoting mathematical learning, which develops self-consciousness in individuals and groups. Evaluation is understood as a task that does not end with a final answer or product.

Students discussed their points of view, justified and supplemented them, they were able to bring into play different cognitive processes, assessing the importance of collective effort as more than the sum of individual efforts, the research opened the way for understanding social and cultural processes. The construction of students' knowledge consolidated their awareness. We worked to develop a critical view of education that promotes the development of democratic powers in the classroom and beyond (Amit and Fried, 2002; Bishop, 2007).

The constitution of working groups led students to commit themselves not only to learning but also to their peers. The collaborative work emerged as an essential, valued by students who believed that through a joint effort they could achieve “a more pluralistic vision of mathematics”. Students appreciated that beyond the application of a technique there is an understanding of the why and wherefore of things. Also, they stressed the need to connect the mathematical knowledge of prospective teachers with the world around them and with their life. For this, an important element is the use of the student experience.

The ideas of interaction and integration are explained by a student who talked about the group strengthening and the importance of breaking with the single conception of individual work: “it also breaks that vision we, mathematics teachers, have or the ones that are preparing for that, of individual work, individual work that we see for what it's worth, I think at least in the case of my group, I think the fact that the group

has emerged strengthened, **we realized that it was the group, that it wasn't a question of one individual** (highlighted in the original)". With these positions the strength of true dialogue has emerged as an essential nutrient of learning as an experience that allows an enriching exchange.

Students who have experienced a teaching model in which teaching practice and assessment practice are considered as integrated instances, giving importance to the group without neglecting the individual aspect, were able to develop a set of values where responsibility is not only placed on the teacher but students also made their own responsibilities. They had the intellectual honesty to put into play self-regulatory mechanisms that did not originate as rules imposed by the teacher, but arose through the commitments that were perceived as important.

SOME FINAL THOUGHTS

To carry out a proposed model to make an impact on assessment in mathematics teacher education it is essential to have a profound transformation of the mental models that are present in many educators and students, and in society itself. To avoid the possible danger that the proposal will be neutralized it is necessary to define new operators. The formation and progress of these new operators will be led by the concept of authentic assessment (Gulikers et al 2004, Rennert-Ariev, 2005).

Gulikers, Bastiaens, and Kirschner (2004) propose the following definition of authentic assessment: "An assessment requiring students to demonstrate the same (kind of) competences, or combinations of knowledge, skill and attitudes, that they need to apply in the criterion situation in professional life" (p. 69). This definition is a good starting point, but we believe that a revitalized notion of authentic assessment is necessary (Rennert-Ariev, 2005).

We must go beyond the proposal that the students "demonstrate the same (kind of) competences". It is necessary for authentic assessment to enable the student to develop new emergent competences to face a changing society. Moreover, the application of knowledge and skills to the realm of "professional life" must be closely linked to the construction of a "social life".

The development of authentic assessments must correspond to what we have called "new operators" which include things like: ways of learning, forms of appropriation of knowledge, field organizers and evaluative context. From the critical insight that we have assumed in this research we must pursue a proposal for mathematics assessment in teacher education with an emancipatory directionality, in the sense that there must be a shared goal of moral and ethical standards, understanding that authentic assessments are set within a social, cultural and political context. The proposal should be assumed in a context of equity and social justice, with tolerance for one another, involving the breakdown of hierarchical structures but nourishing a sense of responsibility and individual and social commitment.

The forms of learning should consider:

* *Collective learning* that involves ways of converging, managing and appropriating mathematical knowledge and its use in different contexts, by the groups.

* *Individual learning* to make effective the internalization of knowledge production processes and allows consideration and understanding of the different ways that individuals within a group "own" a certain knowledge.

* *Social learning* that involves a commitment to effective fields of production and access to knowledge beyond the formal school environment.

* *Self learning* conducive to self-awareness by both individuals and groups. This form of learning would strengthen the assessment as a task that does not end with a final answer or product within a formal classroom context.

These forms of learning are conceived as entities that provide feedback to enable the production of a continuous learning process.

Finally, we believe that this research, as it develops elements for an alternative model of assessment in mathematics, can become a point of reflection for the specific training of mathematics teachers. This concurs with the points made in the conclusions of the Inter-American Conference on Mathematics Education (1995): "It is imperative to have an adequate knowledge about the formation of mathematics teachers. And this training should be redirected, along the lines arising from the Mathematics Education". We recognize that the road may be arduous but, nevertheless, is necessary if we continue in the quest for the always cherished possible utopia.

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INNOVATION OR NOT? CONSISTENCY IN THE CURRICULUM PRESCRIPTION IN THE NEW CURRICULUM IN MOZAMBIQUE

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The study aims at exploring the interrelation and consistency in the new Mozambican Grade 8 curriculum across the proposed teaching methods, activities and evaluation criteria in relation to the innovative focus on the development of students' reasoning. It also seeks to understand the possible effects of the proposed changes on the classification and framing of school mathematics. The analysis of the new curriculum document produced some evidence of consistency between the aims, the goals and the objectives, and a dissonance of the teaching methods, tasks and activities with the innovative aspects of the new curriculum. In addition, the proposed pedagogy can be described as a weakening of the framing in some aspects. It is unlikely that this document is suitable to guide the innovation of teaching practice as intended.

INTRODUCTION

The curricula in Mozambique have experienced transformations during the last decades. The last reform process started in 2000 with the primary school curricula, and new curricula are being gradually introduced in the following grades. In 2008 the new curriculum was launched in grade 8. Presently, the mathematics primary and lower secondary school syllabuses cover the following topics: Number sets and operations, functions, equations and inequalities, Euclidian and spatial geometry, trigonometry and statistics. As the analysis deals with the grade 8 mathematics curriculum, I will describe the most important changes for this grade in some detail.

The grade 8 mathematics syllabus in Mozambique has been changed four times after the independence in 1975. In general the content in those syllabuses remained the same. In the third version, introduced in 2004, only slight changes in relation to the former were made in the objectives. They were subdivided into objectives of knowledge and objectives of competences. The rest of the text remained the same. In the historical context of curriculum development in Mozambique, the recent curriculum (2008) has to be interpreted as progressive. In contrast to the previous one, it generally promotes a learner-centred approach and is partly competency based. It tries to overcome a focus on mastering mathematical techniques and also intends to change the social base of instruction from lecture type to more students' involvement. The new version is to be meant to guide a reform of teaching practice.

In comparison to the previous versions, the new Grade 8 Syllabus exhibits two innovative features: the incorporation of mathematical competences centred around the development of students' reasoning and the use of heuristic methods and procedures that help students to construct his (her) own knowledge assuring the

meaningful understanding of the content. So, I decided to seek the extent of the consistency of the text of the Grade 8 Syllabus across the different sections referring to these two principles, which seemed to represent a shift in paradigm.

THEORETICAL BACKGROUND

As the new version of the Mozambican curriculum is to be meant to guide a reform of classroom practice, the question is to which extent a change in paradigm in content and pedagogy can be traced in the mathematics curriculum documents that are expected to be read by the teachers. The question is not to uncover the implicit values, but to look at the statements explicit in the curriculum document. So the focus is on the content and pedagogy as manifested in curriculum documents. The ways teachers interpret the curriculum documents and the unintentional effects the change might produce are subject of a case study of grade eight classrooms. Are there different identities of learners and their relation to the subject constructed or not? The general question is about how mathematical knowledge is recontextualized in the document and if this affects classroom practice or not. In terms of the, “recontextualizing fields” (cf. Bernstein, 2000, 56 f.), the groups who participated in the process are hard to locate. But the official end product in the curriculum document has to be taken to reflect the preferred modes on the side of the state school authority.

Garcia (2009) notes that the stating of curriculum aims and goals is not a consensus action, it depends on the social context, and is generally a source of debates between groups that want to see their values, views of knowledge, interests and ideologies expressed in the curriculum. Ernest (1991) identified five groups of interests with diverse views of mathematics and consequently different mathematical aims that influenced the reforms in the 80ths in Britain, namely, the industrial trainers, the technological pragmatists, the old mathematicians, the progressive educators and the public educators. The industrial trainers held up the teaching of basics numeracy, the technological pragmatists support the development mathematics useful to industry-based situations, the old humanist mathematicians on their side, proposed the transmission of pure mathematics, the progressive educators are concerned with the creativity and self-realisation of the students, whereas the public educators’ desire is the use of mathematics to develop critical and democratic citizens. These groups of interests may exist in any country and play an important role when educational reforms take place. A curriculum document can be analysed with respect to these ideologies.

The structure of the official curriculum may take different forms. The Mozambican Grade 8 Syllabus resembles a curriculum document rather than a syllabus. It includes the aim and the goals set by the Ministry of Education and are structured as following:

General aims to which all school subjects are expected to contribute (in different ways)

General goals categorized as related to the preparation of students for mastering their private everyday life, professional training and to the career options of the students, development of the students' personality (such as working attitudes), the functioning of the society (norms and values), and cultural heritage of the society.

The subject-specific part is organized in the following way: the introduction to the discipline, objectives of the discipline, a table of content specific objectives, detailed topics and student outcomes, methodological suggestions (content based), performance indicators, and assessment framework.

The general goals statement is an indispensable part, as it transmits the philosophy, the rationale and the aims of the educational system, that is, the ideology. The aims more or less steer the whole curriculum management process. Subordinate to them are the content standards, the set of subject topics and the abilities and skills students are expected to master, the pedagogy, which refers to teacher practices, the evaluation criteria that presents alternative forms of assessment, the performance standards with the indicators of expected students' achievement.

The conversion of a vague language, in which the goals are stated, into a specific set of tasks or rules cannot be considered as straightforward. It is not just a translation from general aims into more specific aims and detailed suggestions for topics and classroom management, but a series of redescriptions in another discourse. However, if the general aims valorize the development of mathematical reasoning, one would expect to find the concept "reasoning" or evidence of issues, proposed activities or methodological instructions that may reflect the criteria of what counts as mathematical reasoning in all parts. The mathematical content can be described by the internal and external classification, but it is only in the section "methodological suggestions", in which proposed changes in framing can perhaps be identified (Bernstein, 1975).

The specific goals can be related to different types of what is expected to be "known" by the students in each sub-area, such as understanding specific concepts, methods and principles or mastering distinct procedures and knowing a selection of facts. The move towards mathematical reasoning marks a move towards a more principled school mathematical discourse which makes more explicit the principles on which it is based.

Both, objectives and content may influence the choice of the pedagogy and the three elements support the instruments proposed to assess students' performance, teachers' methods, school materials and so forth. However, in each of these redescriptions and specification steps there is huge space for interpretation, in which different ideologies can come to play. Eventually, such a document can contain different messages conveyed to the teachers. The study tries to explore the consistency and coherence

across the aims, goals and objectives outlined by the educational system and the pedagogy and assessment proposed in the documents.

METHODOLOGY

The study aims at tracing two innovative aspects in all parts of the document, on the one hand, and the extent to which the proposed activities for classroom practice imply changes in classification and framing (Bernstein, 2000).

To attain the first target I attempted to compress the curriculum text in fewer categories that allowed me to infer the extent to which the different parts of the curriculum build a non-contradictory consistent chain of suggestions.

Content analysis is a powerful technique that fits to the objectives of my study, which is to look at the main trends and patterns (Stemler, 2001) and how they are interrelated crossways in the different components of the curriculum. As any research method content analysis techniques attracts critics. Subjectivity of the interpretative exploration (Oliveira et al., 2003), reduction of the study into a simple words count (Palmquist, 2001), draw of erroneous conclusions due to the use of more available words in the curriculum developers' lexicon (Stemler citing Weber, 1990) may weaken the study.

Trying to strengthen the reliability of the technique I was aware of the occurrence of synonym words, expressions with an ambiguous meaning or with a dual meaning. The words written with multiple meanings were pulled out and analyzed in the context where they were written. Moreover, a set of explicit recording instructions that rules the coding was developed (Stemler, 2001; Palmquist, 1980).

The texts were broken-down into simple words and paragraphs - the coding units of analysis, used to assay and interpret different characteristics of the message.

Inferences on the classification and framing can only be drawn from the parts that contain concrete suggestions for types of tasks and methodological setup of classroom practice. As to the ideologies, it seems possible to infer these to some extent from the text. This however amounts to the construction of "imaginary" recontextualisers because the different groups cannot easily be identified as agents in the process of the development of the new curriculum in Mozambique.

RESULTS

The following aims and goals to be achieved emerged from the text:

Aims: development of reasoning and debate of ideas, development of autonomous and critical thinking, and formulation of judgments, use of knowledge, abilities and values to propose alternative solutions, and being an active subject in the knowledge construction assuring the understanding of the meaning of the content.

Goals: use mathematics in order to properly think and reason, express and argue opinions, formulate judgements, give definitions and enunciate proprieties, interpret

tables, graphs, mathematics expressions and symbols, and transform the natural language to symbolic and vice-versa.

The goals principles seemed to be derived from the stated aims. There is an interrelation of meanings between both components. Two important dimensions that are present in the aims and goals statements are related to “students’ communication” and “students reasoning”. The extent of the prevalence of these in the objectives could be traced by the phrases or sentences that are related to “students’ communication” and “students reasoning” in those statements. The curriculum document provided the following verbs that can be seen as redescriptions of the two dimensions (for the number of occurrences see Table 1):

- Students’ communication: to discuss, to confront ideas, to explain, to justify
- Students’ reasoning: to observe regularities to relate intuitively to interpret to translate the natural language into symbolic language to characterize to estimate to define to demonstrate to enunciate theorems

The verbs in the first line, linked with communication situations, were used in the objectives section. However, the occurrence of such verbs varied. The verb 'to argument', which describes an important action in the communication, appeared twice in the general aims and goals but it was absent in the rest of the components as displayed in the table below.

	Explain	Confront of ideas	Argument	Justify	Discuss
Objectives	5	1		1	2
Methodology	3			2	2
Performance	1	1			
Assessment	1				
Total	10	2	0	3	4

Table 1: Occurrence of related verbs

In the second line are the verbs I assume to be linked with classroom activities usually used to engage the students to elicit their grasp of concepts.

The examples of tasks provided in the document consist of questions that step by step take the students to the stated goal, and were generally structured to be accessible to the students. However, the teacher was in most of the tasks suggested to summarize the discussions. In few cases, students in collaboration with the teacher are asked to do this and in others it is omitted who has to do. Furthermore, the tasks seemed not to be sufficiently rich in cognitive demands in order to trigger the appropriation of mathematical content.

Besides the tasks, the curriculum recommends students to engage in activities, using tools such as paper, pencil, graduated straight line, weight instrument, tables, diagrams, manipulative materials, compass, angle measure instruments and rulers. Generally, the activities do not involve much of a principled mathematical discourse. Students are required to read a table or a diagram, measure segments or angles, make some calculations.

An example: According to the curriculum instructions the students are by means of an “empty number line” expected to discover the order relationship of integers. However, to scale a straight line students have to master the order relationship already or the teacher has to scale the line, which includes a change in the social base of the activity. An activity with the use of a weighing scale, which might be a powerful tool, is proposed without any explanation of its relation to the topic. Similar to the other activity, it is suggested that the students carry it out in collaboration with the teacher or that the teacher demonstrates the activity to the students.

Tasks and activities are proposed to be completed individually or in group work with comparison of results and discussions about them, what presumes that students tackled the problems and drew conclusions. However, in most of the activities the role of summarizing, drawing conclusions, enunciating the theorems, stating the properties is assigned to the teacher. The methodological suggestions are concerned with the problem solutions recommending teachers to moderate students’ discussions where the results are compared. There was emphasis to look at different results rather than in the justification of the procedures and its relation to the underlying mathematical concepts or the relation between the solution and the parameters (cf. Carpenter and Lehrer, 1999).

In the assessment the main aims and goals of the curriculum seemed to be expressed in the sentences that suggest teachers to emphasize not just the memorization of rules and procedures. The recommendations highlight the evaluation of understanding of concepts, the development of reasoning, but also the “know how to do”. Furthermore, it is proposed to make use of students’ portfolio, however, without any explanation of how a teacher may use the items to assess in quantitative terms the achievement of a student which is the final requirement at the end of each trimester.

As a subject, school mathematics remains to be strongly classified: it has its own timetable and it appears organisationally insulated from the other subjects and is taught by one teacher. However, in the new document it appears less strongly classified than in the older versions in terms of the content as it proposes some integration of non-academic issues. The curriculum establishes along with the topics the lessons allocated to each topic, and the methodological sections describe in detail the transmission strategies proposing steps to follow and possible examples. In doing so, it seems that teachers may have little room in relation to the selection of the content sequencing and pacing. However, there appears to be space for discussions,

students may be given opportunities to choose different forms of communication. In this aspect, there is a weakening of the framing as compared to the previous one.

DISCUSSION AND CONCLUSIONS

The first aim of this study was to explore, the extent of the prevalence of two main innovative aspects of the new Mozambican Grade 8 curriculum, namely the development of students' reasoning and the support of meaningful understanding (through a change in the base for communication) across the curriculum components. These aspects are outlined in the general subject-related aims. It turned out that at the more detailed goals statements indeed tend to explicate and specify the ideas stated in the aims. In addition, the verbs signifying students' activities in the descriptions of intended outcomes seem to specify actions that may accomplish the curriculum intentions. So there is a chain of related meanings, in which the innovative aims are redescribed as goals and as objectives. The document is written in a way that makes it possible to read the goals statements as an instruction of how to read the more general aims, and the objectives as an instruction to read the goals. The meanings remain consistent at these levels with regard to the innovative aspects of the new curriculum.

However, the tasks and the activities proposed in the parts containing the methodological suggestions are not structured in a way that matches the intention to the change in the social base for the communication. While it is suggested that the students, for example, engage in drawing conclusions, searching for patterns and seeking generalisations, the teacher is advised to enunciate the pre-defined outcomes of these activities. This suggests that there should eventually be strong framing over the criteria, though on the other hand, while the students are suggested to work in groups on some more open activities, this could be weakened. The innovative aspect of changing the procedural discourse into a more principled one is not reflected in the criteria for the activities to be carried out when solving the tasks, which are proposed. Altogether, the lack of theoretical challenge of the tasks and activities and the suggested teaching strategies do not reflect the evaluation criteria for the activities proposed in the aims.

The section about assessment, which is written without reference to mathematics as a subject, reflects a change in pedagogy. Teachers are, for example, urged to use portfolios to gather information about the development of students' reasoning, abilities, skills, attitudes and values. It is not possible to link the proposed modes of assessment to the other parts of the curriculum in terms of criteria. The techniques listed in the section can be interpreted as a move towards implicitness because some of the suggested activities (for example students' documenting their own work orally and in written form) imply a weaker framing over the criteria. This is in contrast to the suggestions from the methodological section.

The new curriculum document for grade eight conveys contradictory messages. The most specific parts, that is the sections with methodological suggestions, are most likely to be taken up by the teachers. In this case, the innovative effect would be minimal. Some parts could be interpreted in a way that suggest to operate in or switch between different more strongly and more weakly framed activities. The constitution of the mathematical activities as an unprincipled discourse that is reflected in the tasks, is in contrast to the main intention to change this towards a more principled one. Theoretically, the teachers have the freedom to select other than the suggested types of tasks and activities and only take into account the general aims, goals and objectives. The school or national tests may reflect the prototype activities and tasks presented in the methodology section and thus set the criteria for what is to be achieved by the students. So, this may take teachers to prepare students for the tests and examination, holding back their initiative to support innovative aspects. The criteria conveyed in the examination papers can impact negatively on the type of more principled mathematical discourse expressed by the general aims, and even on the level suggested in the methodological section, as for example shown by Saldanha and Neves (2006) in they study about the impact of the national tests on the teachers' practice.

The document can be interpreted as an outcome of hybrid ideologies. The assessment framework and the statements in the aims, goals and objectives can be interpreted as a reflection of progressive pedagogy, whereas the section containing specified descriptions of mathematical activities and teaching strategies, if one would apply Ernest's (1991) categories, can be interpreted as reflecting the aims of "technological pragmatists" in a perspective where mathematics is considered a body of technical knowledge, and the students are exposed to applicable mathematics.

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WHERE DID IT ALL GO RIGHT? THE SOCIO-POLITICAL DEVELOPMENT OF GAELIGE AS A MEDIUM FOR LEARNING MATHEMATICS IN IRELAND

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A significant language shift has taken place in Ireland – Gaeilge (Irish) has become trendy. It is difficult to locate where and when this change began but political and social issues have played a significant influence on this development. In particular, the explosion of Gaeilge-medium primary and second level schools has played a crucial role in the rebirth of our native language. This paper provides a historical perspective on the development of Gaeilge and Gaeilge-medium education in Ireland with particular emphasis on mathematics education and some of the socio-political issues influencing this significant development.

INTRODUCTION

For generations raised on ‘Peig²’, a significant language transformation has taken place in Ireland – Gaeilge (Irish) has become trendy. Normality is engraved in sending our children to Gaeilge-medium schools, seeing comedians doing gigs ‘as Gaeilge’, and tuning into television programmes presented by fluent Gaeilge speakers. In a very short period we have progressed from shunning our native language to endorsing it as a fashionable and positive thing for our country. Where did it all go right for Gaeilge? It is difficult to pinpoint exactly where it all started but what is clear is that this impromptu revolution came about through a combination of significant socio-political developments. The most enthusiastic catalyst has been the dramatic changing face of our primary and second level education system – the explosion of Gaeilge-medium schools. Previously Gaeilge-medium education was limited to remote isolated parts of Ireland known as Gaeltachtaí (all-Irish speaking districts) and these institutions were viewed by outsiders as strange and archaic. However, sending your children to Gaeilge-medium education (outside of these Gaeltacht areas) is now as necessary as possessing the latest iPod or mobile phone. Accordingly this paper will address the socio-political development of Gaeilge-medium education in Ireland, with specific emphasis on the effect of this on mathematics education in the country.

THE IRISH CONTEXT

In order to understand the development of Gaeilge as a medium for teaching and learning mathematics in Ireland it is first necessary to give a brief political history of the Irish language in Ireland. For the purpose of this paper and to draw on the research undertaken by the author, she will specifically address the context in the Republic of Ireland, while illuminating comparisons with Northern Ireland where appropriate. Perhaps this is the most apt place to draw attention to the political role

that successive governments have played in the development of Gaeilge and various language policies, but it is crucial to examine the actions leading up to the political divide of the country in 1921. Up until the 16th century, Gaeilge and its associated culture and traditions were dominant throughout the island of Ireland, surviving invasions by Norman and Viking groups. However, English colonisation of Ireland began in the mid 16th century and continued into the 17th and 18th centuries by means of eradicating the Irish from their lands and replacing them with English and Scottish colonists. The persecution of the Irish people was relentless and coupled with the Great Famine that swept the country during 1845 to 1852, a dramatic decline in the number of Gaeilge speakers and use of the Gaeilge language in Ireland was observed. A number of significant Gaeilge language organisations were established during this period in order to halt the decline of the use of the language including the Ulster Gaelic Society and Conradh na Gaeilge (The Gaelic League) who published documents in Gaeilge and promoted its use in everyday and academic settings. Since the foundation of An Saor Stáit (The Free State) in 1921, a divide has been established in Ireland – the Republic of Ireland (26 counties) and Northern Ireland (6 counties). Gaeilge is the first official language (English the second) of the Republic of Ireland and with the establishment of An Saor Stáit (1921) it was intended to restore the Gaeilge language and its use throughout the country. This ambitious aim was never achieved and currently Gaeilge is spoken natively by a small but increasing minority (95,503 people) of the population and specifically in 7 regions (official Gaeilge speaking districts in the Republic of Ireland) known as Gaeltachtaí (CSO, 2006). A number of positive social and political developments in relation to Gaeilge have taken place in the past 10 years including consolidation of the language at constitutional level (Official Languages Act, 2003); legal contexts (appointment of an Official Languages Commissioner); and at European level with the establishment of Gaeilge as an Official Language (2006) of the European Union (Harris, 2007). Other significant revitalisation movements have been largely targeted through education (which will be discussed in the next section); through the media - mainly TG4 (national television station) and Radió na Gaeltachta (national radio station); and an increase use of Gaeilge in the public sector through bilingual (Gaeilge and English) provision of advertising and services. All of these are positive developments for the language and have contributed to its increased use nationally and internationally.

THE HISTORY OF GAEILGE AS A MEDIUM OF LEARNING

The pressure of a few hundred years of foreign occupation, along with the complicated political, religious and economic pressures of the 18th and 19th centuries, had rendered Gaeilge non-existent at the top of the social scale in Ireland. More importantly, it had weakened its position among the entire population (Ó Cuiv, 1969). The State (under British Rule) became involved in the provision of education for the first time in the early nineteenth century, which led to the setting up of the National School system (1831). This education system is described as having “a

British cultural emphasis” and having “crushed the Irish language” (Kelly, 2002, p.4). The introduction and use of the “Bata Scóir” (a tally stick used to hit students depending on the number of times they spoke Gaeilge) by teachers quickly spread as primary schools were set up throughout the country, resulting in the prohibition of Gaeilge as medium of instruction and communication. Parents supported this punishment system, as Gaeilge was associated with poverty and English increasingly with economic prosperity.

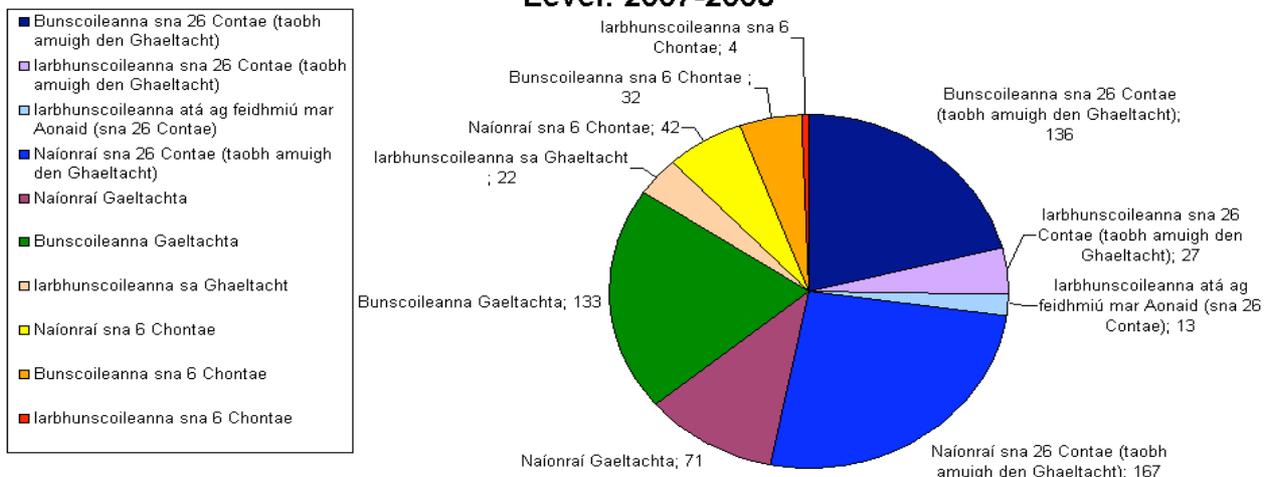
Secondary education, unlike primary education, at that time was reserved mainly for the rich and those who could afford to pay to attend second level education. Secondary schools were not widespread and therefore only a select few continued with second level education. However, like the National Schools, Gaeilge was banned from being taught and spoken within the schools and emphasis was placed on the English language. Therefore, during the nineteenth century it was evident that English was rapidly replacing Gaeilge as the native language, and the strict prohibition of Gaeilge in the education system was perceived as being instrumental in this change (Kelly, 2002).

When An Soar Stáit was established in 1921, Gaeilge was recognised as the first official language, with the intention of restoring it throughout the country (Purdon, 1999). The new state adopted a programme for restoring Gaeilge that was aimed almost exclusively at school children. The plan was to immerse all children in Gaeilge for the entire period of their schooling, so that in the space of a generation or so, the language would be brought back to everyday use (MacAogáin, 1990). The schools and education were chosen to revive the language as it was felt that they had been responsible for displacing Gaeilge with English. Also it was believed that teaching Gaeilge as a subject alone was not sufficient for reviving the language so more drastic measures would be needed and therefore all subjects, including mathematics, were to be taught through this language medium (Kelly, 2002). The debate on using Gaeilge as a medium of instruction in primary schools and a lack of implementation in all schools, continued through the subsequent decades. However, in November 1959, Dr. Patrick Hillary, the then Minister for Education, proposed that schools and teachers should concentrate on teaching Gaeilge well rather than teaching through the medium of Gaeilge (Kelly, 2002). Subsequently, two months later the Government abolished the use of Gaeilge as a medium of instruction in all but a minority of primary schools (Gaeltacht schools). Overall it was felt that Gaeilge, as a medium of instruction, had reduced the standard of education with little improvement in the use and status of the language outside of education (MacNamara, 1966).

At the time of the establishment of the Free State the emphasis on Gaeilge in second level schools was less intense than it was in primary schools. However, from 1927 Gaeilge became a compulsory subject for the award of the key state examination certificates in 1934 (Department of Education, 1975). From 1924 the Government provided additional grants to schools using Gaeilge, with the amount received

dependent on the level of Gaeilge being used and spoken. As a result the number of students sitting Gaeilge in examinations increased by 15% within the first ten years of Independence (Kelly, 2002). This trend continued through the 1930s and 1940s with the number of schools teaching through the medium of Gaeilge and the number of pupils sitting the Gaeilge examination increasing steadily. Clearly and negatively what was sustaining this were the financial rewards that the Government offered those willing to use Gaeilge to the greatest extent possible within the schools. However, Gaeilge medium education at second level was primarily limited to the Gaeltacht regions of Ireland. Since the 1920s secondary schools (and primary schools) were obliged to teach Gaeilge. Compulsion was the most “consistent trait” of any of the language policies introduced (Kelly, 2002, p.14). Gaeilge was a compulsory curriculum subject, a compulsory examination subject and a requirement in order to receive certification. It wasn’t until 1973, when Richard Burke was the Minister for Education, that the requirement to pass Gaeilge in order to pass the Leaving, Intermediate and Group Certificate examinations at second level was dropped. However, an honour in Higher Level Gaeilge is still required to enter primary level teacher-training colleges. So the element of compulsion is still present for many students.

Oideachas trí Ghaeilge ón Réamhscolaíocht go dtí an Dara Leibhéal/ Education through Irish from Pre-School to Second Level: 2007-2008



Iomlán=647

1: Níl fáil ar líon iomlán bunscoileanna Gaeltachta atá ag feidhmiú trí Ghaeilge.
 2: Tá 19 n-iarbhunscoil Gaeltachta ag feidhmiú trí Ghaeilge.

Fig.1: Number of Gaeilge-medium pre-schools (Naíonraí), primary (Bunscoileanna) and second level (Iarbhunscoileanna) schools in the Republic (26 contae) and Northern Ireland (6 contae).

A significant development in relation to Gaeilge and Gaeilge in schools is the increase in the number of Gaelscoileanna (primary schools teaching through the

medium of Gaeilge) and Gaelcholáistí (second level schools teaching through the medium of Gaeilge) – Gaeilge-medium immersion education outside of the Gaeltacht regions. In 1972 there were 11 primary and 5 secondary schools providing education through Gaeilge outside of Gaeltacht areas. However, the rise in popularity of immersion education is significant and has seen an increase in excess of 60% over the past decade. Currently, 136 Gaelscoileanna and 50 Gaelcholáistí have been established in the Republic of Ireland with an estimated 33,000 pupils attending these schools (Gaelscoileanna Teo., 2008, see Fig. 1). Combining this with the number of students attending primary and second level schools in the Gaeltacht areas, approximately 7% of the total primary level population and 2.5% of the second level population are learning mathematics through the medium of Gaeilge. Also, coinciding with this is the development of Naíonraí Gaelacha (Gaeilge medium play schools) for pre-schoolers. Immersion Gaeilge-medium education is largely a parent initiated voluntary movement provoked by the lack of success of State language policies since 1922 (Ní Mhurchú, 2001). This suggests that the general public's interest in the native language is still strong, as is their desire for their children to learn through the medium of Gaeilge.

Since the 1970s Gaeilge has been taught as a school subject only (Gaeltacht schools and Gaelscoileanna/Gaelcholáistí being the exceptions). It is part of the core curriculum during the years of compulsory schooling, six to sixteen. Even though Gaeilge as a medium of instruction in the Irish school system has undergone many changes, significant numbers of students are learning mathematics through this medium. However, what is clear and of importance is that the Irish Government has played a significant role in establishing Gaeilge as a medium of instruction in primary and secondary schools. Clearly,

“Decisions about which language to use, how, and for what purpose(s), are political. This political role of language is not dealt with in the literature on bi/multilingualism and the teaching and learning of mathematics.” (Setati, 2002, p.13).

SOCIO-POLITICAL CONCERNS

The decisions of which language(s) are used in education are predominantly political in nature (Edwards, 1994). As demonstrated in the previous sections the history of the Gaeilge language in Ireland and as a medium for learning mathematics has been marred by issues of access, power and dominance. In the 19th and 20th centuries, English was the dominant language of learning within the country, regardless of mother-tongue spoken. With the establishment of the new state (1921) Gaeilge-medium education was central to policy plans for the revitalisation of the language and compulsion was rampant throughout the country but detrimental to student learning (Kelly, 2002; MacNamara, 1966). Therefore the last two hundred years has borne witness to the Gaeilge language experiencing both sides of the coin - it being

against the law *to* teach/speak Gaeilge and it being against the law *not to* teach/speak Gaeilge. Both extremes were introduced by the then governments and were implemented through the use of education and the schools. It is evident that the governments played a crucial role in the position of Gaeilge in schools and as a consequence its status in society. Once this element of compulsion was removed in the sixties, a catalyst for change transpired. At this time access to Gaeilge-medium education was primarily reserved for people living in the Gaeltacht regions of Ireland due to social and cultural necessity, but was looked upon by many outside these regions as backward and restrictive given the English language association with universality and economic prosperity (Kelly, 2002). However, removing compulsion signalled an element of eliminating ‘choice’ for parents outside of Gaeltacht regions and thus the seeds for Gaeilge-medium immersion education were sown.

Clearly, some of the Irish government interventions were not always done with tact or wisdom that might have made them more effective. Take the example of the position of Gaeilge in the schools of six counties in the North. Irish was tolerated as an optional foreign language only and as an acceptable subject in secondary schools. By the 1950s, Gaeilge was as popular and chosen as often as French was. And as Lord Charlemont, the Stormont Minister of Education said, “forbidding it (Irish) under pressure will stimulate it to such an extreme that the very dogs – at any rate, the Falls Road dogs – will bark in Irish” (as cited by Purdon, 1999, p.59). The first Gaeilge-medium primary school in Northern Ireland was established in 1971 which saw an intake of only 9 pupils (Ó Baoill, 2007). But the growth and recognition of Gaeilge-medium immersion education in the North has been as phenomenal as in the Republic, and has led to the development of a small but unique urban community of Gaeilge speakers in Northern Ireland. Similarly in the Republic of Ireland, immersion Gaeilge-medium education was stimulated when the compulsion element was removed, and the students’ option of learning through the medium of Gaeilge (outside of Gaeltacht regions). Social structures emphasise the importance of choice and access to Gaeilge-medium education and that children should not be denied this opportunity for learning. Hence, Gaelscoileanna emerged in the 1970’s and were independent of other primary schools in their locality. Initially, students enrolled in immersion education were restricted to those coming from Gaeilge speaking homes and had a strong grasp of the Gaeilge language (Ó Baoill, 2007). Therefore, discrimination was evident in the early days but policies changed due to demand for access and the general public’s interest in sending their children to immersion education and developing bilingualism (Gaeilge and English).

Although the initial establishment of Gaeilge-medium education (outside of Gaeltacht regions) arose out of social influences (largely community and parental initiatives) and a resistance to political policies, growth and development have been stimulated by financial and facilitative support by various Governments and governmental bodies throughout the late nineties and early twenty first century (Ó Baoill, 2007). Another consideration is that mathematics learning and teaching are

socially and culturally situated, and mathematics cannot be considered culture free (Bishop, 1988) with mathematical knowledge culturally based and embedding social and cultural values of the group. As Barton (2008) sums up adequately “The practical reality is that every indigenous peoples’ context is different” (p. 167). The range of difference is broad. For example differences may exist in relation to language use; differences in relation the political situation in the context; differences in education; and many more (Barton, 2008). Children growing up in the Gaeltacht areas of Ireland are immersed in a different language and culture to those growing up in all English communities. Similarly, children attending the Immersion schools will have different experience to those in Gaeltacht areas and all-English areas. Accordingly, it is anticipated that these two different Gaeilge groups within the Irish context will possess a different world-view, and accordingly a different mathematical world, to those from an all-English environment within the country.

MATHEMATICS EDUCATION RESEARCH IN THE IRISH CONTEXT

Given the significant growth in Gaeilge-medium education in recent times it is surprising that little research has been undertaken in the Irish context. The author’s work is the first of its kind to be undertaken on mathematics education and bilingualism (Gaeilge and English) in the Irish context and its findings are significant and demonstrates that bilingualism is positive for mathematics learning (Ní Ríordáin & O’ Donoghue, 2009). Gaeilgeoirí (students who learn through the medium of Gaeilge) outperform monolingual students mathematically, once an appropriate proficiency in both languages has been achieved. Bilingualism provides students with the ability to undertake mathematical thinking in two languages and thus provides a cognitive advantage over monolingual students. The author’s findings are consistent with work carried out internationally (e.g. Clarkson, 1992; Cummins & Swain, 1996; Dawe, 1983) and contribute to the robustness of Cummins’ Threshold Hypothesis (1976). Other factors examined included the difficulties encountered with the English mathematics register; an assessment of where problem solving through the medium of English breaks down for Gaeilgeoirí; and an investigation of the qualitative aspects of a transition to a new language of learning for mathematics (Ní Ríordáin, 2008). Clearly, this research is language orientated and has provided positive information on the Irish context. However, this is not sufficient to provide a comprehensive perspective on learning mathematics through the medium of Gaeilge, nor sufficient to sustain research in this field of mathematics education in Ireland.

In particular, if the author is to discuss her work in relation to other cultures it is necessary to examine issues other than just the linguistic aspect. There is a need to move from a language orientation to greater issues of a socio-political and cultural nature. Mathematical discourse is concerned with the ways in which ideas are expressed as well as what those ideas entail. The ways of representing, thinking, talking, questioning, agreeing, and disagreeing is central to students learning and understanding of mathematics. How this is conveyed through the medium of Gaeilge

will be different to how mathematical discourse is conveyed through the medium of English or any other language. However, what is of concern to the author is the process through which the mathematics register was developed through the medium of Gaeilge (and continues to be developed) and accordingly the influence that this has on the mathematical discourse taking place in Gaeilge-medium education. There is a need to look further at how learning mathematics through the medium of Gaeilge can be developed in order to ensure positive bilingual outcomes for all students learning through the medium of Gaeilge.

Two theories that provide the justification for examining further the cultural and socio-political aspect of mathematical discourse in Gaeilge are those of ‘alterity’ (Brandao, 1997 as presented by Domite, 2009) and ‘listening to’ (Freire, 1996)³. Alterity refers to the process of defining “other” groups in relation to ones own group. Key points that need to be considered in relation to mathematics education from a cultural/political aspect include that the process of alterity generates awareness of our own perceptions and beliefs; it creates social and cultural order; and through engagement in the process we may fail to take into account information from outside our own cultural assumptions. Thus, the process of alterity highlights how group differences may occur. Paulo Freire’s (1996) concept of ‘listening to’ is a way of expressing the need to hear others in order to engage in meaningful dialogue with them and to develop a deeper understanding of the ‘other’s’ knowledge and worldview. This concept is closely related to a political attitude in recognizing that we can interfere and generate educational change for human good. It also incorporates a pedagogical attitude in that for example mathematics teachers in English medium education with Gaeilgeoirí in their class can choose to cater for these students through their teaching of mathematics.

The process of alterity firstly permits us to question how Gaeilgeoirí and Gaeilge-medium mathematics education are socially and culturally positioned within the Irish education system. Secondly it generates curiosity in relation to who is developing the mathematics register in Gaeilge and how does the process of translating the English mathematics register into Gaeilge capture the *fior-Ghaeilge* (true/old Irish) mathematics register in use. Therefore, a key aim of future research is to investigate the consequences of these actions on the development of the Gaeilge mathematics register and in turn the potential influence of this on mathematical discourse through the medium of Gaeilge. The concept of ‘listening to’ generates questions in relation to who was involved in developing the Gaeilge mathematics register and who was consulted and ‘listened to’ in order to develop the current register in use. The author seeks to chart the development of the Gaeilge mathematics register through the decades and those involved in its development. In particular the author seeks to establish how the socio-cultural background of those involved may have influenced the current mathematics register and discourse in use in Gaeilge medium mathematics classrooms at primary and second level education in Ireland, and compare to what a *fior-Ghaeilge* mathematics register might look like. There is a

need to address the political and cultural influence on the development of the Gaeilge mathematics register, and accordingly if suggestions can be generated in order to enhance the mathematical discourse taking place in Gaeilge medium education.

CONCLUSION

Gaeilge is more accessible to the majority of people now (media, increase in language provision, advertising, etc.) and attitudes have changed towards the language and its use in daily life in Ireland. This reflects the significant increase in Gaeilge-medium education provision within the country, a provision that continues to increase annually. Clearly, in the past decade or so, Gaeilge and our national heritage have undergone a revival, thus reinstating pride in our native language and culture but this is a relatively new phenomenon and the best is yet to come. This paper has presented the socio-political influences on the development of Gaeilge as a medium of learning and but little research has been carried out in the Irish context in relation to this. The next step lies in extending the mathematics education research undertaken to date by the author to examining the socio-political and cultural influences on the development of the mathematics register through the medium of Gaeilge in Ireland.

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NOTES

¹ National Centre for Excellence in Mathematics and Science Teaching and Learning.

² Core textbook for the Gaeilge syllabus at second level – extremely difficult and hated by all who studied it.

³ See the acknowledgements section at the end of this paper.

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FROM QUESTIONS OF *HOW* TO QUESTIONS OF *WHY* IN MATHEMATICS EDUCATION RESEARCH

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*The educational sciences are generally construed around concerns of providing research that informs practices of learning and teaching in educational institutions. This research emphasizes questions of **how to** and has led to a “technification” of educational research, as primarily concerned with providing solutions to practical problems. In this paper we will show how mathematics education as a research field is not an exception, by analysing how theory is understood and used in the field, to address questions of how. We suggest that, although important, this research leave some important areas unaddressed, namely the ones which can emerge from posing questions of **why**. We argue that making this move implies rethinking and enlarging definitions and views of mathematics education research.*

INTRODUCTION

In recent decades the field of mathematics education research has opened its agenda towards new paradigms and discourses, and it has expanded the field also to include issues of the social, the cultural and the political. Issues of social justice (Gutstein, 2003), critical mathematics education (Skovsmose, 1994), equity (Secada, Fennema, & Adajian, 1995), ethnomathematics (D’Ambrosio, 2002) among others, have become influential players in a research field otherwise and continuously dominated by research exploring psychological and cognitive aspects of students’ and teachers’ engagement with mathematics. Although we consider this move towards the socio-political and socio-cultural a significant one, we also see a need to move the boundaries even further. We thus suggest a move from a research agenda primarily contained within a very specific discourse of the importance of mathematics education, addressing primarily questions of *how* to improve possibilities for teaching and learning mathematics, towards a research agenda strongly concerned with addressing the question of *why* mathematics education. In making this move we see possibilities of opening up the field to alternative discourses and ways of constructing important understandings about the teaching and learning of mathematics in complex social, political and economic settings. We will explore this move from questions of *how* to questions of *why* in relation to the role of theory in mathematics education research. We will argue that the overwhelming majority of theories constructed in the field aim to address questions of *how* and, therefore, do not have the possibility of seeing beyond a technical rationality in order to understand the *whys* of the configuration of mathematics education practices in classrooms, schools and society.

Based on an analysis of recent literature addressing the role of theory in mathematics education research, we start by pointing to the way this research is structured around

questions of *how*. We then analyse some recent trends in mathematics education research (arising out of the so-called “social-turn” (Lerman, 2000)), which has contributed to an enlargement of a field traditionally dominated by a didactical perspective. This research has opened the field to questions broader than those strictly concerned with providing immediate solutions for practical problems. Nevertheless, we will argue that even research presented within the scopes of the social, cultural and political often focuses on questions of *how*. We then proceed to bring in questions of *why*, by exploring new discourses embedded into this simple question. We conclude the paper with some brief comments about the implications of transgressing the boundaries of the existing discourses shaping the field of mathematics education research.

THEORY CONSTRUCTING RESEARCH IN MATHEMATICS EDUCATION

Theory as a key component of mathematics education research is currently on the agenda. At ICME 11 in 2008 one of the survey teams developed a study on the notion and role of theory in mathematics education research. This survey team had the task of identifying, surveying, and analysing different notions and roles of ‘theory’, as well as providing an account of the origin, nature, uses, and implications of specific theoretical directions pertaining to different research developments in the field. Similarly, the *Second Handbook on Mathematics Teaching and Learning* (Lester, 2007) contains two articles addressing issues of theory (Cobb, 2007; Silver & Herbst, 2007). In CERME there has been a working group linking, contrasting and comparing the wide variety of theoretical approaches found in the field in order to tackle the teaching and learning of mathematics. In 2008 the international journal *ZDM* published an issue of some of the results of the CERME working group. Finally in 2009 the theme of PME 33 was “In search for theories in mathematics education”. These examples point to a widespread desire of the community for understanding the role of theory in mathematics education research and a wide acknowledgement of the variety of perspectives brought into the field through theoretical expositions. As Silver and Herbst (2007, p. 41) state, “the moment seems propitious for a serious examination of the role that theory plays and could play in the formulation of problems, in the design and methods employed, and in the interpretation of findings in education research.”

We wish to make a modest contribution to this discussion by engaging in a critical analysis raising questions of *how* and *why*. We wish to understand in more detail how research perspectives in general and theoretical perspectives in particular construct and/or ignore particular discourses and, in this, our possibilities for addressing these basic yet powerful questions.

As the “linguistic turn” in the social sciences has touched mathematics education research (Lerman, 2000), it appears increasingly important to pay attention to the discourses that mathematics education research constructs about itself and the

contributions and limitations of these constructions. By discourses here we understand the ways of naming and phrasing the ideas, values and norms that emerge from the constant and complex interactions among human beings while engaged in social practices. Academic fields construct particular discourses about themselves and their objects of study. Such discourses constitute systems of reason that regulate what is possible to think and do in a given field (Popkewitz, 2004). Discourses thus both open up possibilities and impose limitations on what we can imagine and construct as alternatives to existing orders. Mathematics education as a field of research is not an exception. As researchers engage in studying the field, they not only define what is characterized as legitimate practices of mathematics education. They also define the ways in which it is valid and legitimate to research those practices (Valero, 2009). We have engaged elsewhere in examining the discourses generated in and by the field of mathematics education research, such as the idea of mathematics education being “powerful” (Christensen, Stentoft & Valero, 2008), the conceptions of students as mathematics learners (Valero, 2004), the concept of learners’ identity in mathematics (Stentoft & Valero, in press b) and the concept and view of ethnomathematics (Domite & Pais, 2009). We have also pointed to some blind spots of some of the theoretical constructions in the field. Considering these constructions of various discourses in the field we argue for the need to broaden the research gaze of mathematics education research to embrace the “noises” that are often ignored, in a search for new imaginaries for our field of study and for the educational practices in mathematics (Stentoft & Valero, in press a).

MATHEMATICS EDUCATION RESEARCH AS A SCIENCE OF HOW

One major assumption in mathematics education research is that its main aim is to improve students’ performance in mathematics. For example, Niss (2007, p. 1293) is very clear when answering the question of why do we do research in mathematics education: “We do research on the teaching and learning of mathematics because there are far too many students of mathematics, from kindergarten to university, who get much less out of their mathematical education than would be desirable for them and for society.” If this is the main concern of mathematics education research, it is not surprising that the field has grown as a space for researching in a systematic, scientific way “the problems of practice” (Silver & Herbst, p. 45), defined as problems relating to teaching and learning. According to Boero (in press) “this is a rather obvious widely shared position” (p. 1). In this framework, the work of mathematics educators is “to identify important teaching and learning problems, considerer different existing theories and try to understand the potential and limitations of the tools provided by these theories.” (Boero, in press, p. 1)

The above quotes demonstrate an emphasis in the field of mathematics education research on the questions of *how*. How can we improve and enhance the teaching and learning of mathematics? How can we help students to learn? These questions are highlighted further when Cobb (2007) addresses the issue of philosophy in

mathematics education as he in a concise manner addresses assumptions engulfing the field of research. Cobb suggests that mathematics education should be understood as a “design science” (2007, p. 7), and provides as an example the NCTM standards. By design science Cobb understands “the collective mission which involves developing, testing, and revising conjectured designs for supporting envisioned learning process” (p. 7). The ultimate goal of a science designed this way is to “support the improvement of students’ mathematical learning” (p. 8). As part of the pragmatic realist philosophy adopted by Cobb, attention is given to the comparison between four significant theoretical perspectives used in mathematics education research, namely experimental psychology, cognitive psychology, socio-cultural theory and distributed cognition. Cobb’s discussion revolves around how these theoretical perspectives could help improving students’ learning of mathematics. We can research at the level of the national educational system, school or classroom, however the goal remains the same. In Cobb’s writing, theory is understood as a tool to give insight and understanding into learning processes with the aim of improving them.

An alluring analogy made by Silver and Herbst (2007) between mathematics education and medicine helps us to understand the meaning of theory as “theory for learning”. The authors play with the analogy that mathematics education can be seen as a science of treatment, similar to medicine: By understanding the symptoms that characterise the difficulties of students’ mathematical learning we can propose the proper treatment. They state: “The evolving understanding of the logic of errors has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the logic of diseases has helped the design of better medical treatments” (Silver & Herbst, 2007, p. 63). In this perspective, students are seen as patients in need of treatment, and the role of mathematics education research is to understand students’ problems and elaborate designs that direct us *how* to treat those learning diseases.

This trend that focuses on learning — enhancing or remediating it—is not exclusive to the field of mathematics education research. Philosophers of education such as Biesta (2005) argue that over the last two decades this perspective has proliferated in broader educational discourses where a technical language of learning has largely dominated and almost overruled a language of education. The “learnification of education”, in Biesta’s terms, has narrowed the possibilities to think and do education and educational research. The disagreements about the role of school and the goals of education that fuelled part of the educational debate during the last century¹ seem to have been overcome. We appear to have reached a consensus on the benefits of schooling: we need to make it more effective and, therefore, we live an apparent consensus about what concerns education. The problems with schooling and school subjects are no longer to be political or ideological, but have become primarily technical or didactical. In most cases, solutions to educational problems are being reduced to the devising of better teaching and learning methods and

techniques, to improve the use of technology, to assess student's performance, etc. Educational thinking has progressively been reduced to be a controllable, designable, engineerable and operational framework of action for the improvement of individual cognitive change. It is obvious that the research supporting the emergence of this type of discourse is a research essentially concerned with questions of *how*.

Although the prevalence of theory as “learning theory” has allowed us to gain deeper knowledge about the processes of teaching and learning mathematics, we suggest that it has left important discourses faced by the educational communities in their everyday practices unaddressed. We will argue that in order to bring these discourses seriously into the gaze of research, we need a broader theoretical palette which allows us to understand theory not just as “theory of learning”, but also as “theory of education”. This leads us to propose another type of question for the research agenda, namely the questions of *why*.

TOWARDS QUESTIONS OF *WHY*

As mentioned above, the “social turn” (Lerman, 2000) in mathematics education brought to the field new concerns and new theories that progressively de-emphasise cognitive psychology as the only interpretative framework and instead favour socio-cultural theories. In this we have witnessed a move from an understanding of children's learning focused on the individual subject and his cognition to an understanding that perceives learning as a product of social activity, where not only the cognition of the subject is at stake but also his relations with other individuals and their shared discourses.

This trend is not merely related to a displacement of the way we perceive processes of learning. According to Lerman (2000) this trend also emerged as a result of growing political concerns about the ways mathematics education could be linked to reproduction of inequalities through the structures of school. Several studies in recent years have contributed to an understanding of mathematics education in association with issues of social exclusion according to race, gender, language, social class and culture. Those studies have opened up a space of critique about the way mathematics education could be contributing to systematic social exclusion of some groups carrying particular characteristics. The critical role of mathematics education in society is also addressed in research on ethnomathematics, particularly in studies aiming to understand how mathematics in society conveys hegemonic discourses and oppressive practices that promote exclusion and domination (e. g. Powell & Frankenstein, 1997). Skovsmose (1994), analyses the way mathematics formats reality, by creating models that end up ruling our decisions and daily lives. This “mathematics in action” is critical since it is not neutral, but ideologically loaded, conveying economic, military or national interests. Finally, another way of analysing the critical role of mathematics in society is by raising the issue of power. Valero (2004) and Skovsmose and Valero (2002) have developed a theoretical framework to

engage with the issue of power in mathematics, namely, to understand how the idea that “mathematics empowers people” is conceived in mathematics education.

Popkewitz (2004), in his incursion into mathematics education research, applied a Foucauldian perspective on mathematics as a school subject. He brought out the mechanisms through which the alchemy of school mathematics constructs a set of learning standards that are more closely related to the administration of children than with an agenda of mathematical knowledge. This alchemy is carried out by pedagogy (psychology and social psychology that generate knowledge about children) that appropriates the mathematical content to transmit competences, behaviours and attitudes (e.g., being participative, critical, having self-esteem, etc.). In this perspective, school mathematics serves as an alibi for the appropriation of behaviours and modes of thinking and acting that make each child governable.

Some of the research outlined above, bearing social, cultural and political connotations, has opened up the field of mathematics education by conceiving theory as more than “theory for learning”, and posing questions that do not imply a “technical” response or solution but rather an intellectual and philosophical reflection. This is research which, instead of “facilitating” the work of intervention in the mathematics education process (particularly students and teachers), points to potential and unexplored problems within the field, and raises more questions than answers. This kind of research has an intention to “complicate” and to dislocate “certainties” assumed in the field.

However, despite this invigorating openness, we argue that a significant part of research in mathematics education labelled socio-cultural-political research shows a tendency to understand mathematics education in a didactical sense and to aim primarily to address questions of *how*: How to teach in multicultural classrooms? How to teach for social justice? How to educate teachers for social justice? How to integrate immigrant students in the learning of mathematics? How the socio-cultural contexts of students influence the learning of the concepts of chance and probability? These questions were found in the proceedings of the Mathematics Education and Society, MES conference in Albufeira, Portugal in 2008 (Matos, Valero & Yasukawa, 2008), and shows how even in a research environment where the emphasis is on the political, the research persists on the question addressing the technicalities of the field.

IMPLICATIONS OF RESEARCHING QUESTIONS OF *WHY*

We acknowledge the importance of raising questions of *how*. The research that comes from raising such a question is one that intends to give solutions to the problems faced by those involved in the teaching and learning of mathematics. It is what we can call comfortable research. And all of us need some amount of comfort in our lives. Asking questions of *how* opens up to discourses concerning the individuals navigating with and in mathematics. First and foremost it invites

propositions of how students can learn, with some underlying assumption that it is important for the student to learn mathematics. Second, it invites perspectives on teaching and the teacher as a key player to assist in meeting the hypothesis of the importance of mathematics education. Third, questions of *how* invite a broader socio-political and socio-cultural perspective when they address issues of resources, gender, political agendas etc. The question can in this respect hold a strong political agenda when it asks *how* we distribute resources best to ensure that all receive mathematics education. Questions of *how* navigate within an implicit discourse assuming and attributing some kind of importance to mathematics education. Although potentially political these questions do not touch upon fundamentals or put a question mark on the nature and content of the research field itself. In other words, questions of *how* take mathematics education and mathematics education research for granted and consequently they lack a scope for what can be termed radical alternatives.

As we argued at the beginning of this paper, the ultimate goal for mathematics education appears to be improving students' mathematical learning. The idea described previously of mathematics education as a therapy, a design science or a science of *how* constructs education as a technological endeavour, where mathematics education is understood as a technical engineering of students' mathematical thinking and learning. We acknowledge the contributions that this learnification has brought to our understanding of what happens in a mathematics classroom at a micro-scale. Nevertheless we argue that reducing the possible meaning of "mathematics education" to "mathematical learning" can narrow our perspectives. And thus it becomes impossible to think and act in ways that could open spaces of possibilities inside and outside mathematics education research. Cobb (2007) is well aware of this. When referring to the theory that informs the researcher he mentions that "the constraints on what is thinkable and possible are typically invisible" (2007, p. 7). This awareness also emerges strongly in much research and it is obvious that addressing mathematics education from the narrow perspective pointed out here, reconfirms the fact that "if we look strictly at events as they occur in the classroom, without consideration of the complex forces that helped to shape those learning conditions, our understanding is only partial [and] the solutions to the problem [are] ineffectual" (Rousseau & Tate, 2008). Very few researchers, however, have addressed these limitations.

The MES conference appeared more than ten years ago with an intention of broadening the research field by developing and applying new approaches, new methodologies and new theories to the problems faced in mathematics education research. The MES community acknowledges the need to address these problems from cultural, social and political approaches that situate the problems in a broader context than classrooms and schools. However, assuming a social and political

perspective of mathematics education as a research field also involves developing research where the field itself is under critical scrutiny, and where we can formulate questions that are not directed only towards *how* to develop better ways to teach and learn mathematics (in cultural settings, for social justice, in a critical way, etc.). This kind of research raises the question of *why* the theories, methods and discourses that research constructs and is embedded into. Ultimately it raises the question of *why* mathematics education, which implies an analysis about the discourses setting the scene for its very existence.

Core questions such as the goals of mathematics education, the *whys* and for whom, are political issues that should not be left unattended. The field of mathematics education is not simply a technical field, where the teacher should improve his/her teaching skills and where researchers should develop designs to improve teaching and learning possibilities. To say that education is political means to bring to the field a discussion on the construction of subjectivities through mathematics education. It means addressing the issue of which kind of people are being formed by the learning of mathematics, and for what and why are people to engage in the teaching and learning of mathematics? Ultimately, we can engage in a discussion of which kind of world is being constructed and sustained by the research in mathematics education? Therefore, a theory of mathematics education (and not just for mathematics learning) that places educational practices in a wider political context, where mathematics and mathematics education are neither neutral nor intrinsically “beneficial”, makes it possible to raise deep educational questions about the teaching and learning of mathematics in the social, political, economic, cultural and historic contexts in which they are immersed.

NOTES

¹ For instance the discussions fueled by the work of John Dewey, Ivan Illich, Louis Althusser or Paulo Freire.

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METHODOLOGY IN CRITICAL MATHEMATICS EDUCATION: A CASE ANALYSIS¹

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In this article we engage in a critical analysis on how the notion of “critique” is being used in mathematics education research. After clarifying our theoretical and methodological position – which assumes the need for a systematic critical reflection on our own research – we argue that the notion of critique suffers from a process of “domestication” resulting from a superficial deployment of the radical ideas that emerged in the middle of the last century. After providing clarification of the notion of “critique”, we jump into the critical analysis of a case of research in critical mathematics education using data collected in a typical mathematics education research environment: teachers in the classrooms working with students.

INTRODUCTION

In order to reproduce itself the present capitalist² society demands for perpetual reforms by means of integrating what could be new and potential emancipatory acts into well established social structures. The word “critique” usually becomes a common place in educational research and curricular documents, being used as a signifier implicitly conveying different ideologies about what it means to be critical. Today we find notions of ‘critique’ in a variety of contexts such as school curricula (“educate people to become critical citizens”³), in teacher education (“Tips for teaching critical thinking skills”⁴), professional education (“Education and Knowledge in Safety-Critical Software”⁵), online education (“Role of critical thinking in online education”⁶), etc. One consequence of this extensive use is an obvious loss of meaning. That is, words begin to function as empty signifiers, representing no more than a way of joining the apparent mainstream talk instead of directing the audience into specific and soundful shared meanings. Very often, the use of these words lacks a deeper concern for understanding what could be the ideologies filling the empty space conveyed by them.

We will argue that the notion of critique suffered from a kind of “domestication” in the field of education by focusing on the case of mathematics education. In the last twenty years a strong critical emphasis has emerged, particularly from the work of Ole Skovsmose. By considering a typical research environment in this field – teachers with their students learning mathematics in the classroom – and the will of the teacher to promote a critical mathematics education in her classroom, we explore, firstly, how the notion of critique can be lost when inserted in social frames (like schools) that aim not at emancipation but reproduction; secondly, we highlight what could be a methodology that specifically addresses the emerging tensions and avoids to “clean” the research from them. The exploration of this example will allow us to make visible how ideology is effective in integrating what is presented as

emancipatory actions into existing social structures, such as the capitalist mode of production. Our goal is to address the radicalism involved in a critical educational methodology by confronting it with methodologies that reclaim themselves as “critical” while they do not seem to keep all the substance of such a notion. From a research project involving teachers and students working with issues of critical mathematics education, we explore what could be a methodology that takes seriously the notion of critique, in contrast with one that suspends it for the sake of research. We conclude that when adopting a critical methodology research needs to bring into the research practice the ways in which the “domestication” is achieved.

THEORETICAL AND METHODOLOGICAL STANDPOINT

We share the idea of Valero (2009) that mathematics education as a research field needs to develop research where its own principles and practices are put under scrutiny. She argues that “developing awareness on the research perspectives that I adopt has, therefore, been as central to me as generating particular understandings and interpretations of the practices of teaching and learning in mathematics classrooms” (p. 2). Therefore we claim the need of a constant critical analysis of the way we engage with research and how we understand its results. This kind of analysis demands looking at research from a socio-political perspective (Valero, 2004) that explicitly searches for connecting the role of research – in particular in mathematics education – to the discourses and ideologies that fuel our current society. In order to understand the dynamics of the teaching and learning of mathematics and the way research results influence what is happening in mathematics classrooms, we need to contextualize these practices and the social modes of living that characterizes the world today.

We take the standpoint that a critical methodological approach in research in education has not just to do with the way the researcher engages with the participants, but also the way the researcher makes sense of the empirical reality addressed. Reality is seen as contradictory, full of curves and spins, and a critical methodology is the one that tries to find a language to express these contradictions in a way that does not neglect them, nor clean the research from them, but takes them as part of the core focus. In order to enlighten this tension between a research than “cleans” reality from contradictions and a critical one, we will bring in the example of the work of Ana, a mathematics teacher in a Portuguese secondary school. In the empirical part of her study Ana was confronted with several difficulties while trying to implement critical mathematics education in the context of a mathematics class. For the sake of the research, she decided to obliterate them from the final report (the Master thesis), concluding that despite all the constraints she felt, it is possible and fruitful to bring critical mathematics education into the mathematics classroom. We see the difficulties faced by Ana not as marginalities, things to be avoided, details of a school system, but as core problems of the current school systems and societies that keep suspending what could be a radical emancipatory mathematics education.

Therefore we assume that difficulties and constraints of research are not things to avoid but central issues of the research.

RECOVERING THE MEANING OF “CRITIQUE” IN CRITICAL MATHEMATICS EDUCATION

Although we are aware that in our days “critical mathematics education” is a trend in mathematics education research with several ramifications⁷, we base our analysis on the ideas of Ole Skovsmose, for two reasons. Firstly because his work is one where the notion of critique is used with a philosophical background based on the Critical Theory as it was developed by some of the Frankfurt scholars (especially the further development undertaken by Jürgen Habermas). Secondly because his theory on critical mathematics education was the one used by Ana in her work.

Put briefly, Skovsmose (1994) understands critical education as one that addresses the conflicts and crisis in society: “critical education must disclose inequalities and oppression of whatever kind” (p. 22). In such a task, there is a desire for emancipation, where a critical education must not simply contribute to the prolonging of existing social relations. Skovsmose substantiates his idea of emancipation in the work of Habermas, who connected emancipation with a critique of the positivist way of researching in social sciences, and the need for social sciences to be founded on an interest in emancipation.

By so doing [fall into the trap of logical positivism], social sciences will be colonised by the technical-manipulative research paradigm, according to Habermas. It is not possible to find any platform of neutrality. Social sciences must be ‘committed’. A pretended neutral registration of facts will result in an acceptance of the social status quo. (Skovsmose, 1994, p. 12)

But how did Critical Theory understand the “existing social relations” or the “social status quo”? In other words, which was the core focus of the social and political critique developed by Critical Theory? The answer is capitalism. Despite major differences between members of the Frankfurt School in their assessment of the development of capitalism, it may be noted from the outset that their respective analysis were informed by Marxian tenets (Held, 1980). According to Benhabib (1994) the core feature of critical theory, as it emerged in the works of Horkheimer, Adorno, Marcuse, Löwenthal, Pollock and Benjamin, was the realisation that a revolutionary transformation of capitalism from within capitalism itself was doomed to fail. Critical theory was confronted with the enterprise of thinking a “radical alternative”.

Although initially the critique was focused on political economy, with time it gave place to a critique of instrumental reason, as a response to a positivist paradigm which restricted research to the activity of outlining correlations between well-defined phenomena. These two critiques did not coincide; rather the critique of instrumental reason surpassed the critique on political economy:

The transformation of the critique of political economy into the critique of instrumental reason signals not only a shift in the *object* of critique, but, more significantly, in the *logic* of critique. (Benhabib, 1994, p. 79)

The work of Habermas exemplifies this shift on the object of critique. In his work political economy is not just a matter of *superstructure*, of class struggle, but a matter of administration and technique, due to a change in which politics becomes the sphere for the technical elimination of dysfunctions and the avoidance of risks threatening “the system” (Held, 1980). This split provoked a displacement of the way the political was conceived: capitalism became “naturalized” and accepted, and transformation started to be conceived inside capitalism. Capitalism is no longer seen as the fundamental core of the problem, as the system we have to emancipate from, but the social and political background in which emancipation can take place.

Despite the apparent fall of capitalist principles in the last two years, we are facing the emergence of reforms that keep unaddressed the core of the societal problems. All emancipatory actions are thought and put into action within capitalism. If we recover the critique of political economy developed in the first years of the Frankfurt School, we can say that emancipation from capitalism failed completely. No radical alternative was made. But this fact contrasts with the proliferation of the idea of critique, especially in education. We can read in the curricula all around the world the word critique, how important it is to allow students a critical education, to become critical citizens. It is in that sense that we argue that the word critique has become “domesticated”, it has lost its most radical meaning. It is a case of what Žižek (2005) calls “progressive amnesia” (p. 9): we recover critical theory but deprived from its true transformative core. It is fine to take a critical stance as long as you do not raise questions that could undermine the foundations of society – we are allowed to be critical as long as we do not criticize the capitalist system itself.

A RESEARCH IN CRITICAL MATHEMATICS EDUCATION

We will now look at a piece of research in critical mathematics education, trying to make visible how a potentially emancipatory theory can end up reproducing the same ideologies that it tries to criticize. For this purpose, we understand schools in the Althusserian way as crucial ideological state apparatus in the reproduction of capitalism (Althusser, 1994).

The interest of Ana into critical mathematics education is partly the result of a concern with the way mathematics is traditionally taught in schools: as something disconnected from students’ reality. Being committed to pupils’ education for citizenship, Ana sees her role as a mathematics teacher as an important factor in allowing her students to become participative, active, competent, critical citizens. The ways she found to accomplish this aim are diverse, being one the development of activities with students where they can uncover and understand the role of mathematics in different social situations. She adopts Skovsmose’ idea of

mathemacy, as the competence to analyse and reflect upon the mathematics behind a world strongly structured around mathematical modelling (Skovsmose, 1994). As a final product of Ana's experience (which took place during the first period of 2006, with a class of 9th graders), she developed a Master thesis where she explored the implementation of this critical mathematics education experience.

The Portuguese curriculum gives her space to work with such topics in the classroom, by explicitly mentioning that “mathematics education has the purpose of helping students to uncover the mathematics behind the more diverse situations, promoting the education of participative, critical and confident citizens” (ME-DEB, 2001, p.58). Everything seems prepared and even willing to implement a critical mathematics education in the classroom. What issues are involved here?

The first issue is the decision of Ana of not implementing her critical mathematics education experience in the regular schedule of the mathematics class. She decided to invite some students and form a club, outside the hours destined to mathematics, where they could work with topics of critical mathematics education⁸. Ana justifies this decision because students of the 9th grade will have a final exam at the end of the year, on which their final grades will depend and the approval to enrol in 10th grade next year. Here we can notice the contradiction between the official discourse (present in the curriculum – involving students with topics of critical mathematics education) and the real practice where it is the exam which delineates the teaching content and form. Ana is well aware of this contradiction:

É assim visível que, mesmo sendo uma professora com preocupações ligadas à educação matemática crítica e ciente de que a desocultação das estruturas matemáticas presentes em fenómenos sociais constitui uma forma de aprendizagem potencialmente mais significativa para a maioria dos alunos, a pressão do sistema organizacional envolvente (escola, pais e alunos) levou-me a tomar esta opção, o que ilustra as primeiras dificuldades que um professor tem de enfrentar quando se pretende implementar este tipo de trabalho no contexto de aula de Matemática. (Alves, 2007 p. 57, 58)⁹

The idea conveyed here is that it is good and innovative to implement such topics but there is an inner and rather invisible pressing into conformity that the teacher is aware of and that makes her to put in practice activities that do not directly challenge the school system (and do not change any core features of the school structuring activities). On the other hand, it turns explicit that critical mathematics education is not part of the curriculum and in pupils' minds creates the idea that perhaps is not really mathematics.

Another aspect of the research of Ana that we want to highlight is the criteria that she used to choose the students to interview. She opted for those who had shown more interest and enthusiasm along the sessions, and justifies this choice by mentioning the visibility – “choosing those who appeared more involved and participative in the sessions was a way of guarantee the collection of data (...) I choose the students who gave more visibility to their involvement” (Alves, 2007, p. 66, our translation). This

is an option that most researchers do (finding the ‘best’ informants) as they need to provide clear evidence of their claims. In the case of Ana, what did she want to make visible in her research? She wanted to highlight the potentialities of critical mathematics education for developing citizenship. Therefore, it was not appropriate to choose students who in a way or another did not engage so enthusiastically with such experiences. On the other hand, the selection of the students was also related to the aim of her research. This type of “selection” is a case of what Vithal & Valero (2003) call the “cleaning” of research – putting aside the conflicts and the constraints so that research is presented in a harmonious and positive way.

What about if what Ana wanted to turn visible was precisely the artificiality of bringing to the structure of the school system ideas that conflict with that structure? What would be the more visible things in this case? The issue of visibility has always to do with what we want to make visible and ultimately with the research problem we formulate.

Finally, Ana justifies the lower involvement of some students because, on the one hand, they were not familiarised with the way they work in the club (which was more unstructured and free than in the classroom environment) and, on the other hand, they were still attached to a vision of mathematics as a static science having nothing to do with real life situations. Although these arguments could be true, we suggest that other issues are at stake. We take the risk of saying that the lower involvement of the students could be due to the fact that they knew that these activities would not contribute directly to prepare them to the tests and to get a good mark at the end of the year. Using Vinner’s (2007) description of school as a credit system, we could say that students felt that those activities will not give them much credit.¹⁰ Just remember how many times a teacher who wants to flourish some explanation (a little bit of history, an application, a connection with other themes, a more insightful explanation) heard the students promptly ask “will that show in the test, teacher?” And the teacher is forced to say “well, yes” if maintaining students’ attention is in the agenda.

OPENING POSSIBILITIES FOR A CRITICAL METHODOLOGY

The research developed by Ana shows methodological concerns that are characteristic of a critical methodology. The most evident one is the assumption by the researcher of her subjectivity. Ana expresses her concerns about the difficulties of implementing critical mathematics education in schools, the resistance of the students to such topics, the pressure to fulfil the entire disciplinary program, the need to prepare students for the final exam. This is an example of what Valero (2004) calls “making the researcher visible” (p. 19), which opens to the critical examination of the reader the products of the research process, the intentionality of the researcher, and the paths that the researcher decided. It was this openness in the work of Ana that allowed us to develop such a critique on her work.

But, despite all the difficulties, Ana assumes that it is possible and desirable to develop with students tasks of critical mathematics education, and suggests that this could be a way of promoting a bigger societal transformation. Ana conceives transformation within the school structure. What we think remains problematic in such approach is the absence of a critical analysis of schools as institutions of reproduction which tend to incorporate all potential emancipatory reforms into the mainstream ideology fuelling schools which we identify as capitalism. When we say, like Ana does, that students should become active, interventive, competent, critical citizens we should also ask what it means to educate people to be participative in a more and more consumerist society? When we abstract these desirable features for the students from the concrete social spaces in which they are made operational, we take the risk of just being part of the language game with empty words ready to be filled with the dominant ideology.

We argue that a critical methodology in mathematics education research needs to bring to light what Žižek (2005) calls the symptoms – the points at which the hidden truths of a system emerge, and to avoid engaging in salvation discourses which, by blindly misunderstanding the true problems facing mathematics education, only perpetuate existing realities. In the case of the research developed by Ana one of these symptoms is the fact that critical mathematics education collides with the assessment system, which forced her to implement the critical mathematics tasks outside the mathematics classroom. Ana sees this contradiction as a difficulty, as a problem she had to surpass in order to open a space to promote critical education to her students. But what this contradiction shows is that what the system points as the most important role of the teacher is in fact to prepare students for the final exam. It is good to work with students on these “radical” topics as long as they do not change the smooth functioning of schools as credit systems (Vinner, 2007). This way a potentially emancipatory attempt to educate students according to a critical education is completely inserted and transformed into a small change. Maybe this is a case of what Paulo Freire called the “superficial transformations”: when he suggests that the elites are anxious to maintain the status quo by allowing only superficial transformations designed to prevent any real change in their power of prescription.

From a critical theory stance this ‘marginal’ problem makes visible the inconsistency of the system itself and may force the radical teacher to face the challenge of being an ‘outsider’ within the system and the Trojan horse inside.

This realisation is well acknowledged in critical mathematics education research, although we continue to act as if it were not. It is common to acknowledge that critical mathematics education research requires social and political approaches that commonly situate the problem of “change” in a broader context than the classroom or schools (Gutstein, 2003; Gates & Zevenbergen, 2009; Valero, 2004). Although studies in a critical trend acknowledge this social and political dimension of emancipation (especially in the beginning and the end of the texts), we argue that they manifest signs of persistence as if the problem of allowing students a “critical

education” could be understood and solved within mathematics education. It is as if we realize that the problem has a social and political nature well beyond the classroom, but, since we are mathematics educators, we should investigate it in the classroom.

We understand critical methodology as explicitly addressing these borderline problems, which truly connects mathematics education to the political sphere in which we live. Our claim is that a critical methodology should imply the responsibility for the researcher to develop a critical stance towards his/her own work and results, by framing his/her research in the social and political discourses in which he/she moves.

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2 The word capitalism to refer the current mode of living seems to be falling into disuse. In the social sciences’ discourse it is usually absent, as it is in mathematics education literature. Dowling (1998) refers to it as “a word which should remain unspoken” (p. 19). Indeed, even in mathematics education research that takes a social and political dimension, society is usually described as being “neo-liberal”, “market economy”, “imperialism”, “post-colonialist”, “post-modern”, “consumerist” and other euphemisms. If we recall how ideology works (Žižek, 1988), the obliteration of the word capitalism from mathematics education discourse (but also in general in social sciences discourse) is a symptom of the way we “naturalize” the way we live – we do not need to mention that we live in a capitalist society because we actually live in a capitalist society, without any other imaginable alternative. Nobody seriously considers possible alternatives to capitalism any longer. This is, of course, ideology functioning at its best. This is why we opted for using the word capitalism (instead of liberalism, for instance). Although we criticize the consumerist society, the neo-liberal ideas that takes to its extremes the individualization of social life, and so on, we think that we can go further and realize how behind all these epithets relies the capitalist system. For instance, Chinese society is not organized around liberal tenets, nevertheless it is profoundly capitalist. Capitalism dresses diverse clothes in order to keep reproducing, and no matter how different the “philosophies” of political organization could be around the world (monarchy, socialism, religious fundamentalism, dictatorship, neo-liberalism, etc.) what is common in all them is that, despite the different apparent “clothes”, the human relations are based on capital. By explicitly mentioning capitalism we want to point to the very core of the problem – this (so often) unaddressed reality that permeates all social relations and for which we seem to have no alternatives.

3 Portuguese, Colombian, South African curriculums.

⁴ <http://www.modernghana.com/news/203119/1/tips-for-teaching-critical-thinking-skills.html>

⁵ <http://ercim-news.ercim.org/content/view/446/699/>

⁶ http://www.masternewmedia.org/education/critical_thinking/educational_role_of_critical_thinking.htm

⁷ For instance the work of Marilyn Frankenstein in the United States; or the work of the *Critical Mathematics Education Group* at Sheffield Hallam University, United Kingdom.

⁸ Ana explored with students two situations: “Supermarket promotions” and “A taxi trip”.

⁹ Translation: It is though visible that, even being a teacher concerned with critical mathematics education and conscious that uncovering mathematical structures present in social phenomena is a way of learning potentially more meaningful to the majority of pupils, the pression of the school system (school, parents, pupils) took me to decide for this alternative [critical mathematics education developed not in the regular class but in a club] showing the first kind of difficulties that a teacher faces when one wants to implement this kind of work in the context of school mathematics.

¹⁰ It is always useful to remember the research made by Baldino & Cabral (1999), where they show how students in school are primarily worried to pass (and not necessarily to learn).

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SIMÓN RODRÍGUEZ AND THE CRITICAL DIDACTICS OF MATHEMATICS

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The following article intends to explore the legacy of Professor Simón Rodríguez as a pioneer of critical thinking in the world. His work is one of the most influential in what is currently known as Critical Didactics in Latin America. Using his work allows one to see how the meeting of different subjects and the present sociopolitical moment becomes evident, specifically in Mathematics. Simón Rodríguez' visionary intellectual ability makes him one of the theorists who has transformed Western European thinking and proposed an epistemic purge.

*The general interest is claiming for a public instruction reform; America is called because of the circumstances to start it ... Men are not in the world to interdestroy each other, but to interhelp themselves.
Rodríguez (1975a)*

Simón Rodríguez (1769–1854) was the creator of interpretations of reality and development approaches which drive the creation of Venezuelan, Latin American and Caribbean identity. His educational thinking gave him an honorable place as the most important pedagogue of the Americas in the XIX century. Critical Mathematics Education is a pedagogic, didactic and curriculum conception which humanizes and socializes the Mathematics so that it can play a leading role in the development of deep processes of political and social consciousness. Through this, reflexive, argumentative, critical and decisive human beings who understand and develop their skills and abilities are formed to make it possible to have a constructive and participative performance within the collective to which they belong. The ideas of this great Venezuelan, who named himself Samuel Robinson when living in North America and Europe, were vital for the strengthening of the new National Curriculum project after the wars of independence from the Spanish colonization. One of Simón Rodríguez' premises was to promote originality, especially in such an innovative reality as the American one. Therefore, he wanted education to create new ways of learning in order to break with the repetitive scheme inherited from the colonial classic discourse. He developed a philosophy in which revolution meant to put an end to the characteristic colonial mindset of Venezuelan, Latin American and Caribbean education, present in the pedagogical practices of that time, and still dominating current educational practices in the continent.

American societies (published in 1828) and *Lights and social virtues* (published in 1840) are Simón Rodríguez masterpieces. During his long and conflicted life, the

Venezuelan philosopher spread his pedagogic work all over the continent. Nevertheless, he was little understood in his native Caracas, and was also chased by failure in Colombia, Ecuador, Perú, Bolivia and Chile. Perhaps, for García Bacca (1981), the society of that time was not able to understand his theoretical basis, summarized in one main idea: It is not enough to create republics politically; it is necessary to *invent* the citizens to make them real. Anything else is just a fiction. Maybe the lack of attention to this deep idea can be taken as one of the sources of the many problems that Latin America has experienced since its republican creation.

The eighty-four-year-long life of this teacher from Caracas is divided according to Rumazo González (2005) in four clearly differentiated stages:

- a quarter of a century in Caracas;
- another quarter, in seven European countries;
- a third period of seven years, from his return from the “old world” to Bolívar’s death;
- and another quarter at the end of the century, in which he published his work.

According to this Equatorial humanist, there are no gaps in his existence, even if he traveled a lot. The periods signal the force of a plan which developed in a stringent unit, all hostilities, poverty and adversity subjugated. He worked endlessly, discovered, innovated, created, knew by intuition and foresaw the future one hundred years in advance. But nobody seemed to have understood him and, therefore, his giant drama. Briceño Porras (2005) writes about Rodríguez: There are ideas he expresses analyzing his own problems that are not from the present time, even if they are modern, and not in fashion, even if they are new: “For teaching more of what everybody learns, few have understood me, many have despised me and some have even taken the time to offend me” (Rodríguez’ words quoted by Briceño Porras, 2005: 32). From the age of twenty until the age of eighty, this Venezuelan teacher taught almost without interruptions.

One hundred and seventy-nine years ago, Rodríguez (1975b) described the first Project of Popular Education [1] in the then recently created Republic of Bolivia, in the following manner:

The project of Popular Education has the misfortune of resembling what in several places has been started with this name — and it is practiced in different ways, with a small number of individuals, especially in the big capital cities. All the institutions are made out of pity...Ones for “expósitos”[2]—Others for orphans—Others for girls from noble families—Others for military sons—Others for handicapped. In all of them it is clearly stated: they were not made for the greater good but for the founder’s sake or for the Sovereign’s ostentation. The Establishment, started in Bolivia, is social, its combination is new, in one word it is THE REPUBLIC: there is in what is seen in the others, because

it is a masterpiece — there are men, who are the subjects —agents, who are the workers —places where the work is done, which are the workshops —Principal, who is the teacher —and Inspector (the Government) which is the owner. All the clocks have wheels and springs and they are not the same. (: 55)

Simón Rodríguez states the sense of Establishment and the sense of Republican. He (1975b) expresses:

If the Government of Bolivia, in the year 1826, had taken the time to examine the plan, it would have known its importance; If it had demanded from the ones who disapproved the reasons it should stand for and forced into silence those who opposed it with frivolous excuses, the High Peru would be nowadays an example for the rest of the Americas; truly new things might be seen there:

- a. A fund, directed to what everybody calls welfare, raising instead of decreasing.
- b. A low People, condemned (like everywhere) to misery and disorder, transformed into decent people.
- c. A military force composed of 12.000 young people (at least) with no costs to the public treasure, armed and their hands filled with work and paying a personal contribution to the State instead of requiring a salary.
- d. In the four years passed from January 1826, when the Establishment started, there would be (at least) 25.000 busy people (with property as a consequence) educated in their moral and social duties (consequently, republicans and adhering to the Government), the fields would be cultivated, and the farmers would have well built furnished and clean houses, they would be decently dressed, they would amuse themselves wisely and, they would understand about society ... In one word, they would be citizens.

It is undeniable that some might lose in the change. The donkeys, bullocks, sheep and hens would belong to their owners; from the new people you would not get servants for the kitchen or to carry the rugs behind the young ladies; entering the cities, even if they were artisans, they would not allow them to be held by their necks, to go clean the stables for the officers, or sweep the public squares, or kill dogs; the gentlemen of the cities could not get little Indians for the priests, since the muleteers would not sell them on the road... The rest is known by the tenants of the lands. (: 58)

In this quotation, *Rodríguez* in his visionary role presents the proposal of endogenous development. *Subirats* (1981) emphasizes the concept of popular education around three suppositions:

1. Economical implications: *Rodríguez* talks about fund increase in “what everybody calls welfare”, Young people “with no cost to the public treasure” and “paying a contribution to the State”, about “the donkeys, bullocks, sheep and hens belonging to their owners”, about “new people not being servants” and no personal services would be imposed (sometimes done with the help of priests).

2. Social organization: a youth military force was foreseen “armed and their hand filled with work”, it might have happened that “the fields would be cultivated, and the farmers would have well built furnished and clean houses”.

3. Attitude changes: the low People, condemned (like everywhere) to misery and disorder, transformed in decent people, they would be people “educated in their moral and social duties” and “in one word, they would be citizens” (: 221)

In 1828, Simón Rodríguez reveals the main ideas that he shared with Simon Bolivar about the education for work of the new generations as well as the conviction that in the new South American republics “children should be accommodated in comfortable and clean houses, with rooms as workshops and equipped with instruments and directed by good teachers” (Rodríguez, 1975b: 52). Herein Rodríguez explains

The intention was not (as it was thought) to fill the country with rival or miserable artisans, but to educate and accustom men to work in order to make them useful, give them lands and help them to the establishment... It was to colonize the country with its own inhabitants. Instruction and occupation were offered to women so they did not have to prostitute themselves out of necessity or marry just to guarantee their survival. (:53)

And in them, the teachers would have the responsibility of “making the children know the value of work, so they can appreciate the value of the results”. According to Gomez Gutierrez and Alonso Rodríguez (2007)

The most amazing aspect of these theoretical statements is the fact they were produced twenty years before the publishing of the Communist Manifest, the programmatic document in which the revolutionary philosophers Karl Marx and Friedrich Engels, not only proclaimed the need of establishing a free and public education for all children and the abolition of childhood exploitation, but proposed an “education regime combined with the material production” from which “educate in and for work”, became a pedagogical principle and an expensive aspiration of the socialist education. (: 109)

Simón Rodríguez’s intellectual and visionary ability makes him one of the theorists who transformed Western European thinking and proposed an epistemic purge as shown in *Rodríguez* (1975b) in a text published in 1830:

America is called (IF THOSE WHO RULE IT ARE ABLE TO UNDERSTAND) to be the model of the good society, with no more work to adapt. Everything is done (especially in Europe). Take the good – leave the bad – imitate wisely – and for what is missing INVENT. (: 51)

Professor Simón Rodríguez insists, ten years later, in his spirit of decolonization:

The PUBLIC INSTRUCTION in the XIX century asks for more philosophy that the general interest claims for a reform and that America is called because of the circumstances, to start it daring paradox may seem... it does not matter...the facts will

prove, it is an obvious truth that America must not IMITATE submissively but be ORIGINAL (: 51)

For Rodríguez (1975b) there is nothing Europe has to teach the Americas, what these need has to be found within their own reality. The current political ideas sustained by Europe are no other than their urge for domination over other people which escape their influence. For this reason, they spread ideas such as the superiority of a race towards the inferiority of others. Ideas about some people representing civilization and some other people sank in the barbarism. Europe has also spread the idea that the Latin people may only be ruled by despotism. "The great work of Europe was made with no plan, it was built by pieces, and the improvements have been piling up, not by imposition; the art shines more in the skills than in the matching; the most sublime things mixed with the most despicable ones make a contrast...beautiful, for the perfection of the parts; but unpleasant, for the inappropriate of the whole" (: 65). And he adds, turning the criticism even more severe: "Never will Europe reform its morals as it reforms its buildings: the modern cities are a model of taste and comfort (...), but the inhabitants are always the same, they know better than before, they do not do better" (:65).

If the Congresses achieve the colonization plan proposed by some people interested in the luck of homeless Europeans, and in the success of the new Republics, Americans will have the pleasure of having drawn... for leisure of their children... a more lively picture than the one presented by Europe: that one is natural and simple—this one?!... it will be embellished with new *Characters*— decorated with Indi and African *arabesques* and colored with the different tones from the earth...; ¡what a beautiful SOCIAL CARICATURE!... Leave for the reader, the satisfaction of enriching it with episodes that his imagination may suggest....if it can be fixed. Men from other times, who are unable by nature, or who have been damaged by education, may not be recommended either for what they know or for what they must want to be done: recommend yourselves.... make yourselves respectable... which is more protecting than what should be done.... COLONIZE the country with... its OWN INHABITANTS and have DECENT COLONISTS, EDUCATE THEM in their childhood. (:66)

Simón Rodríguez was one of the most influential critics of pedagogy, and a fundamental contributor to what is nowadays known as Critical Education in Latin America. To Molins Pera (1998: 232) the first school should provide a social education since this one will allow “the making of a prudent nation”, a body education (physical) “to make it strong”, a technical education “to make it expert” and a science education “to make it thinker”. Education and instruction, in Simón Rodríguez thinking, were related in the society-school environment with a total and integrating dynamic. This relationship, according to Murguey (2004), makes it possible that education and instruction are “vital and human needs” for the construction of the society and the stability of its institutions, among them the school, and at the same time, society and school are agents [3] which drive and guarantee education and instruction that must be complete and non-exclusive, so that there

might be social, moral and productive life and in order to have citizens. For Rodríguez, the teacher must be able to teach, to learn and help to understand. This great educator considered essential the learning of Logic, Language and Mathematics as well as civic and religious education (Molins Pera, 1998). This author comments that

The politico-educational conception of Simón Rodríguez may be placed among the “utopian socialist” thinking. Besides, education for civil society was also a proposal of other authors (For instance, Horacio Mann). Moreover, Rodríguez de Campomanes, Pestalozzi, Fichte and Adolfo D. also promoted the education in (or for) work. (:242)

And precisely about Mathematics, perhaps one of the best examples of **Critical Didactics of Mathematics** comes from 1840 and it is due to Rodríguez’ genius. This pedagogue was appointed by Simon Bolivar as General Director of Public Teaching in Bolivia. Rodríguez stated that “the Americas are not to imitate submissively, but be original” and that “the country must be colonized by its own inhabitants”. These conceptions pushed him to structure an interesting educational experience in the *Model Schools* in Sucre where girls and boys had the possibility of learning arts and occupations in a theoretical and practical way.

The idea of Republic for Rodríguez (1975b) is the result of many *combinations*: it is the most simple expression that the *study of man* has reduced all the *social relations*. His formula is:

$$\text{PEOPLE} \quad \times \quad \left\{ \frac{\textit{Particular interests}}{\textit{Particular interests}} \right\} = 1 = \text{REPUBLIC}$$

It is necessary to tell those who do not understand calculus how to read this formula — and it is read so: People multiplied by particular interests and divided by particular interests equals one, equals Republic; and for those who the language may seem dark, the speech will be widened by saying that men meet for their interests, that everyone is looking for their own benefit without taking into account the benefit of others, cause the end of the union, because their interests crash into each others. This is the reason for all conflicts and these, the cause of wars that the lights acquired by experience have made think—that thinking it has been discovered, that everybody thinks in the greater good and that this greater good is the Republic. With no knowledge, man does not come out of the circle of brutality and, with no social knowledge, he is a slave. The one who rules people submitted to this situation, becomes brutal with them. Believing he rules because he orders already proves he thinks little. To maintain that, only because of the blind obedience, government survives proving he does no longer think. (:111)

The Critical Mathematics Education is a pedagogic, didactic and curriculum conception which humanizes and socializes Mathematics so that it can play a leading role in the development of deep processes of political and social consciousness. Through this, reflexive, argumentative, critical and decisive human beings who

understand and develop their skills and abilities are formed to make it possible to have constructive and participative performance within the collective to which they belong.

In the three first lines of this quotation, Rodríguez recurs to didactics. When he explains in a critical way that “men who meet for their interests, look for their own benefit without taking into account the benefit of others, cause the end of the union, because their interests crash into each others” he is socially applying the neutral element in multiplication (or symmetric element) in the best style of critical pedagogy and Critical Mathematics Education. Then, he emphasizes the criticism in his didactics when he insists on his explanation “This is the reason for all conflicts and these, the cause of wars that the lights acquired by experience have made think—that thinking it has been discovered, that everybody thinks in the greater good and that this greater good is the Republic”. A whole lecture written 83 years before the Frankfurt School was founded.

In 1794, Rodríguez presents his reflections at Caracas City Hall, on the flaws which spoiled education and how the first curriculum for elementary school in Venezuela should be dictated. It was about a critical treaty on colony teaching. In the next quotation Rodríguez (1975b) emphasizes the importance of geometry learning and teaching and criticizes the colonial didactics used at that time.

The need in which some People are of knowing what LIGHTS and SOCIAL VIRTUES are, People which believe they are JUDGES of their social fate and OWNERS of that of their children. (...). It is impossible to find someone who denies that the ignorance of the social principles is almost general. Who does not know it is a providence to the government, whatever its shape is, or does not convey in that to make real institutions of general interest, his knowledge must be general. Then, get used to the man who has to live in Republic, to seek from his childhood, **reasons and proportions** in what can be exactly measured so that through them, he learns to discover **reasons and consequences** in the providences and procedures of the government, for him to learn how to approach the moral infinite: so that his probabilities are not gratuitous or his opinions imposed. Therefore, it is said that GEOMETRY rectifies REASONING. The lack of Logic in the Parents, of Jealousy in Governments, and of Bread in the Teachers make the children waste their time reading with no mouth or sense, painting with no hand or drawing, calculating with no extension or number. Teaching is reduced to annoy them by telling them, at every moment and for years, so—so—so and always so, without making them understand why or what for. They do not exercise the faculty of thinking, and they are allowed or forced to spoil the tongue and the hand. (:98 and 99)

Rodríguez postulates on *the political historical construction of free citizens of the Americas*, implies the possibility of developing a republican pedagogy, an original project for the upbringing of free citizens. For Rodríguez, the historical and political being is made through the interaction of subjects within different power relationships: free cooperation, solidarity and the greater good or collective goal of

which he is individually beneficiary. The human and socio-critical character is present when he expresses "*think of all so that all think of you*" (Rodríguez, 1975b). His pedagogical proposal has an ultimate goal: "feeding the hungry, dressing the naked, hosting the pilgrim, curing the sick and distracting the sad from his sorrows" (Rodríguez, 1975a).

RODRÍGUEZ' IDEAS NOW AND TOMORROW

I have pointed to some important characteristics of the ideas of Simón Rodríguez that have a value for thinking about education and mathematics education in the context of Latin American societies nowadays. I will shortly summarize some of them and comment on how I see them having relevance in the current project of construction of a new education in Venezuela.

1. Education as a means of moving away from a colonial past.
2. Education is a political activity that allows connecting individuals with their social and collective environment.
3. Education is a critical activity
4. Mathematics education is....

NOTES

1. In the book *El Libertador del Mediodía de América y sus compañeros de armas defendidos por un amigo de la causa social* (published in Arequipa in 1830), Simón Rodríguez includes "Notes on the project of popular education". He points out some considerations concerning educational matters, his performance as a teacher in the Gran Colombia and the application of the Project of Popular Education.

². Rodríguez was expósito, it meant an illegitimate son. The "expósitos" were born in the secret of the family and were abandoned secretly too, usually carrying a note which indicated his caucasian condition so that he would not lose social quality.

³. Independent Agents: the school, in making the man in the social behavior to live in society and the society, in supporting and founding the school for everybody.

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MATHEMATICS, DEMOCRACY AND THE AESTHETIC

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Starting from Josiah Ober's etymological exploration of the origins of the term 'democracy', placed as one of a series of words identifying specific 'regime types' in ancient Greece, we shift the setting to that of mathematicians and their practices as well as mathematics classrooms, while adding in questions of the aesthetic in relation to questions of whose taste, whose judgements, whose decisions are attended to there. Ober identifies democracy as centrally having to do with power in the sense of the 'capacity to do things' and we explore different forms of power related to the notion of 'taste.' We examining to what extent questions of mathematical 'beauty' are amenable or even accessible to school students and how this key issue interacts with Ober's ideas concerning democracy as a concept.

INTRODUCTION

Relying on etymologies can prove a complex business. For one, it can seem to suggest that earlier meanings are in some sense 'truer' or 'more fundamental', as well as perhaps reflecting a conservative wish to restrain variation or divergence from traditional (even 'natural') meanings. Nevertheless, as political scientist Josiah Ober (2007) is at pains to point out in his recent informative account of the ancient Greek sense of 'democracy', on occasion there can be some benefit. He writes:

Of course, we are not bound by any past convention, much less by the inventors' original definition: if we can devise a better meaning for a political term, it should be preferred. But if common modern uses are not particularly good, in the sense of being "descriptively accurate" or "normatively choiceworthy," then there may be some value in returning to the source. Reducing democracy to a voting rule arguably elides much of the value and potential of democracy. (p. 2)

Ober examines 'democracy' within the small semantic field of Greek terminology for regime types, many of which either have *-arche* or *-kratia* suffixes (e.g. 'monarchy', 'oligarchy', 'democracy'). Monarchy and oligarchy refer to how many are empowered in the ruling body (respectively, the one and the few), whereas the composite *demos+kratia* combines "a collective body" and "power", hence offering a root meaning of "the power of the people". [1] As a result of Ober's analysis, he concludes, "*kratos*, when it is used as a regime-type suffix, becomes power in the sense of strength, enablement and 'capacity to do things'" (p. 5).

Not all regime words fall into one or other of these suffix groups. One of the most interesting groups for our purposes here comprises those terms that begin 'iso-' ("same or equal"), signalling a "distributive fairness in respect of access", of which *isegoria* (used to paraphrase 'democracy') involves the right to make use of "deliberative fora: equal right to speak out on public matters and to attend to the

speech of others” (p. 5). This notion alone is of considerable significance in a mathematics classroom. (For one example, see Jenny Houssart’s (2001) powerful account of rival classroom discourses and a partial ‘speaking out’ in mathematical resistance and protest by a somewhat marginalised group of four boys.)

In attempting to display how rich a relation there is between democracy on the one hand and law, action and public goods on the other, Ober cites a passage from Demosthenes concerning a court case, pointing out how the latter “employs a rich vocabulary of strength, control, ability and protection” (p. 6). And it tempts us to wonder in what sense mathematics is or should be seen as ‘a public good’. [2]

In our paper, we propose to draw on Ober’s helpful re-characterisation of ‘democracy’ as invoking power in the sense of having the ‘capacity to do things’ rather than solely referring to ‘majority rule’. This observation immediately motivates the question of who has, or who can assert, the power to do things in mathematics and on what basis. One interpretation of this question will take us into explicit discussions of the notions of disciplinary and human agency (e.g. Pickering, 1995) in regard to mathematics and mathematics teaching.

We simply raise this question here, however, in order to help us to re-examine the specific and complex connection between mathematics and aesthetics [3], in an educational setting, the central focus and concern of this piece. In particular, we wish to challenge directly the presumed elitism inherent in aesthetic concerns in this domain a presumption both readily and explicitly claimed by some professional mathematicians (see, e.g., Borel, 1983; Hardy, 1940; Poincaré, 1908/1956; Russell, 1967), as well as by some mathematics educators (e.g. Dreyfus and Eisenberg, 1986; Krutetskii, 1976; Silver and Metzger, 1989). It is they (the professional mathematicians) who will be our titular few (*hoi oligoi*), who perceive mathematics as an oligarchy (and also act as if it is one, with them as the oligarchs), a rule exercised among other means through the explicit notion of ‘taste’ (with them as the bearers of it), which we address in the next section (see also Pimm & Sinclair, 2008.)

Our opening conceit, then, is to conceive of mathematics as a political regime (echoing, perhaps, Foucault’s (1980) notion of a ‘*régime* of truth’, which mathematics exemplifies *par excellence*) and to enquire, at least with regard to aesthetic considerations, what form or type of political regime is in place. In what sense, then, can mathematics be considered a *democratic* regime, one in which all (*hoi polloi*) have the capacity to do?

THE POWER OF TASTE

Taking the idea of democracy as invoking power in the sense of “capacity to do”, we can read the role and nature of aesthetic considerations in mathematics in at least three different ways, corresponding to three distinct interpretations of power: power viewed as intrinsic to mathematics, power seen as the oligarchic imposition of ‘good taste’ and power as relational capacity. The first, well represented by subscribers to a

Platonic view of mathematics, sees power as intrinsic to the subject itself and therefore allocates agency and action to the discipline. It is, thus, the discipline itself that possesses the “capacity to do” and that capacity may then be transferred to individuals. If mathematics is good, in the sense of Plato (in terms of mathematics offering the only path to knowledge) or in bestowing successful engagement with the global economy (seen in more contemporary rhetoric), then mathematics has power, and one can obtain it by obtaining knowledge of mathematics. As Valero (2005) points out, a central problem with this view is that mathematics is not a *social* actor, and therefore can neither possess nor transfer power – at least if one subscribes to the epistemological and ontological positions of critical theories.

This intrinsic view fits well with traditional ideologies found in mathematics culture, including the notion that mathematics is somehow pure and detached from the rest of the world – that mathematics has a life of its own that might very well continue without human interference. Mathematicians such as Hardy (1940) have described some aesthetic consequences of this view, including the value of pursuing pure mathematics only and resisting the temptation of applications outside mathematics.

Hardy, as well as several of his contemporaries, has also espoused a more human (if class-driven) view of the power dynamics related to mathematics. This second view sees power in terms of the exertion of influence of one class over another, giving rise to an elitist view of mathematics and mathematical aesthetics that dominates today. If one recognises that mathematics is a discipline in which choices must be made, in which there are no objective grounds for deciding which questions or ideas count as fruitful or important, then one is forced to admit human agency in mathematics. In other words, mathematics itself cannot decide what is worth studying; mathematicians must do this. According to mathematician Henri Poincaré (1908/1956), only mathematicians are privy to the aesthetic sensibilities that enable these kinds of choices and, indeed, to any kind of mathematical creativity. [4]

There are a number of quotations that could do duty here. One of the clearest comes from John von Neumann (1947), while also attempting to offer criteria for salvation from mathematics becoming:

more and more purely aestheticizing, more and more purely *l’art pour l’art*. This need not be bad, if the field is surrounded by co-related subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. (p. 196)

And so power can be seen as being in the hands of those who decide, those who are the arbiters of taste. Hardy was clear about his belief that only the great mathematicians could play this role. He even scoffed at the possibility of admitting those outside *hoi oligoi*: “There is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds” (p. 61).

In terms of exposition, and popularisation more specifically, Latour (1987) notes that the difficulty that mathematicians have in talking with non-mathematicians stems from the fact that mathematics “is designed to force out most people in the first place” (p. 52). Indeed, Csiszar (2003) sees this phenomenon as a “telling indicator of the discipline’s tendency to exclude all but the very few” (p. 243). It is interesting that both of these commentators ascribe as much agency to “the discipline.” Hardy may well deride exposition, but Latour, at least, doubts the very possibility of communication. With regard to the professional register of academic discourse as an exclusionary barrier, linguists Halliday and Martin (1993) write:

But there is another, deeper tendency at work, a long-term trend - however faltering and backtracking - towards more democratic forms of discourse. The language of science [including that of mathematics], though forward-looking in its origins, has become increasingly anti-democratic: its arcane grammatical metaphor sets apart those who understand it and shields them from those who do not. (p. 21)

Note that they themselves attribute [**an**] anti-democratic agency to the very discourse of mathematics and science.

The capacity to do, and to write about it in approved ways, in this case becomes a capacity exerted by one small group of people (the ‘owners’, perhaps, in a Marxist account of this power struggle) to influence a much larger one (the ‘workers’). Alfred North Whitehead talked about the “literary superstition,” which views the aesthetic appreciation of mathematics as being a “monomania confined to a few eccentrics in each generation” (quoted in Hardy, 1940, p. 85). In this view, aesthetic values are seen as residing with the power of few great men, and the influence of these men is thought to extend to the wider community of mathematicians. This in turn suggests that the aesthetic values of the discipline are somehow shared and consistent throughout the discipline – a belief still held by some. In other words, membership in the oligarchy would imply to some kind of indoctrination or at least inculcation.

This assumption was actually empirically tested in the 1990s by David Wells, who asked the readers of *The Mathematical Intelligencer* to rate theorems according to their mathematical beauty. He drew a number of inferences from the seventy-six responses, many from noted mathematicians, mostly from North America. First, mathematicians do not always agree on their aesthetic judgements – at least not in terms of evaluating the *beauty* of theorems. Instead, aesthetic appreciation, even in mathematics, is contextual, historical and subjective.

In addition to this empirical evidence against a class-driven view of power in mathematics, Valero (2005) points to a more philosophical concern, namely that this view of power can become highly destructive in the sense that few opportunities become available for challenging the existing power dynamics. Vithal and Skovsmose (1997) see this concern playing out in the field of ethnomathematics which, while endeavouring to broaden views of what mathematics is across different cultures and historical periods, can do nothing to challenge the existing power

structure in which this question (about what counts as ‘real’ mathematics) has already been decided – by the members of *hoi oligoi*.

An alternative to the class-driven view of power has emerged from Foucault’s work, one reflected centrally in the way disciplinary regimes become installed in institutions, including regimes of truth:

The important thing here, I believe, is that truth isn't outside power, or lacking in power [...] truth isn't the reward of free spirits, the child of protracted solitude, nor the privilege of those who have succeeded in liberating themselves. Truth is a thing of this world: it is produced only by virtue of multiple forms of constraint. And it induces regular effects of power. [...] (1980, p. 131)

‘Truth’ is to be understood as a system of ordered procedures for the production, regulation, distribution, circulation and operation of statements. ‘Truth’ is linked in a circular relation with systems of power which produce and sustain it, and to effects of power which it induces and which extend it. A ‘régime’ of truth. (1980, p. 133)

His ideas challenge the notion that power is intrinsic to and permanent within disciplines or social actors. Instead, power is seen as a relational capacity of social actors (Valero, 2005), one that is constantly transforming through practice and discourse. In other words, the power dynamics at play in the current mathematical regime cannot simply be the result of the influence of *hoi oligoi*, but must arise out of the participation of various actors in social practices and in the construction and institutionalisation of discourse.

This poststructuralist view of power affords interesting opportunities to understand why mathematical aesthetics has consistently been seen as an elitist enterprise within contemporary cultures. In particular, it challenges us to seek alternative interpretations of some of the consequences of this latter view, which include the following: that the discipline of mathematics, both in academic institutions and in education, is privileged over other disciplines in the humanities and sciences; that mathematics does not have the kind of critical interface between ‘consumers’ and ‘producers’ that other aesthetically-rich disciplines such as art and literature have (elitist views do exist in these fields too, although not to such an extent); that children are deemed incapable of aesthetic appreciation in mathematics.

Each of these positions could be explained in terms of the class-driven or even the objectivist view of power, but such explanations, in addition to being problematic, are also fundamentally without hope. In the following section, we delve into just one of these positions more closely and investigate some of the relations of power that are practiced and constructed in our culture that have led to it. In so doing, we follow Ober’s lead in seeking forms of the “capacity to do” that are more relational and transformative, as well as more widely accorded.

IF MATHEMATICS IS AN ART, WHERE ARE ITS CRITICS?

Many mathematicians have advanced the claim that their discipline is more properly an art than a science. They cite several reasons, most of them related to aesthetics. For example, the mathematician John Sullivan (1925/1956) claimed that mathematicians are impelled by the same incentives as artists, citing as evidence the fact that the “literature of mathematics is full of aesthetic terms” and that many mathematicians are “less interested in results than in the beauty of the methods” (p. 2020) by which those results are found. There is also the argument that, unlike with the sciences, mathematics does not have to compare itself against an outside reality – thus, the implication being mathematicians have choice and freedom when it comes to selecting their objects of interest (though see note [2] about an unease with these terms). Sullivan described mathematics as the product of a free, creative imagination and argued that it is just as “subjective” as the other arts.

These characteristics that mathematics supposedly share with the arts – creativity and free choice, as well as the use of “aesthetic terms” – may sound alluring to non-mathematician, who can recognise them as familiar in other (less exclusive) experiences. Tell a mathematics-fearing artist that the discipline is really about ambiguity (Byers, 2007), creativity and freedom, and their ears will likely perk up. However, these very characteristics only serve to remove the accessibility of mathematics from the non-mathematician further since, like the aesthetic sensibilities of Poincaré, they only belong to a privileged few.

It may be more fruitful to consider the differences between mathematics and the arts in understanding the power dynamics involved. Indeed, the philosopher Thomas Tymoczko (1993) may well have pointed out the most operative difference between aesthetic judgements in mathematics and those at work in the arts: the mathematics community does not have many ‘mathematics critics’ to parallel the strong role played by art critics in appreciating, interpreting and arguing about the aesthetic merit of artistic products. Mathematics may well be a discipline of freedom and creativity, but individual mathematicians have to engage in what Pickering (1995) calls “a dance of agency” between their own agency and that of the discipline – radical, revolutionary new ideas must still find expression, connection and interaction within the accepted forms of the discipline. This is true for art as well; Picasso’s cubism only existed and flourished through its opposition to and promotion of other artistic movements. But in the arts there are critics to interpret and negotiate the meaning and place of creative new products.

In mathematics, however, virtually no one stands on that border between the productive and interpretive aspects of creative work for mathematics (see, for instance, Corfield, 2002, on Lakatos’s legacy in this regard). This is not just a problem for non-mathematicians, who have little help in assessing the importance of new developments in mathematics; it has been problematic within mathematics itself. Gödel’s famous incompleteness theorem almost passed unnoticed when it was first

announced in 1930; that it was not is thanks to von Neumann who first appreciated its significance. This kind of thing is bound to happen in any creative discipline – other mathematicians were blinded by their philosophical commitments to formalism and positivism – but without any mechanisms to aid in interpretation, mathematics closes itself off from others, and sometimes from itself.

For Leo Corry (2003, 2006), this border becomes a distinction between the *body* and the *image* of mathematics, which he sees as forming “two interconnected layers of mathematical knowledge” (2006, p. 135). While the body includes “questions directly related to the subject matter of any given mathematical discipline: theorems, proofs, techniques, open problems”, the images “refer to, and help elucidating, questions arising from the body of knowledge but which in general are not part of, and cannot be settled within, the body of knowledge itself” (p. 135). These questions might include, as they did for Picasso’s audience, the questions that mathematicians David Henderson and Daina Taimina (2006) recognise as often going unanswered: “Why is it true?”, “Where did it come from?”, “How did you see it?” (p. 66).

In that the image involves questions that cannot be settled from within the body, Corry contends that the black letters and symbols on white pages that constitute formal mathematical texts cannot bring forth forms and colours that constitute the image of mathematics. The same can be said with respect to the arts: questions of categorisation and importance, for instance, are settled by outside commentators, most notably art critics who employ aesthetic notions and devices.

In our analysis then, mathematical elitism with respect to aesthetic issues arises from an instructional lacuna that could be filled by the creation of mathematical image-makers or mathematical critics. Like the art critic who helps the interested citizen understand the challenging art of the day – Barnett Newman, John Cage, Damien Hirst – so the mathematical critic would help the non-mathematician citizen understand why, for example, mathematicians continue to take the unproven Riemann hypothesis as true – using it to prove other results – even though high school geometry inculcates the idea that no result, not even the most obvious one, should be accepted without proof. Some may counter that it is easier to explain Newman’s brushstrokes than it is to explain the techniques used in mathematics to prove results. (See Mazur, 2003, and Tahta, 2004, for a related discussion around issues of mathematics and poetry.) But aesthetic decisions are rarely about specific thorny techniques; rather, they relate to issues of the body: why is this true or important, where does it come from and how can you help me see it?

IS/E]GORIA AS A GOAL OF MATHEMATICS EDUCATION

In conclusion, we return to the opening notions of democracy, and in particular *isegoria*, the ‘equal right to speak out on public matters’. Does it make sense to see a mathematics classroom as a deliberative forum? If we are thinking in terms of schooling for democracy in mathematics, and democracy is taken in the sense of

Ober's formulation of 'capacity to do', then what forms of mathematics education should students encounter? That right does not need to be exercised all the time, but it needs to be genuinely felt as a possibility.

Secondly, recalling Noddings' (1994) view of mathematics needing to be made optional once again (past a certain point) for progress to be made, who, then, will see mathematics as an activity, a discipline, a way of thinking, that they can *do* something with? This is not 'utility' seen in terms of a naïve external utility, the stuff of much contemporary debate, nor the equally problematic 'mathematics give[s] access to power' lobby, but a view that mathematics can do something for me, in a humanistic sense, one that repays the careful attention and deep engagement it may require. One that may expose students to a fundamental sense and experience of equality, the 'iso-' root of a number of words that connect to political regimes, and provide them another sense of human commonality. In addition, mathematics can provide aesthetic experiences with which students can shape themselves and orient themselves toward a culture of action and thought that is at least as old as the written.

Lastly, we wish to highlight the tradition and style of mathematics that masks the aesthetic values that underlie judgements at the same time as presuming their universality. This double concealment makes it harder to criticise such decisions and judgments even as we strive to make them manifest. At the heart of the oligarchy is a hierarchy – another key *-arche* term – where *hieron* refers to the sacred. That mathematics has a sacred writing is unarguable. To what extent the members of the congregation – *hoi polloi* – are able to engage with and criticise the sacred writing of mathematics remains to be seen.

NOTES

1. *Demos* meaning "the people" actually referred only to native, adult, male freeborn citizens of a *polis* (state), so is, of course, not exactly all-embracing. Also 'aristokratia' (*hoi aristoi*, the excellent) is another label for a form of governing 'power' that has resonance in mathematics.

2. A distinction between private and public goods is helpful in this context, where public goods, such as environment and culture, health and education, should be decided upon in the public realm. In her Massey lectures, Janice Gross Stein (2002) deconstructs notions of 'efficiency' and 'choice' in the latter two arenas, signalling how these have become ends in themselves rather than powerful means to contested ends. Choice is not a public good. Or, as our colleague Eric Love has eloquently framed it: "People don't want choice; they want what they want."

3. There is a tradition in mathematics of writing in terms of aesthetics. As will become clearer in this paper, we use 'aesthetic' in the following way, though unsurprisingly perhaps, this is not how everyone sees it. For us, aesthetic considerations concern *what* to attend to (the problems, elements, objects), *how* to attend to them (the means, principles, techniques, methods) and *why* they are worth attending to (in pursuit of the beautiful, the good, the right, the useful, the ideal, the perfect or, simply, the true). We have deliberately framed this specification in general terms, so that it applies equally well to mathematics as to art, historically the realm of much discussion of things aesthetic.

4. This stance is reflected in the fact that the Fields medal (equivalent for mathematics to the Nobel prize) can only be awarded to mathematicians under the age of 40.

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‘SOMETIMES I THINK WOW I'M DOING FURTHER MATHS...’: TENSIONS BETWEEN ASPIRING AND BELONGING

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Students draw on educational and social discourses of quality when they aspire to study further maths A-level. Historically these discourses are associated with privilege and inequity since many schools do not teach this subject. One recent government-funded project has set out to redefine knowledge about doing further mathematics. I use one student's account to illustrate how the practices of further mathematics engage with practices of the self. The reasons given for aspiring to study further maths are linked to claims for belonging in maths. They also raise challenges to belonging. Students face the choice of exclusion or aligning themselves with a neoliberal model of self-improvement.

BACKGROUND

There is a small but prestigious qualification for school-leavers in England and Wales that informs access to elite universities. This is Further Mathematics A-level: usually shortened to Further Maths (and officially capitalized). Over the last thirty years, Further Maths has changed in structure but remained at the centre of overlapping discourses about rigorous mathematics and widening participation. This inspired me to examine the ways in which one particular context can inform us about quality and equity in mathematics and how they are linked to western, liberal understandings of aspiring and belonging as practices of subjectivity.

A-levels are the traditional academic qualifications in England and Wales, taken by 40 per cent of students at 18+. Students usually specialise in three or four subjects over two years. Mathematics is one such subject and, uniquely, supports a second qualification. Further maths is a relatively stable, minority subject in a changing educational environment; its longevity heightens its visibility and positions it as an objective educational measure. It is thus given a gold-standard construction of quality: the gold-standard only has meaning because we no longer pay in gold but, by evoking the rationale of calculating back, it continually reinvents itself. So in further maths we have stories of a past golden age in which students were well-prepared in science subjects and competed to enter mathematics degrees; and these stories have currency today, even as we accept that practices have changed.

The practices of further maths also construct issues of equity. Students do not have equal access to studying further maths: they are constrained by ‘individual’ factors such as prior attainment, and ‘social’ factors such as school resources. Alongside official knowledge about A-levels, teachers and the media frequently disclose a hidden, ‘expert’ knowledge that certain subjects have greater exchange value, and these include further maths. White, middle-class students tend to seek more expert

advice about their choices and choose these high-status combinations . Information about further maths is thus differentiated by class and ethnicity. Moreover, student choice is constrained by what schools can offer. Students in state comprehensive schools are 3 times less likely to study further maths as those in independent and selective schools (Vidal Rodeiro 2007). Further maths is thus a context in which individual differences in opportunity, treatment and outcome are made visible as structural differences between schools, and it poses a problem to neoliberal explanations of social inequity as individual choice. My research examines a recent project to promote further maths to students and schools, the government-funded Further Maths Network (hereafter FMN), which ran from 2005-9. The FMN provided branded promotional and teaching resources and regional centres that recruited locally, employed tutors and collected data. These two distinctive features, branding and local expertise, acted as ‘selling-points’ to schools. Heads of maths effectively subcontracted further maths teaching for a group of their students. In return, the centre agreed to teach on a concentrated schedule, typically a weekly 2-hour after-school session.

My research traces how further maths has been reworked through the changing institutional technologies of the FMN, and in the practices of students and teachers. These new knowledges about ‘doing maths’ (Mendick 2006) in turn construct, and are constructed by, the ways that students experience themselves as aspiring and belonging to the identity/ies of further mathematics student. In this paper I use one student’s accounts of his experience to exemplify how representations of further maths as an ‘imagined community’ (Anderson 1991) intersect with liberal ‘practices of the self’ to enable and disable choices about how and whether to belong.

METHOD

My theoretical base is a poststructuralist perspective. I insist that power circulates within local practices: it is at the levels of schools, teachers and individuals that knowledge is constructed and reconstructed about *who* can study further maths and *how*. I aim to take Further Maths out from its capitalised seclusion, and examine how (small-f) further maths is constructed by practices, technologies and talk, and so open to questioning and revision rather than closed off behind an official meaning. Martin (2006) suggests that the best way to understand equity is to ask how students live and explain their day-to day experiences of mathematics in relation to school, community and sociohistorical contexts, and how this interacts with the senses of the self that are have meaning for them. I find this compatible with a poststructuralist methodology, analysing in detail “what is given to us as universal, necessary, obligatory” (Foucault 1991, p45), and how it co-exists with whatever is presented as “singular, contingent, and the product of arbitrary constraints”.

My analytic focus is what Foucault (1990) calls ‘practices of the self’: the knowledges and processes that inscribe what it means to be a successful individual within a particular history or culture. Practices of the self establish the norms and

means by which people explain themselves, govern themselves, and engage with others. I have explored the intermingling of discourses of further maths and discourses of the self by analysing textual data that gives accounts of choosing and studying mathematics and further maths. This data comes from 31 interviews and follow-up e-mail questionnaires with twenty-four students in three FMN sites, chosen for their differing sociogeographic settings and further mathematics teaching practices. The sites were in a London borough, a market town and an industrial city; students either met at a weekly FMN lesson to study AS-level over two years or were visited by a FMN tutor and could pursue the subject to a full A-level. The participating students belonged to seven different ethnic communities; ten were girls and fourteen boys; some were working-class by parental education and employment; others middle-class. There is not space here to discuss effects of gender or community but this overview shows that I found no 'standard' profile of a FMN cohort. Although I go on to consider participation in further maths in terms of individual choices, it is worth noting that it was at school level that students were first excluded from further maths and then included via the FMN.

Students attended semi-structured interviews in year 12, and again in year 13 for A2 schools, with half-termly emails in between. I asked direct questions about choosing AS subjects, how their class interacted in lessons and how they fitted in with others in their class or who they had met at FMN events. I was concerned that students should have opportunities to take up and compare different subject positions so I also included indirect questions that involved talking about school and mathematics in unfamiliar ways. For example, I asked students to select from a list of adjectives (such as *disciplined*, *crazy*, *independent*) to describe themselves as learners, and talk me through their reasons. The emails asked standard questions, eg reflecting on A2 choice at the beginning of year 13, and also allowed me to follow up interesting responses in a reflective email conversation. I coded the students' accounts for descriptions that suggested collectives, such as further maths students, family and friendship groups; then summarised how and in what contexts they positioned themselves as belonging or not belonging.

I have chosen one student, Mario, to discuss here because he often appeared uncomfortable with seeing himself as a further maths student. He positions himself very explicitly in relation to collectives of maths and further maths, and illustrates some of the tensions there are in belonging to those collectives. It is his quote that gives my title:

Sometimes I think Wow I'm doing Further Maths, but then I think am I actually doing any good?

In our conversations Mario positioned himself variously as successful and as struggling, as a natural and as an outsider, and tried out different ways of justifying his decisions to continue. I interpret the ways in which he argues whether doing further maths is 'doing any good' as examples of what he aspires to by doing

mathematics, and thus how he participates in constructing quality. Mario also describes the practices and positions that he sees as threatening his engagement in mathematics. I see these personal accounts as examples of the ways-of-knowing which individual agencies make use of thereby contributing to re-creating discourse and re-forming social patterns.

MARIO: ASPIRING AND BELONGING

Mario lives in the centre of a relatively deprived English industrial city. The somewhat run-down school he attended until age 17 was replaced by a new business-sponsored academy that specialised in mathematics and science. Mario is White and his family show characteristics of both middle-class and working-class cultures (Ball, Maguire, and Macrae 2000): his father is employed in engineering and insurance, and his mother as a secretary, neither attended university, and he receives a government education maintenance allowance. Mario's passion is rock guitar. He and his best friend, Randall, both dream of making the improbable happen and finding a career in music production.

Mario started year 12 with maths, chemistry, physics and further maths AS-levels. After he had chosen those subjects, the further maths teacher fell ill and Grants arranged weekly after-school tuition with the FMN instead. For year 13 the academy appointed a new teacher to take on Mario's maths class of eight students and his further maths group of four. Mario also dropped chemistry before year 13, so that further maths joined physics and maths at the core of his school experience.

Mario and Randall take all the same subjects, and chose to be interviewed together. They interrupt and predict each other's talk in a way that emphasizes how their experiences of learning have been jointly constructed around shared ambitions and interests in and out of school. In particular, in year 12 they tell me that they make choices together: *our choices are like influenced because we love music.*

Aspiring: using maths to be an individual in education and beyond

In his first interview Mario describes his initial subject choices as based around maths – the *four core Maths subjects*. He presents evidence he has gathered to support this claim for the centrality of maths: all university courses want high grades in maths, and maths *comes into everything*. His claims for maths are based on its power as a widely accepted currency and a knowledge that will be relevant even – and especially - when he attempts a more idiosyncratic career linking science and music. Mario then describes further maths as a way of proving that he can make a success in unexpected ways:

First of all when I said about Further Maths my mum was ... 'This is... No, you can't do it.' And I was okay at Maths at GCSE but never like that star, that everyone else was like getting full marks all the time. And when I said Further Maths she was quite shocked and didn't think I could do that. And that made me want to do it even more.

Here Mario ascribes his mother's doubts to measurement in the national examinations at 16. The tests position him compared to others and so can be understood as excluding him. I have suggested above that further maths invokes quality as a gold-standard. Mario draws on this representation to challenge the measurements of GCSE: for him, choosing further maths is a way of standing out as being different but just as good or better than other people. He has however set the stakes high by making a comparison with 'full marks all the time'. I suggest that the sociohistorical context that positioned further maths as rarified because it was restricted to certain schools, has been reinterpreted as a practice of individual choice so that further maths aligns the chooser with extreme personal qualities of ability and dedication. It offers a possibility of measuring oneself against what seems to be an objective standard rather than against other students.

Although the first implication of his choice is to stand away from school measurements, Mario also argues that further maths conforms with school's expertise and technologies. He describes how his work-experience mentor encouraged the academic route to his dream; he could take a physics degree and then work with sound either on stage or acoustic consultancy. Mario cites universities as telling him further maths is the right preparation for physics, and his teacher's view that it gives you a head-start at university. The reasons, and their authoritative institutional sources, suggest that he is aspiring to a breadth-plus-depth construction of quality. He aspires to study further maths because it is broad enough to provide both academic certification and application in a 'real-life' setting, and because it is deep enough that he can use it for a 'head-start', making sure that he is included in the niche he has picked out as desirable.

Belonging: further mathematicians as the archetypal maths student

For Mario, maths is a rational choice in two, connected ways: it is about *right or wrong answers*, and it provides and denies access to certain careers. This absolute binary structure carries over to define collectives of maths students. There are two kinds of students studying maths: those who really belong and those he positions as less rational, the people who just *choose it on a whim*. He sees the dependable, rational structure of maths as making it objective and accessible to him - *you can see what you've gotta learn* – but he also needs maths as a tool for exclusion: not everyone can be equally enabled by this structure. This means that his statements about belonging have to be made boldly and require a positive modality: for example *we are a lot more clever than them*, and *you've just gotta be so dedicated*. For Mario, doing further maths is a way of demonstrating the depth of his involvement in mathematics. He stresses that *it's the hardest subject*, that *it helps, really helps* at university, that by doing it *we are immediately more mature*, that he may get told he is a *geek* and *insane* but *this in turn makes me feel more determined*. Using this kind of statement Mario constructs further mathematicians at the extreme of mathematicians but also at the centre. I picture him as placing them at the very top of a conical heap. By aspiring to further maths, Mario legitimates his claim to belong to

maths itself. At the same time he awards himself approved qualities such as cleverness, maturity and determination.

Challenges to belonging: doing cool and doing clever

There is both pleasure and surprise in Mario's '*Wow, I'm doing Further Maths*'. However he is more frequently concerned about how others construct maths and further maths collectives, and how he might not belong. When Mario's AS grades were lower than he wanted, he was once again threatened by measurement. Other students in the class excused their low grades by citing reduced teaching time. Nearly all stopped further maths; but Mario and Randall continued. At the end of their second year I asked why they had chosen differently. During the discussion (lines 576 to 649), Mario deals with conflicting understandings of what studying further maths means to them and about them:

577 We were a lot more clever than them.

594 I think a lot of people say it's the hardest A level.

596 And everyone knows it as well. Which makes us feel cool...

602 It's just it's... I didn't mean it makes us feel cool, it makes us look stupid.

622 And it doesn't make you... People makes ... think it makes you a genius.

624 We should be really clever, but something about it, maybe a bit of common sense, like we just... sometimes we just, like maybe the time of the day, or what mood you are in, but sometimes we feel really stupid.

In this exchange, Mario is conscious of how further maths can position him and Randall as clever, cool and different-but-not-alone. These feelings are described as resulting from common choices and so personal to *both* of them. Further maths starts as a way for Mario and Randall to mark themselves out compared to others through belonging with 'the hardest A-level'. But Mario ends up worried about the exclusion that is implied by this emphasis on difficulty and depth. He recognises that 'it' – maybe further maths itself, maybe what 'everyone knows' about it – compares him with the 'genius' model he has just helped to build. Mario not only has to defend his sense of belonging against structural threats such as AS grades but also against how he explains his self-practices to himself. If he doesn't 'feel' clever, how can he belong? In line 624 he calls up an explanation which supports two interpretations: either self-doubts are momentary irrational lapses from cleverness, or cleverness is not related to common sense and practical experience, and further maths does not have the 'broad' application that he originally aspired to.

Mario is also concerned by the way that schoolteachers portray further maths. Describing students collectively is a common classroom practice. Mario described his resentment when teachers asked questions which the class could not answer – something that was a pedagogic problem for the *whole* class or for the teacher – and then diverted responsibility onto the further maths students. His physics teacher would typically address a question first to the whole group, then again to maths A-level students, and finally end up asking the further maths students. This progression

reinforces the embedding of further maths students as archetypal mathematicians. It constructs them as more able and/or more knowledgeable than other students but still less knowledgeable, though not necessarily less able, than the teacher. The repeated questioning suggests too that there are reasons for their lack of response, perhaps they are too shy or self-conscious to contribute in class or are not as knowledgeable as they should be; it calls attention to this and holds them to account for it. So Mario, and other individual further maths students, feel that they are the ones positioned as at 'fault' when the larger group cannot answer the teacher's question. They are constructed as failing: either as not really belonging in further maths, or as belonging but embarrassed to admit it, or as belonging to something that is useless. Mario and Randall particularly object to their physics teacher positioning their further maths knowledge as deeper than other school knowledge. They recall their teacher's words and how they felt in response:

M 'You should be learning' ... 'You should know this; you do Further Maths'. But we just learnt about complex numbers, we haven't learnt about...

R Yeah. We just learnt about the root for minus 1, don't they? Not how to... Not what black matter is or whatever, dark matter.

Here they are not only resisting the way that they as individuals are being associated with the teacher's personal representation of what further maths students 'should' be, but the way their collective school further maths experience is being represented in terms of obscure learning such as 'dark matter'.

Belonging as inauthenticity

In the previous section, I showed how Mario articulates the dangers of being potentially excluded by other people's constructions of further maths as well as his own positioning of it as being something worth aspiring to. In both of these it is how further maths is how positioned as achieving quality through depth that serves to exclude. In his experiences of learning maths in school, these positions dominate the discourse at the expense of the alternative position that further maths achieves quality through breadth of application. Images of depth such as 'genius' or 'dark matter' call on wider social images (Mendick, Epstein, and Moreau 2007) of the mathematician as an outsider who risks losing touch with society, whereas Mario had aspired for maths to include him in a specific masculine social role. Mario takes this risk of losing touch inside himself and rephrases it in terms of his personal qualities when he wonders:

whether I'm patient enough to actually go through all the Physics and stuff, and be good, really good at it at the end, or go straight into it and build up experience in it.

Here he is linking education with having to be 'really good', and contrasts having to 'go through' a degree course with actively 'building' authentic, direct experience. Staying in post-compulsory education, and studying more abstract disciplines are the types of choice that re-produce structural class inequalities (Atkinson 2007). Here

Mario is constructing them as threats to belonging based on a potential 'true' understanding himself as practical and concerned with the present. Later in this interview Randall, too, positions studying maths and physics as inauthentic, and suggests that by opting for a university route Mario is excluding himself from their dream:

Randall: I'm going to be there. But Mario's gonna be like working out all these equations.

Mario: And I'm gonna be paid ten times more than you.

Randall: And I'm gonna be the happier one. It's not all about money Mario.

Mario: No. I'm gonna be happy.

Mario has to go on the defensive. He uses maths to make a claim for economic success but Randall excludes him not just from practical experience but from happiness. Gaining self-knowledge so that you can pursue personal happiness is central to neoliberal subjectivity (Rose 1999). Here the construction of further maths as desirable-through-depth reappears as a tension between understanding oneself as excluded either from maths or from one's authentic self.

Working at belonging

Despite these threats to belonging, Mario makes a final push for success in maths and further maths. He presents this as having to work on himself by changing his past practices: *I'm trying just to picture in my head what it's gonna be like getting the results and knowing that I've worked as hard as I could have like now, this time.* He describes motivating himself by recalling previous personal emotions, such as *I know the feeling that I'll get in the summer*, and imagining family diasappointment: *my mum paying for my re-sits*, and he changes his previous attitude to finding a tutor:

through GCSE and AS I never did because I thought, oh I just wanna do it myself, you know get things done. But then I thought comes August and I think, what if I haven't achieved could I have done any better, so I thought I'd give it a try.

Through these and other descriptions of his efforts in further maths, Mario suggests that he can belong through changing who he is. He puts aside the images of 'effortless achievement' (Jackson 2006) that accompany the genius model of further mathematician, and instead firmly promotes effort as the way to align oneself with further maths. Further maths involves personal qualities such as intuition - *you've gotta be very intuitive* to know how to tackle the questions, and *the maths side has to be natural to you.* However rather than accepting these qualities as being located in himself as an outsider individual he now stresses how that performance depends on engagement and work: *I need to be quicker. And that's linked to being natural, having it natural, coming naturally to you. And that comes from experience, and that comes from time revising.* He is starting to see what is natural to him as subject to continued work, demonstrating his adherence to a neoliberal view of life as a project.

Belonging in the further maths collective becomes part of identifying and acknowledging essential characteristics of one's maturing self so that by year 13 Mario says: *I can't believe I even considered dropping it*. Although Mario has been unsuccessful in using further maths to challenge the public gaze of examinations, he now uses it as a way of exercising the autonomy over what appears within his control: in his words, to *mould myself*.

SUMMING UP

I have used Mario's account to introduce representations of mathematics and further mathematics students, and illustrate how an individual can position himself as in relation to these actual or imagined collectives. These representations and positions were not unique to Mario, but he voiced them particularly clearly. He and Randall were the only two students in my study to continue with A2 further maths despite getting some low AS level grades, and certainly led them to question what they were getting out of the experience. I have summarised five themes in Mario's accounts of his relationship with maths, further maths and other collectives. First, aspiring to study mathematics is a means of controlling how one is valued as an individual in education and in a competitive, masculine, 'real' world such as rock music; second, representing further mathematicians as archetypal maths students positions doing further maths as a means of belonging in maths. Thirdly I note that the practices of 'doing cool' and 'doing clever' that support these aspirations equally construct challenges to belonging, and are negated by public effort or muted success. Rejecting further maths as obscure and deep is a way of protesting against these challenges but can position continued belonging as inauthenticity. Finally, accepting the need to work hard in further maths devalues it as a way of claiming success outside education but allows a position as a maturing identity-worker.

Mario and Randall are caught in what Hall calls the 'deep ambivalence of identification and desire' (1992, p255) that results from meeting collective representations of your group that are created by dominant discourses. Identification with such collectives involves internalising the 'self-as-other'. Hall gives the examples of the ways that black men both fear and desire the representations of noble savage and violent avenger constructed by racism, and women produce and contest the representations of weak and feeble females and strong castrating women. So further mathematicians, like Mario, want to belong to further maths in part because they want power over unspecified future knowledge, but they are only too aware that it cannot offer them guarantees and certainties. In Mario's case the image of the able, committed, archetypal further maths student is so dominant that it too produces a doubling of desire and fear. He aspires to wield the power of this position and particularly its privileged access to the kind of niche of cool, individualized masculinity suggested by mathematics, rock music and sound engineering. He reconstructs this representation of further maths in setting out the qualities that attract him as breadth-plus-depth: breadth of application that extends beyond school and

connects him to the physical world, depth of learning that offers him a measure by which he can claim to belong. Problems arise for Mario because the ways in which he can show that he belongs to school further maths - by studying hard and by accepting that he needs help of tutors and teachers – are in tension with the reasons he aspired to do the subject.

Mario described his final push for success as deciding to change how he thinks about himself and his goals. In the process of including himself, he strips further maths of the quality of separateness that once attracted him, complaining that ‘it should just be called different modules’. Excellence and depth are powerful constructions, but knowing how to include ourselves is accompanied by consciousness of how to exclude ourselves. It would be easier to continue further maths if it was practised and promoted as offering quality as breadth.

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RECOGNIZING WHAT THE TALK IS ABOUT: DISCUSSING REALISTIC PROBLEMS AS A MEANS OF STRATIFICATION OF PERFORMANCE

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This paper analyzes a typical classroom discussion of a realistic problem in an eighth grade mathematics classroom in a German high-streaming school. It considers performance in the classroom as not merely relying on cognitive variables but rather on social interactions. The teacher's role in these interactions is highlighted as crucial for defining the legitimate discourse and thereby the conditions for performance. As there is quantitative evidence that the performance on solving realistic mathematics problems is strongly connected to social variables, classroom discussions of these kinds of problems appear to be very fruitful for qualitative research. Therefore, a structural distinction of different discourses engaged in realistic problems is emphasized and the relation between these discourses in the discussion is outlined. Furthermore, two incidents of students' misrecognition of legitimate discourse are analyzed. The analysis shows that the teacher has at least a joint responsibility for the students' misrecognition, as he encodes his favoured ways of performing. Thus, a stratification of performance levels emerges based on the students' ability to decode implicit instructions.

PERFORMING REALISTIC MATHEMATICS PROBLEMS

In recent years there have been several studies on the performance of students in realistic mathematics problems. Roughly a distinction can be made between research on problems demanding to *make use* of the realistic contexts on the one hand and research on problems demanding to *distance* oneself from the context and to use only a few certain contextual details on the other hand. The former (e.g. Baruk, 1989, Verschaffel et al., 2000) sees the students' troubles in making sense of word problems and solving them superficially by just using trained mathematical strategies, even if there is no related sense at all. The latter states that in most situations of assessment, marking schemes are given, that value to make use of certain details of the context to be mathematized, but penalize the inclusion of realistic aspects in general (e.g. Boaler, 1994, Cooper & Dunne, 2000). These studies report that students draw too extensively on the context and fail to reckon the one certain detail which is valued by the marking scheme. Furthermore, there is evidence that this phenomenon is not equally distributed among all students, but that it is influenced by sociological variables, such as socio-economic status (Cooper & Dunne, 2000) or gender (Boaler, 1994). As Säljö and Wyndham (1993) maintain, a student's problem solving behaviour is also dependent on the situation s/he is in while solving the problem. They found that students solved a problem differently when confronted with it in a mathematics class than when confronted with the same

problem in a social studies class. Accordingly, the reasons for both of the phenomena reported above can be assumed to lie in implicit assumptions students have about what is expected of them. Moreover, the distinction between the two lines of research outlined above already shows that realistic mathematics problems may entail different demands. Hence, students' performance is not only dependent on their mathematical skills, but more importantly on their skills to recognize what kind of problem solving behaviour the problem demands.

Gellert and Hümmer (2008) go further and state that a legitimate performance in a mathematics classroom is not only dependent on the broad cultural view of mathematical competence, but is socially constructed by interactions on the micro-sociological level of the classroom. Through implicit and explicit evaluation the teacher establishes criteria for a good performance.

As emphasized above, different realistic problems may pose different demands. Furthermore, legitimate mathematical performance is socially constructed within the classroom, and this construction is controlled by the teacher. Hence, in different mathematics classrooms there might be different demands for a good performance. With regards to realistic mathematics problems that means that in order to perform well, students have to find out on which level reality and mathematics shall be connected to or insulated from one another. Realistic problems are often artificial and only apparently 'real'. The performance demands are less regulated by the original context than by its recontextualization in the classroom (Gellert & Jablonka, in press). This gives rise to the question if a school-mathematics problem can be considered "realistic" or "pure mathematical" at all. A point of view, focussing more on discursive structures, as provided by Bernstein (1999) might help out.

HORIZONTAL AND VERTICAL DISCOURSE

„A horizontal discourse entails a set of strategies which are local, segmentally organised, context specific and dependent, for maximizing encounters with persons and habitats.”
(Bernstein, 1999, p. 159)

Consequently, the validity of knowledge and strategies developed in a horizontal discourse is bound to its original contextual segments. An example for a horizontal discourse is a mundane activity, such as sharing a bowl of pasta with a friend. The validity of a sharing strategy is dependent on situational segmental variables, such as individual appetite or who is invited and who is inviting. There is no general solution to solve such a "problem". As different situations of this kind exist without having an impact on one another, this kind of discourse can be called *horizontal*.

In contrast to this, Bernstein defines vertical discourse as follows:

„[...] vertical discourse takes the form of a coherent, explicit, and systematically principled structure, hierarchically organised as in the sciences [...]"
(Bernstein, 1999, p. 159).

In this case the validity of knowledge and strategies is dependent on knowledge of higher generality. Knowledge needs to be coherent with other context-independent knowledge of the same discipline and not with contextual segments. Hence, new knowledge can be generated from what is already known. The structure is therefore hierarchical and this kind of discourse can be called *vertical*.

In pedagogic discourses, and in school mathematics discourse in particular, horizontal discourses are often used to provide students with access to a vertical discourse (Bernstein, 1999). If one exchanges “pasta” for “pizza” in the example above, it would be likely to be found in a mathematics classroom as an entry point to the vertical discourse of fractions. Mathematics is thereby restricted to its horizontal ‘origin’ and students are led to believe that mathematics is crucial to participate in such situations. Further, fractions seem to reference the situation of sharing food. Even though teachers might be suggestive of parts of pizzas and fractions being about the same thing, out of school they are still describing different things. Hence, the boundaries between horizontal and vertical discourse remain. Therefore, Dowling (1998) uncovers both the aspects of participation and of reference as myths.

As shall be shown in the following analysis, a crucial condition for students’ performance in “realistic” problem solving is the ability to *recognize* the boundary and its strength between the horizontal and the vertical discourse. Using Bernstein’s terminology (1996) this ability shall in the following be called the possession of *recognition rules*, while the strength of the boundaries between the two discourses shall be referred to as *classification*.

“Where we have strong classification, the rule is: things must be kept apart. Where we have weak classification the rule is: things must be put together.” (Bernstein, 1996, p. 26)

However, classification is not created by a mathematical content itself. By shifting the content into a classroom and therefore into a context of transmission and acquisition, the mathematical content is *recontextualized* into school mathematics. Consequently, something or somebody has to control this recontextualization. Again following Bernstein (1996), the strength of this control shall be referred to as *framing*.

“Where framing is strong, the transmitter has explicit control over selection, sequence, pacing, [evaluation] criteria and the social base. Where framing is weak, the acquirer has more *apparent* control (I want to stress apparent)” (Bernstein, 1996, p. 27).

While classification (C_{\pm}) connects a structural level, framing (F_{\pm}) [1] can be seen as its interactional implementation. A change of classification can be reached through framing. If classification is strong, framing can furthermore be used to either explicate or blur classification modalities. The impact of classification and framing on the acquisition of the recognition rules by the students will be pointed out in the analysis and discussed in the conclusion.

THE DATA

This paper analyses a four-minute sequence in an eighth grade (age 13) mathematics classroom in a German “Gymnasium” [2]. The data is taken from the rich data corpus of the Learner’s Perspective Study (LPS) [3]. In sixteen culturally diverse countries three classrooms each were videotaped for sets of ten consecutive lessons, using a three-camera approach (Clarke, 2006). Furthermore, video-stimulated interviews were conducted with students after each lesson, and a questionnaire was handed out to the teacher. The data collection in Germany was carried out in 1999. The sequence has already briefly been analyzed by Gellert & Jablonka (in press). A deeper analysis of the classroom discourse, based on the language of description by Basil Bernstein (1996) and developed further by other scholars (Singh, 2002, Morais, 2002, Tsatsaroni et al., 2004, Gellert & Jablonka, in press) has been carried out by Straehler-Pohl (in press). This work will be summarized in the section on the characterization of the classroom talk, before the focus is moved towards two specific incidents of students’ misrecognition of discursive demands. The chosen sequence is the only sequence in all ten videotaped lessons that shows this class working on a realistic mathematics problem. However, the discussion of that problem and the intricacies emerging for the students can be seen as exemplary for a “Gymnasium”, as Gellert & Jablonka (in press) have pointed out.

THE GIVEN PROBLEM

After having treated binomial formulas in several previous lessons, the teacher draws a square on the blackboard to represent a farmer’s soil and then asks a LPS-researcher to pose the following problem:

Researcher: Now the neighbour comes to the farmer and says right listen (...) that borders on this

Student: What

Researcher: It would be incredibly useful for my planning if I could take away one meter from your one side say the side across on the top if you'd give that to me I'd give you an additional one on the other side instead

Teacher: So

Researcher: And the question is would you agree to that if you were the neighbour?

Afterwards, the researcher hands over to the teacher who proceeds to lead the discussion. Obviously, there are different ways to approach this problem. Several different aspects can be taken into consideration to answer the question of fairness. These aspects can be roughly categorized in three categories: aspects that

- can be mathematized straight away (e.g. area, perimeter),
- could be mathematized with further information (e.g. quality of the ground),
- do not need any mathematization at all (e.g. social arguments).

In the student interview, the researcher formulates her view on the problem:

Researcher: With a so to speak the smallest fence with maximal area is always a square.
That was ... that's what this exercise actually showed [5]

The teacher himself states the aim “representing contextual problems/tasks in linear equations” in the teacher questionnaire. However, an analysis of the whole sequence has shown, that the teacher’s role in leading the discussion is rather coherent with the researcher’s statement than with his own one (Straehler-Pohl, in press). Hence, while the problem itself is set within a horizontal discourse it aims at extracting highly generalized knowledge, clearly settled in a vertical discourse. A crucial condition for successful performance in the classroom discourse will be to follow the path leading from the horizontal into the vertical discourse. In the following section the boundary between these two discourses and the teacher’s guidance on the way from one to the other will be analyzed.

CHARACTERIZING THE CLASSROOM TALK

The classroom discussion leading to the solution favoured by the teacher is about eight minutes long (or short) in total. The analysis (Straehler-Pohl, in press) has shown that the sequence consists of two parts of almost equal length, but of very different character. While the discussion remains close to the problem text and therefore includes horizontal discourse in the first part, it is purely mathematical and therefore purely vertical in the second part. In this second part of the discussion there is no evidence for students’ misrecognitions of the discourse. One could conclude that this shows the teacher’s success in providing access to the recognition rules. However, none of the students showing misrecognition in the first part of the discussion is participating (or even trying to do so) in the second part anymore. This paper does not focus on the second part of the discussion as it is a purely vertical discourse. Instead it aims to analyze the classification and framing of both horizontal and vertical discourse in the first part of the discussion.

By drawing a square on the board before the introduction of the problem, the teacher creates a setting, which can be identified as a school-mathematical setting instead of a realistic out of school-setting (C+). However, it is not explicit to the students, if the square is meant to represent a major frame of reference or just a sketch to support the comprehension of the following problem (F-). In contrast to the square as an institutionalized signifier, the language the teacher is using in the discussion is often mundane. This shows that he is weakly controlling the boundaries between the horizontal (mundane) and the vertical discourse (F-). The classification between these discourses is not clear yet. The weak framing achieved through using mundane language could be implemented intentionally to weaken the boundaries (C-), but it could just as well be an unintended blurring of existent boundaries (C+). Different students’ answers, which can be identified as horizontal discourse, are rejected by the teacher (see the two examples in the next section). This shows that boundaries

between these two kinds of discourse are, in fact, strong, and at the same time they are put into a hierarchy (C+). Answers that draw on horizontal discourse by using the sketch for visual arguments or by asking questions about details of the problem's context are ironically rejected by the teacher or are labelled "irrelevant". The analysis clearly shows that the teacher aims at leaving the horizontal discourse behind as fast as possible and at launching into the 'more relevant' vertical discourse, which he values as legitimate. This becomes very obvious when two students are giving similar, yet not identical answers regarding the decrease of the area by one square-meter. The first student (as the teacher and the researcher before) uses mundane language. The second student makes the same argument, but he uses mathematical terms such as "one m times X" and "one m times X minus one in brackets". While the teacher's comment on the first answer is a short mumbled "um" followed by his looking for further answers, he acknowledges the second answer with an "aha" underlined by a smile on his face. Now he seems to feel confident in having reached the point where he can proceed the discussion on a strictly vertical level. This shows that horizontal and vertical discourse are strongly classified (C+). The criteria for a good performance do not only include drawing on vertical arguments, but also formulating them in the *language* of a vertical discourse. This crucial criterion is not stated explicitly, but the students have to extract it from the subtle differences in the teacher's reactions. The teacher uses mundane language throughout the discussion and thereby creates the impression of weak classification. As classification de facto stays strong, this is a case of weak framing (F-). In summary, the sequence is characterized by a strong classification of horizontal and vertical discourse (C+). Incoherently, the teacher is using a weak framing (F-) over the boundaries and by that making the classification invisible. Hence, the students have to *recognize* the implicit boundaries established by the teacher in his inconsistent interactions to have their performance *acknowledged* by him. Two situations, where students fail to recognize these boundaries and the teacher's reactions to these students' utterances shall be discussed below.

TWO SITUATIONS OF MISRECOGNITION

Student 1: No well because that he's got less (...) You can you can see that in the drawing already that you get less of the property then

Student 2: So I now somehow well I would decline the offer because that is after all (...)

Teacher: Wait now I haven't really understood that now there's such a murmur

Student 2: (...) I would decline it because somehow um you've got a piece you don't have a corner like that (...) property goes around that

Teacher: Well you got uh which one's the new property now. That's no square anymore now but it's still also a rectangle after all so you would decline it because because you'd rather have a square than a rectangle

Student 3: That's great

Teacher: No, right?

Both student 1 and student 2 argue relying on the sketch on the board:

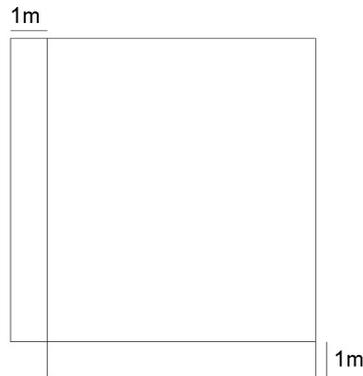


Figure 1. Sketch of the farmer's ground as drawn by the teacher.

Such arguments are bound to the segment of this singular sketch and therefore they are part of a horizontal discourse. Student 1 argues that she can see that the area will decrease, relying on her visual judgement. Visual judgement, however, might be a legitimate strategy for a farmer to solve problems but not for mathematicians. Student 2's argument is different: She misinterprets the sketch and argues from a practical viewpoint, seeing a problem in having an irregular area. Again, this might be an important argument for a farmer needing to steer his tractor around a corner, but not for a mathematician, arguing in the vertical discourse of general areas. So in the end, both students are drawing on different arguments, a visual argument of area sizes on the one hand and a practical argument of shapes on the other hand. Both of these arguments are cases of horizontal discourse. The teacher's ironical reaction shows that he considers none of these answers legitimate. In Bernstein's words: both students do not have access to the recognition rules. Hence, at this moment both students are not in the position to perform well in the discourse. In his ironical comment on preferring squares over rectangles, the teacher introduces vertical discourse and by that gives a hint as to what kind of talk is considered legitimate. In the chronological order of the talk, the teacher seems to react to Student 2's answer. But regarding the content of the answer his reaction is pretty vague and rather seems to refer to Student 1's answer. In the end, the delay in time and the vagueness of the reaction puts the students in charge of finding out, what to change to make their answers legitimate and, thus, to perform well.

The following subsequent answer by Student 4 shows that the teacher (at least for this student) did not succeed to provide access to the recognition rules through his comment:

Student 4: (...) I mean um wanting to take away something on one side and add something on the other side. Where is the neighbour's garden at all? Is it on the left hand side or below it?

Student 5: All around.

Teacher: It is all around.

Student 4: That's a little illogical isn't it?

Student: Well why

Student6: Oh

Teacher: That is now that is now uh

Student: Boy

Teacher: Uninteresting our question uh whether the one concerned well if he should swap whether whether that's favorable for him or whether uh well whether it doesn't matter or that's all this is about whether for that farmer who's making trouble here in here whether for him uh yes

Student 4 seems to have problems imagining the given problem in reality. She, therefore, asks for further information. This shows that she is still 'stuck' in the horizontal discourse of reality, while student 5 seems to have already recognized, that the problem is not about a real neighbour who needs to be positioned somewhere. He is even able to translate the vertical fact that this is not relevant into the language of the horizontal discourse as practiced by Student 4: "It's all around." The teacher, who seems impressed by this performance of Student 5 acknowledges it: "It's all around." So, while communicating meanings of a vertical discourse, they translate them into - and by that participate in - the horizontal discourse. As this translation is invisible for Student 4, she rightly doubts their answers. Being surrounded by the property of a single neighbour would be a very unlikely situation for a real farmer. However, the teacher states that her objection is not of interest. To explain why, he again engages in the horizontal discourse of contextualized language: "*that* farmer who's making trouble *here* in *here*". By that, he encodes his implicit actual message. To perform well in the further discussion, students need to be able to decode this message. It is pretty obvious in the sequence above, that not all students are able to do so. The possession of the recognition rules is a major prerequisite for good performance. At the same time the teacher is not explicating but rather encoding them. This creates a situation, stratifying students into those who have the ability to decode implicit evaluation criteria and those who have not. Hence, hierarchies among students emerge.

CONCLUSION

From the point of view of mathematics education, dealing with realistic problems can have different aims. These problems can be discussed to use mathematics for a serious examination of realistic problems, as favoured by scholars advocating of mathematical literacy (Gellert, Jablonka & Keitel, 2001). A different, yet not less respected, school advocates the use of mathematically rich realistic situations as entry-points for students into a self-employed construction of complex mathematical

ideas, as postulated by Realistic Mathematics Education (RME) (e.g. van den Heuvel-Panhuizen, 1996). These are just two of the many diverse, but respected stances on realistic problems. For teachers, this creates a slightly obscure situation. They might not always know which aim to follow and which problem to choose for their aims. The inconsistent situation of strong classification and weak framing analysed above can be regarded as a documentation of such a situation. While the teacher consciously aims at “representing contextual problems/tasks in linear equations” the practiced discourse is taking a different course. The problem’s inherent aim is rather the development of a complex mathematical insight (see the researcher’s statement above). So it is no wonder, that a situation is created, where the teacher’s aims (C+) and actions (F-) are not coherent anymore. At the same time, the analysis has shown that students’ performances are highly dependent on their ability to decode the teacher’s aims in this classroom discussion. Hence, the unjust situation is created where students need to read an implicit code (of C+) out of the teacher’s incoherent behavior (of F-) to be able to perform in the classroom. In the end, the condition for performance is rather the ability to read the teacher’s codes than the competence of either “representing contextual problems/tasks in linear equations” or using mathematics to solve real problems (mathematical literacy) or to develop mathematical ideas out of real situations (RME). This is an inappropriate situation and it is likely that it is not desired by the teacher himself.

NOTES

1. The strength of classification will be indicated by C+ and C-. The same applies to framing: F+ and F-.
2. The German school-system is separating students into three streams after primary school. “Gymnasium” is the highest stream and graduation from it allows students to enter university.
3. See <http://www.lps.iccr.edu.au/>
4. The researcher refers to rectangular areas. By the smallest fence she means the smallest perimeter.

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PARENTS' SUPPORT IN MATHEMATICAL DISCOURSES

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In our empirical research, we are concerned with parents' support in everyday mathematical discourses. The leading questions for the qualitative analysis are the following: What can be identified as the parents' main support focus? And what do the different foci display regarding the underlying mathematical culture? In this paper, we provide an insight into two case studies that refer to game situations. The analyses lead to the distinction between a support focus on structured learning and one on the game itself. Altogether, the current results of this on-going project show how differently children can be integrated in mathematical discourses at home, assuming that these differences influence their mathematical development.

INTRODUCTION

In mathematics education research, the understanding of mathematics as a cultural practice, which cannot be separated from its specific context, is more and more prevalent. Regarding this culturality of mathematics, two complementary views of learning mathematics can be recognised. On the one hand, learning mathematics means that one becomes part of the mathematical culture, which permeates one's social environment (Bishop, 1988). On the other hand, mathematical learning processes are also an intended acquirement of an apparently unchangeable faculty culture with its specific set of terms, structures, and principles (Prediger, 2003). In our opinion, these two descriptions supplement each other and serve as a useful framework for mathematics education research.

From this cultural perspective on learning mathematics, we have to focus on processes beyond those at school as well. Learning mathematics is not limited to school. "In fact, students are from the beginning of their life a member of a community that extensively employs embodiments of mathematical knowledge." (van Oers, 2001, p. 59) As a result, we should not only pay attention to teachers or administrators, but to parents as well (Warren & Young, 2002). In the majority of cases, they are one of the most important figures in a child's life, serving as a mentor, a model, and providing aid for the child. Because of this importance, parents cannot be ignored with regard to mathematics education. Although this point of view is increasingly accepted by the scientific community, a lack of research is noticed over and over: "Studies of the processes by which parents encourage early numerical development in the context of parent-child interactions during routine, culturally relevant activities at home are scarce." (Vandermaas-Peeler, 2009, p. 67) By means of our study, we intend to make a contribution to this issue. We focus on everyday mathematical discourses between parents and preschoolers and try to learn more about the mathematical culture that children encounter at home. Thereby, we are particularly interested in the support structures that parents provide for the young

learners. What can be identified as the parents' main support focus? In other words, what seems to be their central goal by supporting their children?

In the following pages, we shed light on our theoretical framework, present our methods and, finally, discuss some results.

THEORETICAL FRAMEWORK

In studying early years mathematics, we necessarily do so with a certain conception of what learning mathematics is all about. In our opinion, children do not encounter mathematics itself, but a cultural practice that is recognised as mathematical by capable members of the belonging culture (Sfard, 2002). In other words, we regard mathematics itself as a social construction and, consequently, learning mathematics as a social construction, too. This idea of learning is explicitly described in Sfard's theoretical work. She defines learning mathematics as “(...) becoming fluent in a discourse that would be recognized as mathematical by expert interlocutors.” (Sfard, 2002, p. 5) Pursuant to this latter definition, adults are of prime importance for the child's development because they can spur mathematical discourses. According to the interactionistic fundamentals, we assume furthermore that, in such discourses, the interlocutors also negotiate what mathematics itself is all about. Naturally, the results of such negotiation processes can differ a lot, e.g. in regard to the above distinction (Prediger, 2003; Bishop 1988). Thus, mathematics can be an integral part of the cultural practices used naturally in one's social environment or it can occur as a separated set of symbols and rules, which, firstly, is disconnected from one's everyday life.

Home Mathematics

Leder (1992) describes by means of two detailed case studies how differently children can be integrated in mathematical discourses at home and which influence these different discursive frameworks have on the child's mathematical competence. In her analysis, Leder emphasises that it is especially beneficial when parents involve their children in varied mathematical situations, when they pose numerous high cognitive level questions along the way and when they encourage the children to autonomy.

Focussing to a greater extent on interactional aspects, Benigno & Ellis (2008) state that parents support the child's mathematical learning primarily through regulating the level of involvement. Thus, while promoting the idea of the social origins of numeracy, Benigno & Ellis point to the fundamental relevance of interactional aspects in home mathematics.

In a qualitatively laid out study, Street, Baker & Tomlin (2005) deal with just this issue in detail. They point out that children's experiences with interactional patterns are dramatically different. In terms of mathematical discourses at home and at school, the researchers explain that, for some children, there is a gulf between these contexts: “The school replicates the Primary Discourse of middle class homes whilst

it presents children from other backgrounds with a Secondary Discourse.” (Street et al., 2005, p. 7) At this point, we can clearly see the connection between early mathematics, discourse practices and mathematics education. According to the study just cited, many children are restricted in their prospects to succeed in mathematics education because they are confronted with a problem of discourse: The switch between home and school discourses can be a source of difficulty because of different values, rules and patterns.

All these results indicate that the parents' support in mathematical discourses is of prime importance for the children's mathematical development. This is the reason why we focus especially on the support structures. As a supplement to the presented findings, we aim for a very detailed analysis of the object of research. What is the parents' main support focus in mathematical discourses with their child? And what do the different foci display regarding the underlying mathematical culture?

Support

Wood, Bruner & Ross (1976) revealed by a seminal study that the key function of support is to arrange a situation that allows the child to participate as a competent community member. “This scaffolding consists essentially of the adult 'controlling' those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence.” (Wood et. al, 1976, p. 90)

With regard to the issue of language acquisition, Bruner (1983) devised a theory that points out how parents support their child's learning. Thereby, he coins the concept of 'formats', which refers to standardised interaction patterns. Such a format is provided by capable interlocutors with the objective of adjusting the child's utterances more and more to the cultural conventions. At the same time, the format enables the child to participate in discourses and ensures that the child becomes a competent interlocutor by taking increasingly responsibility and becoming more and more independent.

It was Rogoff (1990) who introduced an advancement of Bruner's theory, namely the concept of guided participation. Much more than Bruner, Rogoff pushes the interactional equality of adults and children to the spotlight: “The mutual roles played by children and their caregivers rely both upon the interest of the caregivers in fostering mature roles and skills and on children's own eagerness to participate in adult activities and to push their development.” (Rogoff, 1989, p. 209) In Rogoff's theory of learning, the process of becoming a competent participant in a specific type of discourse is called ‘appropriation’ (Rogoff, 1990). That way, she emphasises the fact that learning takes places within social activities and is something different than a cognitive individual performance. In the process of appropriation, the child “(...) can carry over to future occasions their earlier participation in social activity.” (Rogoff, 1989, p. 213) In other words, in her opinion, learning is a process of transformation of individual participation in cultural activities. Because of this

analogy to interactionistic fundamentals, we regard the concept of guided participation as especially valuable for our theoretical framework. How can we describe the guidance that children are confronted with in mathematical discourses at home?

METHODS

Pursuing this central question, we intend to study everyday mathematical discourses between parents and their preschoolers.

Sample

For this reason, we currently conduct a study with ten German families who were all contacted with the aid of the children's kindergartens. Predominantly, these families belong to the well-educated middle class.

The beginning of the study has been about one year before the children started school. Thus, they were about five years old.

The parents agreed to take part in the project, which means that we visit each of them five times and videotape all the sessions. Four visits are arranged during the child's last year as preschooler, the last one will take place some weeks after the child has started school.

Materials

In order to spur everyday mathematical discourses between the parents and the preschoolers, we arrange situations that are as open as possible. However, to simplify the sessions, we offer materials to the families (cf. Shapiro, Anderson & Anderson, 1997; Vandermaas-Peeler, 2009). These materials serve as impulses for mathematical discourses and should be more or less typical for the familial context. On this account, we chose picture books and games as materials for our study, which are indeed a common part of most middle-class families' everyday life.

Shapiro, Anderson & Anderson (1997) found that picture books are an appropriate context in which to engage children in mathematical activities. Although the researchers actually did not focus on mathematics in their study, they noticed that some of the mothers attended to mathematics during reading sessions with their children.

Vandermaas-Peeler (2009) used both kinds of material, books and games, when studying mathematical discourses. She comes to the conclusion that parents and children initiate more mathematical exchanges during playing.

Altogether, we consider picture books and games as an appropriate impulse for mathematical discourses in families. For this reason, we bring different books and games to the families and let them choose. All materials contain mathematical aspects that could become topics of discourse, but we do not give any further advice for the playing, the reading or the discourse in general. The families may arrange the

sessions as they wish to. According to this, they are also free to use their own books and games. However, we assume that, in general, basic everyday practices and discourse structures of supporting the young learners emerge even in contact with potentially strange material.

Data Analysis

As a first access to our issue, we identify those sequences in the video that obviously display parental support activities. More precisely, we choose those sequences in which the common playing or reading interactions are interrupted in order to solve an emerging problem. For a start, only these sequences are transcribed and subject to an analysis of interaction. This method is based on the conversation analysis (ten Have, 1999) and was, in reference to the interactional theory of learning, devised by the working group around Bauersfeld (Voigt, 1995; Cobb & Bauersfeld, 1995). Focusing on the evolvement of the topic(s), the analysis of interaction reveals how meaning is negotiated in the discourse between the parent and the preschooler.

This analysis of interaction serves as a foundation for a second step. With regard to our central question about the parents' support, we intend to finally concentrate more on the issue of support itself. Bearing Bruner's (1983) theory of formats in mind, we aim at describing the interaction patterns that provide assistance for the child. What are the characteristics of the mathematical guidance that children get familiar with while talking to their parents?

Our study is laid out as a comparative set of case studies. Thereby, we will choose just a few families for a detailed analysis. The decisive criterion when choosing the "focus families" is their variety in terms of our research question and theoretical perspectives. By this means, we can complete our study by a comparative analysis.

CURRENT RESULTS

In the following, we provide an insight into our first two case studies. As described above, we chose two dyads that are absolutely different concerning the parents' support. That way, the variety of parents' support foci comes to the fore and, at the same time, different ways of integrating mathematics into everyday family discourses shine through.

In both examples, the parent and the preschooler are playing a game of dice called "Max Mümmelmann" [1]. Each player's goal is to collect six cards with the numbers from one to six. On each card, the number and a bunny with the accordant set of dots on its coat are pictured. According to the rules of the game, bunnies with numbers from one to four are the four bunny children, the five is the mother and number six is finally the bunny father. Who has got a whole bunny family at first, wins the game.

In the following examples, the dyads are concerned with the issue of already collected and still missing cards. They interrupt the normal playing activities in order to reflect just on this issue. Thereby, the dialogue partners touch on the subject of the

number sequence. Thus, as sorting the cards by numbers, they gain an overview of the current score.

Example 1: Kiara

This first episode is from the first playing session with Mrs. Falkberger and her daughter Kiara (5.1 years old). Kiara is still missing the card with number five. Her other cards are lying in front of her, sorted by numbers [2].

Kiara: (looking at her cards) I'm still missing the six!

Mother: No, what are you still missing?

Kiara: Hum.

Mother: What comes after four?

Kiara: (4 sec.) Wait!

Mother: One, two, three, four?

Kiara: **Five.**

Mother: Exactly.

Looking at the cards that she has already collected, Kiara states that she is only missing one card: "(...) the six!" However, her mother negates this claim and demands a correction from her daughter. In doing so, she shows that she knows the right answer herself and that she is confident that Kiara knows it as well. Maybe, that is the reason why Mrs. Falkberger is not responsive to Kiara's utterance.

When Kiara hesitates there upon, her mother immediately gives her a direction: "What comes after four?" She obviously acts on the assumption that Kiara is able to produce the successor of a number without reciting the whole sequence that leads to it. According to Fuson (1988) and her theory about the acquisition of the number sequence, Mrs. Falkberger's question requires the breakable chain level, where single elements of the sequence can be produced separately. Thus, Mrs. Falkberger only points out where the right answer can be found ("after four"). Thereby, she establishes the number sequence as a suitable tool for the problem at hand and, at the same time, she is excepting more global, survey-like strategies from the discourse. Hence, Mrs. Falkberger does not emphasise that Kiara is missing a card with a certain number to complete her bunny family, but that, now, the successor of number four has to be found.

In answer to this hint, Kiara waits quite a long time before she continues talking. Maybe, she is thinking about the solution of the problem that her mother posed: Which number comes after four? In this case, the mother's assumption that Kiara can count from any point in the number sequence would certainly be wrong. However, it is also possible that Kiara thinks about her own question again: What is the card that she is still missing? In this case, she would at least for now ignore the mother's aid. After all, she asks for some more time to reflect on the question in her mind: "Wait!"

It is a striking fact that Mrs. Falkberger does not respond to Kiara's request at all. Instead, she expands her help. On following the interpretation presented above, one could assume here that Mrs. Falkberger starts doubting Kiara's knowledge about the number sequence. Thus, she counts up to four and invites her daughter just to continue and to give, in this way, the right answer. In fact, by beginning the count at one, Mrs. Falkberger facilitates the task. According to Fuson (1988), her new question requests a lower level of counting competence, namely the unbreakable chain level. Thus, she seems to assume now that Kiara can view each number word as distinct but cannot begin a count at any point other than one.

At that time, Kiara indicates the solution of the problem: "Five." Her stress on this single word may be an expression of self-confidence. In the end, she identified which card she is missing. Thereby, she seems to accept the previous discourse structure by giving an answer that fits perfectly in.

Intermediate Result: Kiara

Mrs. Falkberger accepts the initial impulse set by Kiara, but, then, follows her own idea of support. She is hardly responsive to her daughter. In fact, she integrates Kiara in an interactional pattern that reminds us of traditional mathematics lessons. The expert interlocutor is the one who decides about the appropriate strategy and directs the discourse accordingly. Mrs. Falkberger is the one who poses the questions and Kiara is obliged to answer. This structure is typical for interaction patterns, which Bauersfeld (1978) describes as funnel patterns. Thereby, the 'teaching person' expects a specific answer and constricts the possible course of actions for the learner until the latter can give the requested answer.

Altogether, in this case, the mathematical practice does not appear as an integral part of the playing activities. The support that Mrs. Falkberger provides for her daughter consists in a separate discourse about the number sequence, which is no longer directly related to the concrete game situation. At this point of the discourse, five is only the number after four, but no longer the number of the specific card that Kiara is still missing. For this reason, this short sequence gives an impression of what it means to support a child in acquiring an apparently unchangeable faculty culture with its specific set of terms, structures, and principles (Prediger, 2003).

Example 2: Paco

The second episode is from the first playing session with Mrs. Czipin and her son Paco (5.4 years old). Paco has already collected the cards with numbers three and six, which are lying in front of him.

Paco: (drawing a card with number two) Oh yes! I've got two children now.

Mother: (pointing at Paco's card with number six) And this is the father. Look, you can arrange them in sequence.

Mother: (arranging Paco's cards in sequence: 2, 3, 6)

Mother: Look, two, three. (pointing between the cards with number two and number six) Then, you're still missing four, five, (pointing on the left side of his card with number two) and here still the one.

Mother: (rolls the dice)

Paco draws a card with number two. He seems to be happy ("Oh yes!") and sums up that he has already collected two children, which means that he has two cards with numbers smaller than five. His utterance can be taken as an interim conclusion and as an indication of success.

His mother directly adds that Paco already has the father as well. Thereby, she picks up his language. She does not speak about numbers, but about (bunny) family members. Subsequently, she suggests arranging the cards in sequence and switches, thereby, to numerical language. As she does not explain why it could be useful to sort the cards by numbers, it is possible that she acts on the assumption that Paco knows about the usefulness of this procedure. Alternatively, it is entirely conceivable that Mrs. Czipin does not focus on the issue of understanding or even learning at all. Maybe, she is not interested in ensuring that Paco knows all about the game's details; in fact, it could be her focus to provide him only with the directly relevant information and to enable him, in this way, to participate in the game. Thereby, she integrates a useful mathematical pattern as a matter of course. She does not emphasise the number sequence as a separate issue; in this case, the practice of sorting by numbers appears rather as a common element of the social environment.

This latter interpretation fits well with the fact that Mrs. Czipin, in the following, arranges Paco's cards in sequence herself. She does not use the opportunity to initiate a discourse about the right order of the cards; instead, she supports her son by doing and saying those things that she obviously regards as necessary or at least as helpful.

Finally, Mrs. Czipin completes this sequence by rolling the dice, which means that she goes on with the normal playing activities. At least by now, we can see that her focus is evidently on ensuring that the game can quickly go on and that Paco can participate in it as an equal player.

Intermediate Result: Paco

Mrs. Czipin sets an impulse that fits with Paco's interim conclusion. She raises the issue of arranging the cards in sequence and, then, carries out what she herself has proposed. In this sequence, the ongoing playing activities are only marginally interrupted. In our opinion, the cause for this perception might be Mrs. Czipin's support focus. She primarily assures that the game can smoothly go on. For this reason, she does not initiate any separate discourse about mathematical aspects with questions and answers, and a right solution in the end. In fact, she integrates a mathematical issue as a natural part of the discourse. For this reason, this short sequence gives an impression of what it means to support a child in becoming a part

of the mathematical culture, which permeates one's social environment (Bishop, 1988).

CONCLUSION

What these two examples show first of all, is the striking difference in parents' support foci. So far, these two cases serve us as prototypes of two opposite foci. Mrs. Falkberger's focus seems to be on structured learning (Example 1). Thus, Kiara is integrated in an interactional pattern that provides her with the role of a student, whereas her mother is the competent interlocutor who poses the questions and assesses her daughter's answers. Thereby, Mrs. Falkberger uses mathematics as a tool for a specific problem that arises from the game situation. However, the cultural practice of sorting by numbers is indeed applied, but isolated from the concrete issue of a missing card.

On the contrary, Mrs. Czipin's focus seems to be on the game itself (Example 2). Paco has not to signal demand for support; his mother provides him with useful hints anyhow. But she does not expand such sequences to explicit learning situations. She rather limits her support to the directly game-relevant aspects. In this case, the mathematical practice of sorting by numbers is an integral part of the game situation and is not discussed separately.

What additionally arises from these results is that the category of middle class has to be taken into account with care (cf. Street et al., 2005). As the analyses above display, there seems to be no standard discourse of middle class homes that can be regarded as a mostly uniform context for mathematics learning. Although our families belong without exception to the German middle class, they display a remarkable variety concerning their support systems and ways of integrating mathematics into their everyday discourses. Provided that parents' support in mathematical discourses is of prime importance for the child's mathematical development, we should bear in mind that the belonging to the middle class does not allow any prediction of an individual's mathematics achievement or failure.

NOTES

1. Rüttinger, J. (1996). *Max Mümmelmann*. Ravensbrüg: Ravensburger Spieleverlag.
2. Transcription rules: (1) **Bold text** marks stressed utterances. (2) (Text in parentheses) refers to non-verbal actions. (3) (x sec.) indicates a pause of x seconds.

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THE SEDUCTIVE QUEEN – MATHEMATICS TEXTBOOK PROTAGONIST

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I reflect on the difficulty of bringing human agency to bear in authoring a mathematics textbook based on curriculum outcomes that seem to demand closed text rather than text that presents open choices for dialogue. The language in the text seductively removes you and I from the subject, leaving only mathematics, the queen of the sciences, as the voice of authority.

During the time I was writing chapters for mathematics textbooks to be used in Bhutan, I was also reading *Teaching Positions: Difference, Pedagogy, and the Power of Address*, in which Ellsworth (1997) investigates the way teachers address students. The juxtaposition of my reading and writing made me ask questions about the form of address in mathematics textbooks, questions that spill over into reflections about my role as an educator, particularly in the context of international collaboration. In this paper I reflect on how my writing influenced my reading of Ellsworth's text.

THE CONTEXT - BHUTANESE TEXTBOOK WRITING

The University of New Brunswick (UNB) recently completed three consecutive five-year projects for the Canadian International Development Agency that supported the development of education in Bhutan, a small mountainous kingdom situated between India and China. One of the other international partners that supported the development of education in Bhutan was the World Bank, which provided soft loans for writing mathematics textbooks that correspond to curriculum changes that were supported by UNB's most recent project. I developed courses for mathematics teachers as part of the UNB project, and I co-taught these courses with the Bhutanese educators who would continue to teach them in subsequent years. As a writer for the *Understanding Mathematics* textbooks funded by the World Bank project, I did my writing in Canada and then co-facilitated a writer's workshop in Bhutan, in which Bhutanese mathematics teachers helped us revise drafts of the books.

Before curriculum reform, Bhutanese schools followed Indian curriculum. India is Bhutan's primary neighbour and trading partner, so it is sensible to have the curriculum to set students up for tertiary education in India. Education leaders in Bhutan sought to develop a uniquely Bhutanese curriculum that addressed Bhutanese contexts and aligned with international foci. The new mathematics curriculum closely resembles New Brunswick's curriculum, which explicitly follows principles and standards established by the National Council of Teachers of Mathematics (NCTM).

POSITIONING FOR SEDUCTION

In the dictionary, seduction is related to attraction. Some definitions suggest an element of intent; people are trying to make themselves or their ideas attractive to others. Some definitions include a sense of manipulation; the person being seduced is manipulated to be attracted to someone or something that she or he would not normally be attracted to. I note that intent and manipulation may or may not coexist in a case of seduction. I would add to these definitions that a seducer fulfils the needs, real or imagined, of the seduced. The seduced person is usually taken as passive.

Elizabeth Ellsworth (1997) used a theoretical lens informed by critical film studies to describe the positioning in typical classrooms. The part that most engaged my reflections as a mathematics textbook author related to the idea of seduction in cinematography. She drew my attention to modes of address in film and in classrooms. The operative question is “Who does this film think you are?” (p. 22) or, in my context, “Who does this teacher think you are as a student?”

Ellsworth’s question relates to Umberto Eco’s accounts of the model reader. Eco (1994) described how texts create a “model reader – a sort of ideal type whom the text not only foresees as a collaborator but also tries to create” (p. 9). In this sense, I think a seductive text says, “I know who you are. I am giving you what you want.” The text addresses the needs of a real reader enough to transform him or her into the reader imagined by the text. I say that the text, and not the author, imagines and addresses the reader because a text constructs a model reader regardless of the author’s intent.

When Eco (1979) developed the idea of a model reader, he noted that there are different kinds of model readers. A “closed text” imagines and constructs a single reader. Only one interpretation is recognized. By contrast, in an “open text” “the author offers [...] the addressee a work *to be completed*. [The author] does not know the exact fashion in which his work will be concluded, but he is aware that once completed the work in question will still be his own” (p. 62). The text invites the reader to choose from a variety of interpretations.

Eco’s sense of closed and open texts relates to the orienting distinction made in appraisal linguistics – linguistic resources can be “broadly divided into those which entertain or open up the space for dialogic alternatives and, alternatively, those which suppress or close down the space for such alternation” (White, 2003, p. 259). White connected texts that open dialogue to Bakhtin’s (1975/1981) notion of heteroglossic interaction, and he connected texts that close dialogue to the notion of monoglossic utterances. Appraisal linguistics contributes tools of analysis (which are also used in systemic functional linguistics) to help us understand how the grammar of texts opens or closes dialogue. Along with Beth Herbel-Eisenmann, I have used appraisal linguistics to analyze mathematics classroom oral interaction (e.g., Wagner & Herbel-Eisenmann, 2008).

When Brian Rotman (1988) analyzed semiotics in mathematics, he noted a distinction among imperatives used in mathematics. This distinction relates to open and closed texts. He distinguished between inclusive commands that require a person to do something in interaction with others, and exclusive commands that direct a person to do something in isolation. He called exclusive imperatives (such as *write* or *put*) “scribbler” (p. 10) commands because one is expected simply to follow directions without questioning. He called inclusive imperatives (such as *explain* or *prove*) “thinker” (p. 10) commands because one is expected to engage with others, which requires a degree of responsiveness.

Ellsworth (1979) described how camera work in film has effects similar to texts that close and open dialogue. Like Eco, Ellsworth made clear that there is a difference between a real viewer and the viewer constructed by a film. “Multiple entry” (p. 27) is a necessity in the film industry because commercial viability depends on drawing in a diverse audience. Ellsworth used the example of *Flashdance*, which appeals to adolescent girls and boys for different reasons (p. 27). However, even while appealing to diverse needs director techniques can arouse the viewer’s “empathy for and imaginative collusion with a character’s intentions, experiences, goals” (p. 30) by, for example, filming shots from the principal character’s point of view. Ellsworth described how in alternative cinema directors often break typical forms to avoid seducing audiences to normalize one point of view: “The revolutionary hope was that changing modes of address in films might change the kinds of subject positions that are available and valued in society” (p. 30). “While audiences can’t simply be placed by a mode of address, modes of address do offer seductive encouragements and rewards for assuming those positions within gender, social status, race, nationality, attitude, taste, style, to which a film is addressed” (p. 28). Mathematics texts might seduce readers in similar ways to assume certain positions. In both mathematics and film, the hope is that alternative forms of address may encourage openness to multiple points of view.

With awareness of the possibility for seduction, I am prompted to reflect on how I engage in seduction when I write mathematics textbooks. How am I an agent of seduction? What tools have I employed to seduce? Do I draw mathematics students into a monoglossic world, positioning them as passive? How does this happen even when I do not intend to seduce?

AVOIDING SEDUCTION

Before authoring for the textbooks for Bhutan, I had done critical analysis of texts, which gave me a sense of some things to avoid in my authoring. In addition to building from real contexts that would be meaningful to the readers and writing to direct students to understand key mathematical concepts and procedures in the curriculum, I attended to grammatical features that had figured in my textbook criticism.

In critical analysis of a volume from the *Connected Mathematics Project* and a chapter from a text produced by the University of Chicago's *School Mathematics Project*, Herbel-Eisenmann and I drew attention to personal pronouns (Herbel-Eisenmann & Wagner, 2007). In particular, we noted that these volumes contained few personal pronouns and no first person singular pronouns; there were some instances of *you* and *we*, and no instances of *I* or *me*. The absence of first and second person personal pronouns masks human agency in the mathematics. When the oral discourse of mathematics classrooms includes these pronouns, teachers and students become aware of human agency in mathematics (Herbel-Eisenmann, Wagner, & Cortes, 2008). When the text shows people making choices, the reader sees that she or he too can make choices in mathematics. Thus the text is relatively open or heteroglossic. The absence of *I* and *me* is even more powerful in obscuring human agency than the absence of *you*, *we* and their related pronouns because *you* and *we* can be used in a generalizing sense. Rowland (2000) has described how *you* can mark generalizations in mathematics, not referring to anyone in particular, but to everyone in general. Pimm (1987) has described a similar generalizing sense of *we*, and Herbel-Eisenmann and I have given examples of this in our analyses.

LOCATING SEDUCTION

In the *Understanding Mathematics* series, for which I was authoring as I was reading Ellsworth (1997), I was pleased to find that the lead author had mandated the presence of an *I* voice in most of the lessons. The general structure of a lesson was a quick exploration called Try This, an Exposition, a return to the Try This, some Examples, and a series of questions called Practising and Applying. I will use excerpts from the Grade 7 book (Small et al, 2008) to consider how the structure of the text opened and closed dialogue. All of the excerpts are from chapters I authored.

Each Try This addresses the reader directly with imperatives and questions. These forms are intended to get students to do something, so there is a sense of student agency. But even with action the degree of agency can vary with the space allowed for making decisions. For example, in a lesson called "Area of a Trapezoid" the Try This shows a trapezoid drawn on dot paper. The instructions go like this:

This polygon is drawn on 1 cm dot paper.

A. i) Find its area by dividing it into a rectangle and two triangles.

ii) Find its area by dividing it into two triangles.

iii) Show another way you can divide the polygon into two triangles.

(Small et al., 2008, p. 144)

Using Rotman's (1988) distinction between inclusive, thinker imperatives and exclusive, scribbler imperatives, this Try This begins with scribbler imperatives (*find* and *divide*) and works toward a thinker imperative (*show*), at least nominally. There is only one way of performing part i and there are only two ways of performing part ii. A student in isolation can find the required divisions of shapes. He or she is only

following directions, so the imperative in these parts are scribbler imperatives. For part iii, the student is expected to show another possibility to someone else (perhaps a peer), so it seems to have a thinker imperative. However, because there is only one remaining possibility for dividing the polygon, there is little opportunity for different ways of thinking about the situation. Though the structure of working from independent scribbling to interactive thinking is sound (in my view), it still leads the model reader down a narrow path. Thus it is a closed text.

If the author's voice were recognized in the text (not hidden behind imperatives), it would be more open because the reader would be more likely to realize that someone decided what to instruct him or her to do. For example, if the text said, "I would like you to divide the polygon into two triangles" the reader might wonder why the author chose to give this direction. In oral mathematics classroom interaction, we see a person directing activity in this way all the time (Herbel-Eisenmann, Wagner, & Cortes, 2008). But when the author is masked, it is harder to question the text.

The Exposition is even more closed than the Try This, as it comprises a series of assertions about what trapezoids are and what the formula is. No person is shown to be naming the shape or developing the formula; the shape and formula simply exist. In the work to develop understanding of the trapezoid (still in the Exposition), I tried to draw the student in by using a *you* voice, asking the reader to imagine two congruent trapezoids fit together. With this thought experiment, I told the reader how to manipulate and think about the pair of trapezoids to develop the given formula. These are all scribbler commands. There is no interaction, and I presented only one way of doing things as if it were the only way. It is still a closed text.

For me, the most interesting part of the grammatical structuring of the mathematics comes in the Examples that follow the Exposition. Here the *I* voice appears. I do not think having a closed text up to this point would be so problematic if there were movement toward an open text, because the part of the text that is open would invite multiple points of view and serve to turn the earlier closed text into what White (2003) calls a retrospectively dialogic text; once a reader's attention is drawn to alternative possibility, even a text that is structured to be closed is open for question.

The Examples in this mathematics textbook were structured such that a question is followed by a two-column table showing a Solution in the left-hand column and the related Thinking in the right-hand column. In the top right corner of each Thinking section there is a photograph of a Bhutanese child apparently doing mathematics. There was a bank of six different photos to draw on, three girls and three boys. In each photograph the child is looking down at an open notebook and holding a pencil up to his or her right temple. (The one exception shows a boy holding the pencil in his mouth.) Each child is intended to embody thinking. Some of the Examples have more than one Solution and associated Thinking, each with a different student photograph.

Some of the curriculum outcomes were difficult to exemplify with open text. For example, the outcome “apply the formula for the area of a trapezoid” is by nature a scribbling task, using Rotman’s (1988) distinction. Thus the Thinking for the one Solution to an Example in this lesson does not open up alternative possibilities:

- I knew it was a trapezoid because the arrow marks showed that it had exactly two parallel sides.
- I identified the bases and the height. I noticed that the 6.8 cm side length was extra information that I didn’t need.
- I used the formula.

(Small et al., 2008, p. 145)

All the verbs are exclusive, scribbler verbs because the student can do this in isolation. He or she “knows,” “identifies,” “notices,” and “uses.”

Other outcomes were easier to author as open texts. For example, for determining the area of composite shapes, I gave one of the Examples two Solutions. In one Solution, my fictional student used addition, dividing a polygon into a rectangle and three triangles. In the other Solution, my fictional student used subtraction, drawing a rectangle around the shape and identifying the triangles outside of the polygon. By showing two methods, I meant to suggest that multiple approaches are possible, but a reader could also assume that there are exactly two ways of performing the task. Again, the relevant curriculum outcome is performative; the outcomes are structured as imperatives, many of which are scribbler imperatives. In this case, the outcome reads, “estimate and calculate the area of shapes on grids.”

In general, outcomes that required understanding or generalization were easier to develop with open text. They used thinker imperatives. For example, to address the outcome “determine if certain combinations of [triangle] classifications can exist at the same time” my Example asked, “Is it possible for a right triangle to also be isosceles? How do you know?” (Small et al., 2008, p. 112). The respondent has significant latitude. The Thinking I wrote went like this:

- Before I tried to draw it, I thought about whether it was possible.
- I knew an isosceles triangle had two equal angles and a right triangle had a 90° angle.
- I also knew the sum of the angles of a triangle was 180° .
- So, in a right isosceles triangle, there had to be a 90° angle and two 45° angles.
- I sketched the triangle and it looked possible. My sketch also helped me draw the triangle.

(Small et al., 2008, p. 112)

I had resigned myself to writing mostly closed text for scribbler curriculum outcomes. But then relatively open Examples like this one became interesting to me from the perspective of appraising the way text can seduce a reader.

HOW SEDUCTION WORKS

Ellsworth (1997) pointed out that realist representations can develop and maintain an illusion of difference. She focused on the illusion of dialogue in classrooms, and claimed that the use of dialogic forms of interaction masks an undercurrent of control. With this illusion, understanding is the goal and conscious intention and consensus are valued while desire, conflict, and ambiguity are scorned: “By presenting themselves as desiring only understanding, educational texts address students as if the texts were from no one, with no desire to place their readers in any position except that of neutral, benign, general, generic understanding” (p. 47).

My question is whether something similar is at work in the *Understanding Mathematics* textbook, which is explicitly oriented to develop understanding. Do the dialogic, or open forms of text, such as the multiple *I* voices in the examples (and the thinker imperatives in other parts of the text), work as an illusion? In other words, does the text seduce the reader by suggesting open dialogue while maintaining closed positioning?

Any real reader is different from the model reader projected by a text. Even if the text is closed and seductive, a reader can resist being positioned in the way the text initiates the positioning. This is an important principle of the positioning theory developed by Harré and van Langenhove (1999). However, as an author I am interested in how the text initiates positioning. It was as I was writing the Thinking portion of the Examples that were structured to be the most dialogically open that I was especially conscious of the normalizing force of these sections.

In the Thinking about isosceles right triangles shown above, for example, the model student portrayed in the Thinking listed things he knew (the picture for this one is a boy) and he drew a sketch. His sketch included markings that showed the things he knew. I promoted these actions because I saw them as effective strategies to help students develop an understanding of the parameters and then form a generalization related to the parameters. It seems to me that educators are usually expected to promote effective strategies in this way.

A good way to reflect critically is to think about alternatives. In this case, I could have promoted certain choices explicitly by writing in the Exposition, “It is a good idea to draw diagrams when you think about a mathematics problem. You should list what you know and mark those attributes on your diagram.” This alternative would explicitly draw attention to the choices, whereas the Thinking format normalizes the behaviour without drawing attention to the choice being made.

The normalizing *I* voice is similar to the seductive camera technique Ellsworth (1997) described. Writing from the point of view of a model student is like positioning a camera from the point of view of the protagonist in a film. The seduction comes from the ease with which readers or viewers can see themselves in the place of the model student or protagonist.

Reflecting on my observations about open, closed, and seductive text in this mathematics textbook, I notice that the underlying structuring power of official curriculum. If a textbook is to “follow” the official curriculum, which is the norm for textbook structuring, then it seems inevitable that closed texts will result from performance-based curriculum outcomes. It seems clear to me that imperatives that Rotman (1988) would call scribbler imperatives call for performance of narrowly-defined procedures, and would thus result in closed texts. Thinker imperatives may also be deemed as performance-based outcomes, but they are different. They seem to invite more latitude, and thus give an author of a curriculum-following textbook more latitude in structuring text. Though in my earlier analysis of mathematics textbooks I have decried the lack of an *I* voice, my experience as an author trying to use an *I* voice leads me to recognize that there are further subtle dangers. Using this dialogical form may seduce students who read the text to take up the one point of view I present to them even though I might intend for it to represent the possibility of multiple points of view.

RELATED SEDUCTIONS

My sense is that the closed and sometimes seductive nature of typical text in mathematics textbooks, and in particular the text that I authored, is connected to other seductions. Thus I ask what other apparent needs seem to be met by the process of my authoring, and how the fulfilment of these needs relates to the seduction or narrowness of mathematics as it is presented.

I have a personal need to be relevant. In particular, being relevant in the development of mathematics education in an exotic location feels good. The exotic connections seem to support my reputation as an educator. How does this need connect with the text I produced? For the teachers who mediate the text in mathematics classroom, I lend them expertise and authority so that they too can be relevant. And part of the explicit purpose of education is to equip students to be relevant to society.

With all this valuing of relevance, I have to ask what relevance is. It seems to be closely related to particular values though the word itself seems values-free. In this way, the goal of relevance is seductive. Relevance in general seems like an unquestionable need but any particular act of trying to be relevant would index a particular value set that is masked by the grammar and lexicon of objectivity. For example, me “helping” Bhutan revise its curriculum suggests that the curriculum needs revision, and it suggests that I know what Bhutan needs. So it seems that Bhutan’s culture is being privileged, but at the same time my culture and experiences are privileged even more. The superiority of my culture is reinforced by the idea that the Bhutanese need my kind of help to foreground their own culture.

Related to the exotic connections, I have noticed that I need to experience other cultures – other points of view – to develop my understanding of the people in the world around me. This need is what draws me to travel and work alongside people in

Bhutan and elsewhere. I found it challenging to write text that provides readers with the experience of diverse points of view while writing to follow a curriculum that aims for particular kinds of performance mandated for “all” by the NCTM.

This tension is central to intercultural work and also to the work of an educator. The key questions are: Am I standing alongside the people I work with as we address together the needs in a local context? Or am I standing in front of them, leading them (“helping” them) toward outcomes promoted outside the local context?

POSSIBLE PROTAGONISTS IN MATHEMATICS TEXTBOOKS

The difficult experience of writing mathematics texts for school in a dialogically open way prompts me to consider possible alternatives. It seems that the central problem to structuring an open text is the normalizing power of official curriculum. I see two ways of overcoming this power. One is to ignore it, as done by some innovative texts. Stocker’s (2006) *Maththatmatters* is such an innovative text as it comprises a collection of lessons that centre on social injustices and direct students to do mathematics that helps them understand the injustices in a particular way. As much as I approve of Stocker’s attention to social justice concerns that I share, I recognize that ignoring curriculum would be difficult for a teacher who is required by law to “deliver” curriculum. Stocker’s book would have to be supplemented by other resources.

A second way to overcome the normalizing force of curriculum would be to challenge it in the text. This approach, which resembles writing under erasure as described by Derrida (1976), allows for the possibility of presenting the curriculum while at the same time questioning it. This would require an authentic *I* voice – an author who reveals him- or herself to be reflecting on the things society expects of students. It would require the revelation of values outside mathematics itself as a stance from which to think critically about mathematics. Social justice concerns, such as those raised by Stocker, might provide an appropriate orientation.

This is an approach I am trying to use in this paper (though the content is not mathematics, but rather mathematics education). I am using an *I* voice. I am presenting analysis not in a detached way that implies objectivity, but rather as self-critical reflection. In this way, the text is open to dispute and different interpretations. I am vulnerable. I raise sore points, which may undermine my authority because of the of moral complexities of authoring pedagogical text, but may also substantiate my authority by positioning myself as a self-aware author.

If mathematical texts are closed and seduce students to accept unquestioningly a single point of view, the student is passive and cannot be an active subject or protagonist. And if the text hides its author, the author is not a protagonist either. I argue that only one protagonist remains: mathematics, which Gauss named the queen of the sciences. This is the queen who reigns and sits in judgment.

I prefer to think of the queen as the chess queen, which is the most powerful tool in chess a player's repertoire. The player uses the queen (mathematics) to exercise his or her intentions. The player is not subject to the queen, but rather the queen is subject to the player. This is the way I would like to see mathematics portrayed in mathematics texts. This kind of portrayal requires a radically different stance on curriculum.

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SOCIOMATHEMATICS: A SUBJECT FIELD AND A RESEARCH FIELD

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Complexity is a characteristic of problem fields related to mathematics education and, in any study; the researcher has to focus one of the problems without ignoring the others. Diversity (gender, ethnicity, social class etc.) in the subject area calls for multi- and inter-disciplinary studies and for different research methodologies. The concept of sociomathematics is developed and suggested as a name for a subject field where people, mathematics and society are combined, and for the research field where the societal context of knowing, learning and teaching mathematics is taken seriously into account. This is done through a critical dialogue with ethnomathematics.

COMPLEXITY IN MATHEMATICS EDUCATION

Le sens du problème est le moteur du progrès scientifique (Bachelard, 1927).

Research is always a response to a problem. The epistemological assumption that the sense of the problem is the motor of scientific progress is a frame of reference for this paper. Thus, understanding, selection, interpretation and formulation of problems and research questions are seen as vital activities in the researchers' practices. Complexity is a characteristic of mathematics education research and none of the problems in or related to teaching, learning and knowing mathematics can properly be isolated from the others. In any educational study, the researcher has to focus on one of the problems without ignoring the others. Diversity (gender, ethnicity, social class etc.) in the subject area calls for multi- and inter-disciplinary studies and for different research methodologies. However, the focus and the methodology of any study are determined by its purpose, theory and research questions. For example Evans and Tsatsaroni (2008) have argued that research into gender within a social justice agenda requires both quantitative and qualitative methods. When the problem is formulated as a research question and the method and the sampling strategy are to be decided, the researcher has to choose among a series of factors and dimensions to reduce complexity. The societal context is one of the aspects to decide upon. In some studies, *society* is a dimension *in the foreground*: the study is designed to investigate society and mathematics education – meaning that the societal context is addressed in the research question and suitable theories and methods are chosen. In other studies, *society* is *in the background*: societal factors are just independent variables among others. *No society* in the study means that information about the societal context is not available in the data.

In a study of students' motives (motivation, reasons, rationale) for learning or not learning mathematics, society can be in the background or ignored (Hannula, 2004;

Wæge, 2007) or in the foreground (Alrø et al., 2009; Mellin-Olsen, 1987; Wedege & Evans, 2006). The notion of *landscapes of learning* is introduced by Alrø, Skovsmose and Valero (2009) as a tool to capture and structure some of the complexity and to guide their study – with society in the foreground – of students’ motives for learning mathematics in the multicultural mathematics classroom. Landscapes of learning is a notion with a double meaning which brings together a research perspective and a research field:

First, it represents an interpretation of (mathematics) education as a complex network of social practice that is constituted by different interrelated dimensions. Second, it makes possible to identify specific – but interdependent – dimensions of an empirical field to do research (Alrø et al., 2009, p. 330).

Based on recent research, the authors have selected nine dimensions as relevant aspects to be considered for a better understanding of the social complexity of classrooms. Among these dimensions are for example “students’ foregrounds as an experienced socio-political reality”, “teachers’ perspectives, opinions and priorities of teaching”, “the (mathematical) content for learning”, and “public discourses about immigrants, schooling and multiculturalism”.

Complexity is highlighted through the notion of landscapes of learning with students’ motives for learning mathematics as an example of a subject field for mathematics education research. The problem field of the mathematics classroom is opened to the societal context and simultaneously restricted by the lens of the landscapes of learning, which reflects the problematique of Critical Mathematics Education. Some research questions are legitimized and possible to formulate from this perspective, other questions are not (Wedege, 2006). In this paper, I develop a conceptual framework around the concept of *sociomathematics*, which has a double meaning like the notion of learning landscapes. First, sociomathematics is a field to be researched (a subject field) where problems are formulated bringing people, mathematics and society together. Second, sociomathematics is a research field where the societal context of mathematics education is seriously into account, e.g. scientific studies with society in the foreground.

THE TERM “SOCIOMATHEMATICS”

I found my inspiration for the term “sociomathematics” in *sociolinguistics* i.e. relationships between language and society constituted as a scientific field within linguistics. However, sociomathematics is a field within mathematics education research where people’s relationships with mathematics in society are studied, not a sub-discipline of mathematics.

Previously, the substantive “sociomathematics” has been used by Zaslavsky (1973) in a meaning similar to “ethnomathematics”. She explains *sociomathematics* of Africa as “the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon their evolution of their

mathematics” (p. 7). In mathematics education research, the adjective “sociomathematical” is used at the level of the social context of the classroom. Cobb and his colleagues developed the concept *sociomathematical norms* in an interpretive framework for analyzing mathematical activity at classroom level from a social perspective (classroom social norms, sociomathematical norms and classroom mathematical practices) and from a psychological perspective (beliefs about roles and mathematical activity in school, mathematical beliefs and values, and mathematical conceptions and activity) (Cobb & Yackel, 1996). In this framework, the social category of sociomathematical norms is combined with the psychological category of mathematics beliefs and values. In my terminology, studies of sociomathematical norms in a classroom would be regarded as sociomathematics only if the students’ relationships with mathematics *in society* are explicitly on the agenda; for example related to the students’ gender, ethnicity or class.

FROM ETHNOMATHEMATICS TO SOCIOMATHEMATICS

From my research on adult mathematics education, I realised that a concept of the kind developed in this paper was needed. The concept of ethnomathematics has been a very important notion in my studies of workers’ mathematics in the workplace (Wedegé, 2000). In both meanings of the term as defined by Gerdes (1996): ethnomathematics as a subject field (a field studied in mathematics education research) in contrast to “school mathematics” and as a research field, ethnomathematics reflects an acceptance and a consciousness of the existence of many forms of mathematics, each particular in its own way to a certain (sub)culture. Ethnomathematicians argue that the techniques and truths of mathematics are a cultural product and stress that all people – every culture and subculture – develop their own particular forms of mathematics. D’Ambrosio (1985) contrasted *academic mathematics* (the mathematics taught and learned in schools) with *ethnomathematics*, which he describes as the mathematics “which is practised among identifiable cultural groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes, and so on” (p. 45). Ethnomathematicians emphasise and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics (Gerdes, 1996).

I have argued that ethnomathematics paved the way for researching workers’ mathematics in the workplace. However, in the Danish vocational context, we have never used the word “ethnomathematics” instead we talk about “workplace mathematics” or “everyday mathematics”. In many languages and situations, the prefix “ethno” has connotations with reference to biological characteristics, colour of skin etc. At the Second International Congress on Ethnomathematics, 2002, Skovsmose referred to his strong reservations about the use of the word “ethnomathematics”. However, his reservation is not to do with the meaning of “ethno” in the literature of ethnomathematics where, according to D’Ambrosio, it simply refers to “environment”, e.g. culture and society: mathematics is acted out in

many different ways in different cultures and by different groups. What is emphasised in ethnomathematics are the connections between culture and mathematics: Mathematics is always culturally embedded. Thus “engineering mathematics” and “mathematics in semi-skilled job functions” also represent different branches of ethnomathematics (Skovsmose, 2002).

I found that “sociomathematics” could be an answer to this terminological problem. However, sociomathematics is not just a translation of the word ethnomathematics into a “cleaner” word. It is also a notion that makes explicit the power relations in mathematics education. On the basis of previous studies of people and mathematics in society – others’ and my own – I have given a preliminary definition of sociomathematics as an analytical concept addressing relationships between people, mathematics and society, which encompasses the studies of for example numeracy, ethnomathematics and workplace mathematics in a single term (Wedeg, 2003). In his discussion of socio-political functions of mathematics education, Skovsmose (2006) has found that the conceptual suggestion of sociomathematics “might help to establish further relationships between the ethnomathematical programme, and those very many studies which share a number of the same concerns, but which might find it awkward to operate with the notion of ethnomathematics” (p. 275).

THE CONCEPT OF SOCIOMATHEMATICS

By *sociomathematics* I mean

- a subject field combining mathematics, people and society,
- a research field where problems concerning the relationships between people, mathematics and society are identified, formulated and studied.

As a subject field, sociomathematics is defined by a specific perspective on the subject area of people, mathematics and society – as it may be found for example in notions of ethnomathematics, folk mathematics, mathematical literacy, adult numeracy and mathematics-containing qualifications (see figure 1). As a research field sociomathematics reflects an acceptance and a consciousness of the influence of the societal context in the knowing, learning and teaching of mathematics, i.e. society is in the foreground when research is designed. *Sociomathematical problems* concern:

(1) people’s relationships with mathematics (education) *in* society and vice versa.

People’s relationship might be seen as cognitive, affective or social according to the given perspective of a specific study. A problem to be studied could be: What does it mean to know mathematics in society? The issue is the relationship between people and mathematics in society but to investigate problems in this field one has to involve two other problem complexes:

(2) functions of mathematics (education) in society and vice versa, and

(3) people learning, knowing and teaching *in* society.

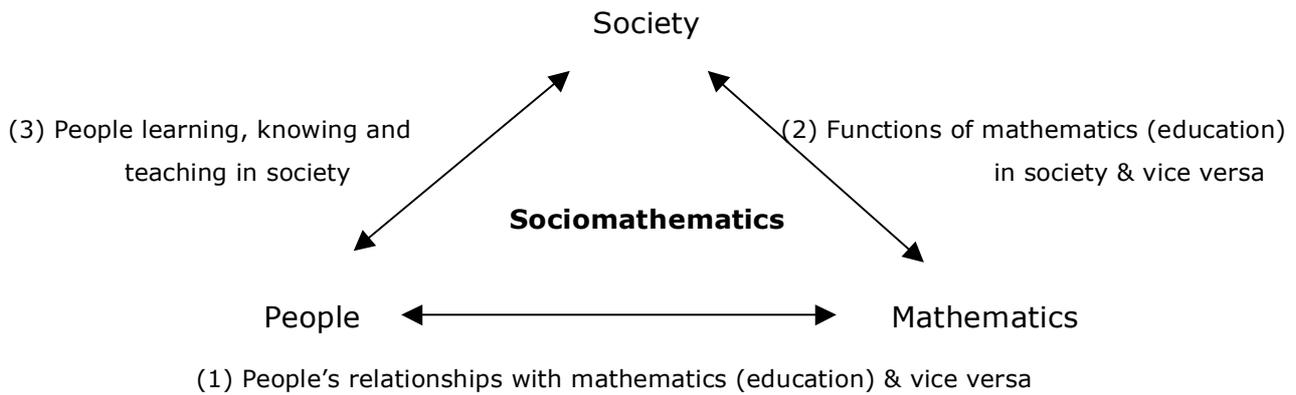


Figure 1. Sociomathematics as a subject field (Wedegge, 2003, p. 2).

In Skovmose's studies of students' learning obstacles in mathematics, one finds an example of a *sociomathematical concept* construction. He does not find the cultural background of the students sufficient to account for the situation but also involves their *foreground*, i.e. the opportunities provided by the social, political and cultural situation: "When a society has stolen away the future of some group of children, then it has also stolen the incitements of learning" (Skovmose, 2005, p. 6). The issue of *mathematisation* presented by Jablonka and Gellert (2007) as describing and analysing the social, economical and political processes in which relationships between participants in society become increasingly formal, is an example of a *sociomathematical problem*. An example of a *sociomathematical study* is for example found in my inquiry of adults learning mathematics (Wedegge, 1999). I go beyond the local situation given by Lave's socio-psychological concept of community of practice and involve Bourdieu's sociological concept about *habitus* meaning a system of dispositions which allow the individual to act, think and orient him or herself in the social world:

(...) the habitus of a girl born in 1922 in a provincial town as a saddler's daughter, of a pupil in a school where arithmetic and mathematics were two different subjects at a time where it was "OK for a girl not to know mathematics", and the habitus of a wife and mother staying home with her two daughters is a basis of actions (and non-actions) and perceptions. Habitus undergoes transformations but durability is the main characteristics (Wedegge, 1999, p. 222)

People's habitus is incorporated in the life they have lived up to the present and consists of systems of durable, transposable dispositions as principles of generating and structuring practices and representations (Bourdieu, 1980).

According to the definition, the mathematics shared by a cultural subgroup of only two persons could be regarded as "ethnomathematics". I would not call a phenomenon like this "sociomathematics". However, the critical approach to ethnomathematics defined by Gelsa Knijnik (1998) is a clear example of a perspective that could

be termed “sociomathematical”. Her study of landless paysans’ mathematics is not just about people’s competences in a well-defined cultural context but about a larger political context, where power relations are made visible. In a sociomathematical study, societal rights and demands (from the labour market, educational system, democracy) are made explicit as well as consequences for people belonging to different social classes. Gellert (2008) has shown what difference a sociomathematical perspective can lead to in interpretations and understanding of data from a mathematics classroom. He has chosen a short transcript of sixthgraders collaborative problem solving. From a structuralist perspective, classroom practices are regarded as social representations that are more or less accessible to students, depending on their social backgrounds, and this is made visible in his analysis.

TO KNOW MATHEMATICS IN SOCIETY

Today it is scientifically legitimate to ask questions concerning people’s everyday mathematics and about the power relations involved in mathematics education. In other words, it is legitimate to ask “What does it mean to know mathematics in society?” In all three dimensions of the triangle (figure 1), power is a central socio-mathematical issue. In his book “The Politics of Mathematics Education”, Mellin-Olsen (1987) stated that it is a political question whether folk mathematics is recognized as mathematics or not. He presents the book as a result of a twenty year long search “to find out why so many intelligent pupils do not learn mathematics whereas, at the same time, it is easy to discover mathematics in their out-of-school activities” (p. xiii). FitzSimons (2002) states that the distribution of knowledge in society defines the distribution of power and, in this context, people’s everyday competences do not count as mathematics. In policy documents in educational systems, in teachers’ practices, and in research in the teaching and learning of mathematics, the power of mathematics and mathematics education is clearly assumed. However, it is not clear what is really meant by the terms “power” and “mathematics”, particularly when it is being used differently by the multiple actors involved in giving meaning to the practices of the teaching and learning of mathematics in society (Valero & Wedege, 2009).

Definitions of mathematical literacy and related studies are concerned with the relationships between people, mathematics and society, and any construction of a concept of mathematical literacy appears as an answer to the question “What does it mean to know mathematics in society?” Thus, mathematical literacy – for example in PISA – ought to be a sociomathematical concept. The latest definition from the theoretical framework of the survey sounds as follows:

Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72)

According to the framework, the PISA mathematical literacy “deals with the extent to which 15-year-old students can be regarded as informed, reflective citizens and intelligent consumers” (Ibid). According to this definition, the approach of PISA, which pretends to assess mathematical literacy of students near the end of compulsory education, should start with the needs of the individuals in society. However, the concrete construction of the eight mathematical competencies composing mathematical literacy (thinking and reasoning; argumentation; communication; etc.) is general starting with mathematics and ending up with mathematics. In the test items the so-called real world situations are only a means for re-contextualising mathematical concepts and in the end “it is not the situations themselves which are of interest, but only their mathematical descriptions” (Jablonka, 2003, p. 81). Thus, I claim that in spite of the declared purpose of PISA this survey is not sociomathematical. However, one may find that society is in the background with social class as an independent variable.

PERSPECTIVES

The problem behind the development of the conceptual framework of sociomathematics is located in term and content of ethnomathematics. In this paper, sociomathematics is defined with a double meaning: As a subject field where people, mathematics and society are combined, and as a research field where people’s cognitive, affective and social relationship with mathematics in society is investigated. As mentioned above, Gerdes (1996) has also identified ethnomathematics as a subject field and a research field. In addition, he argues that the ethnomathematical paradigm includes principles for educational practice. For example, ethnomathematicians look for cultural elements and activities that may serve as a starting point for doing and elaborating mathematics in the classroom. Within science education research, a notion of *socio-scientific issue* has been developed and Ekborg, Ideland & Malmberg (2009) have shown that a way to increase Swedish students’ interest in school science is to bring in a humanistic perspective, i.e. issues with a basis in science which are important for society and are dealing with moral and ethics. In Sweden, a similar experiment has been made in bringing up sociomathematical issues in compulsory mathematics for social science students in upper secondary school (Course A). This course is on the borderline to more advanced mathematics studies and it covers elementary mathematical content such as arithmetic, geometry, algebra, statistics and basic-level functions. Normally, the problem is that the mathematical content is not connected to other subjects in the social sciences programme and the students find the mathematical activities meaningless. Integrating mathematics with social science offered the possibility for students to exercise a degree of personal agency and many students expressed their experiences of meaning in this experimental course (Andersson & Valero, In press).

Recently Yasukawa (2007) argued that the UN declared decade of education for sustainable development (2005-2014) presents an opportune moment for mathe-

mathematics education educators and researchers to “reflect about the effectiveness that mathematics education has had in creating citizens for a sustainable future” (p. 7). Introducing the term “socio-mathematical issues” in mathematics education corresponding to “socio-scientific issues” in science education could be a possible answer to this challenge. At the same time, the conceptual framework of sociomathematics could be extended to include principles for bringing socio-mathematical issues into educational practice.

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EXPERIENCING A CHANGE TO ABILITY GROUPING IN MATHEMATICS

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This paper reports on the experiences of children, teachers and parents at one school of a change in the way that children were grouped for their lessons. Mathematics was central in the decision to make this change. The change itself, and the discourse that produced the perception of a need for it, is visible in the way people talk about themselves, each other, and the business of education.

INTRODUCTION

This paper draws on research in a school in which parents are very much involved in all aspects of strategy and policy. When the school was established, mixed ability grouping was central to its organisation. Just over a year ago the school moved to a system where Year 7 became streamed: children now worked in groups that were seen as fairly homogeneous in terms of ability. The grouping system was described as follows: one 'Accelerated' group, two 'Accelerating' groups, four 'Standard' groups, and one 'Nurture' group. The main criterion for grouping students was average level of attainment across subjects of the National Curriculum¹.

The choice to move from mixed ability to streaming was taken by senior managers at the school who stressed it had been constrained by real and practical considerations over which the school had no control. This decision was based on their observation and monitoring, consultation with teachers, and perception of the views of parents. Parents had made no direct request for this change in grouping, or to opt for the particular choice of streaming, but, this school, like most schools in England, is engaged in a struggle within a discourse of performativity (Ball 2008), and there was a clear perception of pressure - from parents, from Ofsted², from government

The powerful discourses at work here serve not only to frame discussion, but also to shape the identities of the people involved. The naming of these ability groups is an indicator of one of these, suggesting a language for describing education framed in terms of pace and hierarchy, rules that Bernstein (2004) argues apply in any pedagogic context. Indeed, in their discussion with me, most parents, teachers and children either could not, or did not speak of each other without evoking some image of speeding through the business of schooling.

ETHICS, LIMITATIONS AND FOCUS

I should be clear about what is the focus of this paper and what is not. The focus is not the school or what might be seen as its particular and distinctive features, or the process of choosing to move from mixed ability to streaming for Y7, which, from here on, I will call the 'choice'.

Whatever the circumstances of the school, here were a set of people experiencing changes in their lives that have resulted from the 'choice'. In this paper I focus on some of the Y7 children and what impact the 'choice' appeared to be having on their lives: how they experience school, how they see themselves as learners (and learners of mathematics), how they are seen by others. I seek to contextualise their experiences through the parallel, though less detailed, exploration of the experiences of some teachers and parents. The 'choice' is present, in some sense, as a constraint that has shaped both people's experiences (of school, of education, of themselves in these contexts), and as a lens that has shaped my perspective and that of interviewees as we have probed these experiences.

Ethical considerations constrain my selection of data from the mass of interview transcripts. This leads me, inevitably, to selection and editorialising, but then such selection and editorialising is ever present in any kind of study, and in particular in any socio-cultural study: the fact of my 'editorialising' needs very much to be kept in mind as this paper is read. I talked about my editorialising with all participants as we worked together to make our texts: I would have to make choices, in some cases because of my own agenda, in others because of my perception of the agreed ethical constraints upon the research activity. I explore 'editorialising' further in the section on methodology.

THEORETICAL PERSPECTIVE

Mathematics has figured prominently in much of the research literature on ability grouping. Of the departments at the school, the mathematics department found mixed ability most problematic, and, as might be expected, mathematics figured prominently in the accounts of those involved. Here I use these accounts to map the part that mathematics (or conceptions of mathematics and its importance) in particular and wider experiences of schooling, mediated by a range of artefacts, has played both in experiences of the 'choice', and, more generally, in the lives of some of the parents, teachers and children involved.

Here I am using notions of identity worked out in recent writing, but also influenced by Gee (2000).

'Being recognized as a certain "kind of person," in a given context, is what [Gee] mean[s] ... by "identity." In this sense of the term, all people have multiple identities connected not to their "internal states" but to their performances in society.' (p. 99)

I continue to make use of a view of 'identity' as the aggregation of the smaller 'becomings' (or identities) identified with a learner's participation in a multiplicity of communities of practice, local and not so local, some of which are locatable within school classrooms and most not (Winbourne and Watson, 1998; Lave & Wenger, 1991). Holland et al (2001) point out that 'Identities become important outcomes of participation in communities of practice in ways analogous to our notion that identities are formed in the process of participating in activities organized by figured

worlds.’ (Holland, Skinner, Lachicotte, and Cain 2001, p. 57), and I make use of figured worlds, too, where I think this is helpful.

Here I set out to provide a sense of the communities of practice, seen as a problematic notion, and the broader institution that both contextualise the activity of children, their parents and their teachers, and constitute and are constituted by aspects of their developing identities.

EXPERIENCES OF ABILITY GROUPING

This section serves partly as a brief, selective review of some of the literature on ability grouping, but mainly as the start of the presentation of the study. I talked with two parents³, three teachers⁴, one deputy head teacher, the head teacher, and five children⁵. Some of these people wanted to talk about evidence for and against ability grouping in their conversations with me, in some cases, quite reasonably, wanting to know where I stood.

Extract from interview with Martine, whose son is in an ‘accelerated’ group:

Martine [not her real name]: What do you think, as an adult person as to ability groups?

Peter: I think that these things are a product of particular ways of thinking about schools and teaching and learning; they’re kind of an inevitable product of ways that we understand schools. ...There’s quite a lot of work that’s been done looking at the effects of various types of grouping and there’s not a lot of evidence in favour of ability grouping really. There’s some; there’s a lot of research done, but the evidence is not clear-cut.⁶

Extract from interview with Clare whose daughter is in an ‘accelerating’ group:

Clare [not her real name]: last week I actually went to see the head about a number of things, which included talking about how the children were being taught. The night before I went, I talked with [my daughter], it was kind of the first time I had talked to [her] about it really, and she was actually saying that she actually preferred it the way it was before. She said that her and two other girls are like the top of the class, which is quite unusual, because [my daughter] and these two girls aren’t actually that bright, really. What she seemed to be saying was, in essence, that it wasn’t very motivating because there wasn’t any kids above them. And she said when they were in the mixed ability groups – this is obviously a 12 year-old – but, when they were in the mixed ability groups she felt more inclined to help the ones that were struggling and she doesn’t now.]

Peter (continues): My own view is that it’s very much a product of particular ways of looking at school. If we have to look at school, in terms, perhaps quite rightly, of results, then this may seem like what you have to do. But

there's also evidence ... that if you look at school in other ways⁷, like in terms of democratic participation, then the results also come...

Clare had used the internet to read up on some of the evidence herself. She had used the DCSF site and searched under streaming.

Clare: There was a research study on there, but there wasn't much there ... and that study said what you've said, that streaming wasn't necessarily indicative of good results, and it was all a bit inconclusive actually.

I am not sure what research Clare had looked at. However, materials prepared for Ruth Kelly, when she was Secretary of State for Education, and which were used as background for a speech she gave in 2006, suggest the extent to which grouping by 'ability' is central to official discourse, in spite of the lack – well known to her department - of supporting research evidence (DCSF, 2006).

METHODOLOGY

In this study I wanted to see how what could be thought and said about schools is framed and constrained by current educational discourse. I wanted to get some insights into some of the possibilities for being and becoming, glimpses of the formation of discourse identities (Gee 2000), that might be afforded by talking with people about the changes in practice that have attended the 'choice'.

I felt that some insights of this kind might be possible through inviting interviewees - teachers and parents mainly, but children to the extent that this might be possible - to entertain the possibility of the world of education being other than it is; in particular, how might this world look were we to believe that, rather than re-group children into 'ability' groups, we should establish groups that were as diverse and as heterogeneous as possible.

Processes of text negotiation and editorialising were central to my way of working and made explicit to interviewees from the start. I used a process of text negotiation, similar to those I have used with colleagues in recent research (Wilson, Winbourne, and Tomlin 2008), and about which I have written more explicitly elsewhere (Winbourne, 2007). My aim was to enable participants to feel that their voices would be heard legitimately in this text and to secure greater validity. All the teachers and children I spoke to said that they trusted me to choose which of their texts to include.

The following exchange, during the second interview with the five children, exemplifies the text negotiation process:

Peter: You see, ...when I was talking to Eugene, Chris and Aaliyah, I asked this question, 'Have people in this school had problems?' and Chris said, 'Yes, people in the lower group'⁸. If, in what I write, I just put that, what are people going to think, if that's the only bit I wrote about ...?

Eugene: That it's a bit boring...

- Chris: That I'm in it. That I am in the lower group, cos I'm not.
- Eugene: But that's why we changed our names, cos they don't know that you're Chris.
- Peter: That's entirely my point: if that's what I write, Chris said that they might think that he was in the lower group...
- Chris: I didn't just say that, I also said that sometimes people are kind of bad, yeah, but when they're in their class they're proper smart...their friends might think they're neeky cos they're smart in class.

In the example I have just given, as the children discuss the text, they speak of each other in terms of how others would see and speak about them; they reveal some aspects of their sense of who they are: within some of the practices that intersect at the school and the figured worlds which are the spaces for their development of self. Below I present more texts that I have assembled through my coding⁹ process, aiming to represent aspects of children's experiences of the 'choice', looking for further signs of how they see themselves, how their identities are produced in practice and in discourse. I use extracts from interviews with teachers and parents to provide a sense of the broader institution and community. In my selection I have been guided by the need to show the position that mathematics might have in all of this.

TEXTS AND CONTEXTS

Being standard, accelerating and accelerated

Aaliyah's world is one where being seen as high achieving is important; Eugene makes clear it is important for him too:

- Aaliyah [accelerating group]: I am not the highest, I am the one underneath it, but I am at the top of my... I am going to move up into the highest one here...
- Eugene [standard group]: Yeah, I want to move up into another class...
- Aaliyah: ...I am going into the highest, yeah, all the teachers are telling me that I am on my way.

The 'choice' may not have changed Aaliyah's ambition or view of herself, but it has afforded her new ways of talking about herself, clearly framed in terms of hierarchy and with a sense of movement and pace. Her omission of 'in' ('I am not the highest') may be due to her way of speaking, but it is striking in this context. Kwame makes similar omissions and very clear reference to pace and speed:

- Peter: when there was that broader mix in class, were people expected to help each other?
- Hera [accelerating group]: Yeah.

Kwame [accelerating group]: Yes. But sometimes it slowed down the learning. If, let's say in maths there's something called accelerating, accelerated and standard, yeah, and if some people are standard, accelerating and accelerated, and the teacher would have to explain all of them, so we wouldn't get that much work, but, if, now, you're in the highest group for maths, accelerated, they don't have to go through all the accelerating, accelerated and standard a same thing (for all of these groups). It makes more time to learn...

Kwame used the names of the groups as identifiers for himself and other children; he seemed to think (and he was right) that these names were used only in mathematics.

Being Gifted and Talented

Probing to see if children in the school were generally happier now - after the 'choice' – elicited responses like this:

Chris [standard group]: We would always have jokes with teachers. Now we're an ordinary class.

Aaliyah: My class is totally different. More people.... I feel proud of myself because I am in the second to highest group. We all kind of know that we are gifted and talented and can produce excellent work.

Being identified as 'gifted and talented' is important to Aaliyah, but the new grouping highlights some confusion. Not long before, a trip to Sussex had been organised for children identified as gifted and talented:

Eugene: I would like to go to Sussex..

Peter: Do you know who the children are who are Gifted and Talented and on that trip..?

Eugene: I know most of the people who are going to Sussex.

Peter: Which groups are they now in [after the 'choice']?

Eugene: Some of them's in my class...

Aaliyah: ..most of them are in the top set and my set

Eugene: ...some of them, two are in my class, I know, yeah, cos [boy] was going to Wales, but he didn't want to go Wales, so [other boy] is going in my class.

Peter: So, he's in your class [, Eugene,] and he's gifted and talented; how does that work?

Aaliyah: I don't understand that bit. There must be gifted and talented in all different sets; but obviously he's a gifted and talented for his level; but maybe he's a gifted and talented for his level, but he can't face the gifted and talented at my level, I don't know, I don't know.

Kwame's Friend: the all-rounder

I didn't meet Kwame's friend, Darrell [name changed by me], but he 'turned up' first in conversation with the children and was subsequently 'identified' for me by Brett, the head of mathematics. He appeared, I think, both as Kwame's true friend and as a character in the children's figured world; his combination of respect on the street and recognised academic 'ability' – enhanced as a result of the 'choice' - captures, I think, some of the essence of that world.

In the first interview, Kwame said:

I don't miss all the people in my old class...just that people do miss their old friends from their old class. If my friend (Darrell).. if my friend was still in my class, yeah, ...

And later, in the same interview:

Kwame: Sometimes, yeah, in my (maths) group, yeah, there was five people went to a different classroom, like to do higher stuff sometimes, to do extra maths. Four of them..are in my class...but, my friend, Darrell, I think he got like, I don't know, I think he got 4-4-5¹⁰ or 5-4-5 , as well, then he improved, but I don't know why he's not in my class.

Darrell 'turned up' again on the morning of the children's second interview:

Chris: Some people didn't expect some people to be in the like the group that they're in, like the higher group that they're in...

Kwame: Like some person, loads of people thought he wasn't that smart, cos how he acts outside of classes, ..people think that he's not all that smart...

Aaliyah: I think we're all talking about the same person..

Peter: How, why are they changing their view of him now..?

Chris: Cos people see what class he's in..

Aaliyah: He can get even more respect now...that's showing that he's like an all-rounder..

Later that morning, Brett was reading some of my notes from our first interview. I had drawn his attention to what he had said about the 'few children (seven) for whom the change had really not worked and mixed ability would be better, but most are happier and feel like they're learning which is important. Those seven were happy and were working in a pocket of children including brighter children and were being brought along quite well.'

Brett (reading on from my notes): 'One was the brightest, and is now in a competitive environment in his new group. It turns out to be Kwame's best friend, who is now in second group (and has said he missed. – see K's text.)'

Brett also reads the comment I have added where I draw attention to what Kwame had said about Darrell and asks, ‘What’s that?’

Peter: Oh, about, cos one of the things that Kwame said was that one of the brightest boys, in fact it’s Darrell, is in this different group..

Brett: Initially he was going to be in one of the standard groups – this is why I feel like we’ve got the groups really right actually – Initially he was going to be in one of the standard groups and he’s got the highest CATs and I thought he was going to die in there. I used quite strong language, like that, because I think it is like killing a kid.

WHAT ELSE COULD SCHOOL LOOK LIKE?

The world of the school is, indeed, such that the ‘choice’ may better be seen as an inevitable product of the powerful discourses to which I referred above. These discourses extend to the children’s homes where they also powerfully shape identities (Hughes and Greenhough, 2008). In some sense children ‘bring’ these identities with them to school (Winbourne, 2008). Discussion with Martine provided some insight into the constraints upon such shaping of identity.

Peter: To think out of the box a bit, imagine, perhaps...what might be different about [school] if, rather than thinking, oh yes, ...we must put people of the same ability together, we were to think the opposite: which is to say we must have a range of abilities in each group?

Martine: Well, what level do you teach at?

Peter: I was just thinking what the school would look like if that’s how you felt you needed to respond to it, do you know what I mean? ...what would school be about?

Martine: Oh, I don’t know.... If children have the ability, it makes no difference...

Peter: Yes, I suppose, what I was saying is, if it were the reverse, you know, somehow because of the way school would be in this different world, as it were, we would feel, yes, we have to mix the groups up..

Martine: I think yes perhaps....What, are you talking in ability?

In our conversation, I found that Martine and I could not talk of school as other than structured through ‘ability’. Brett and I were able to do so. In response to similar questioning, Brett offered a vision of school:

...that dealt with first, maybe, learning how to learn - with actual learning that was measurable secondary to that, still assessable, but later on. Not speeding through stages of development, setting up structures. Involving children in how to learn.

This is a vision strongly supported by Boaler’s research (2008 and forthcoming). It is also a vision which Brett shares with his colleagues and which, in their

conversations with me, is seen as out of reach. Were it to have been realised in these children's experiences they might have spoken very differently about themselves and appeared differently in this paper.

NOTES

- 1 'Accelerated' group – average level 5b; 'Accelerating' groups – average Level 4b; 'Standard' groups – average level 3b; 'Nurture' group - N's or B's in English.
- 2 Office for Standards in Education
- 3 I spoke with parents by phone; I did not interview their children. Interviews with staff were one-to-one. For the first interviews with the children I spoke with Aaliyah, Chris and Eugene together, and then with Hera and Kwame together; for the second I spoke with the five children together.
- 4 Brett, Head of Mathematics; Juliet, Head of English; Sophie, teacher of PE and mathematics.
- 5 The children and the teachers decided the names by which they would be known in the text.
- 6 I was thinking here of reviews such as those of Sukhnandan and Lee (1998), University of Brighton (2005), and Ireson, J. & Hallam, S. (1999).
- 7 Here I was thinking of Boaler (2008)
- 8 At this time, we were all looking at the transcripts of the first interviews from two weeks before.
- 9 I transcribed the recordings of interviews and coded them using TAMSAalyzer (<http://tamsys.sourceforge.net/>) and open coding.
- 10 These three numbers are the National Curriculum levels that Kwame thinks Darrell reached in the tests they all took at the end of their Primary schooling.

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A COMPARATIVE APPROACH TO THE TEACHING OF MATHEMATICS AT THE TIME WHEN PUPILS BEGIN ATTENDING GREEK PRIMARY SCHOOL: THE CASE OF THE EDUCATIONAL MATERIAL OF THE 1982 AND 2003 MATHEMATICS CURRICULA

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In this study we have utilised Basil Bernstein's theoretical framework regarding pedagogic discourse and have aimed at a comparative approach to the way school mathematics were taught in the period when 6-year-old pupils begin to attend Greek primary school between 1982-2009.

INTRODUCTION

The time when children leave kindergarten and begin attending primary school marks a critical stage in their schooling and is justifiably a focus of attention of educational research (e.g., Dunlop & Fabian, 2006; Woodhead & Moss, 2007). In Greece, attending the first grade of primary school begins at the age of 6. Pupils come in contact with a curriculum, in the form of a collection code (Bernstein, 1991). That is to say, school knowledge is divided into specialised subjects, of which the most prestigious – in terms of teaching hours per week – are Language and Mathematics (CTCF, 2003).

In 2003 in Greece, a reform of school mathematics took place with the introduction of the Cross Thematic Curriculum Framework for Compulsory Education (CTCF, 2003). This reform was implemented in 2006, when the new mathematics textbooks were published to replace the ones previously used as part of the curriculum of 1982 (Ministry of Education, 1982).

In this paper we have utilised Basil Bernstein's theoretical framework regarding pedagogic discourse and have aimed at a comparative approach to the way school mathematics were taught during the 1st school trimester of first grade of Greek Primary School between 1982-2009. This is to say we are interested in the period when 6-year-old pupils begin to attend Primary School. Moreover, the historical period under examination, which coincides with the modernisation of school knowledge in Primary School, is covered by the curriculum of 1982, which was used until the school year 2005-2006, and that of 2003, which was first implemented during the school year 2006-2007 and is in force to this day.

THEORETICAL REMARKS

The creation of curricula is an ideological process through which the powerful political and social groups endeavour to control official knowledge in order to

promote their own aims (Apple, 1999). In Greece, where the educational system is strictly centralised, changes are introduced at the top of the hierarchy pyramid, i.e. by the State through the Ministry of Education (Kazamias, 2004). According to Bernstein (1990), the State, taking into account the general regulative discourse and the particular traits of each historical period, constructs the Official Pedagogic Discourse, which is expressed through laws regarding education, curricula and textbooks. Therefore, the educational reforms we will be examining set different aims: in the case of the curriculum of 1982, its proclaimed aim was to promote the democratisation of education (Bouzakis, 2000). In the case of the reform of the compulsory education curricula of 2003, the aim was to adapt the decisions of the European Union in order for the Greek educational system to follow the new trends in education as these occur in other EU countries, especially after the poor performance of Greek pupils in the PISA2000 test (Alahiotis & Karatzia, 2006).

The creation of school knowledge takes place through a process of recontextualisation, i.e. it is a conscious social act through which, the State and its mechanisms (the official recontextualising field), which in Greece's case is the Pedagogical Institute, oversee the selection of knowledge and data from the primary scientific field of various disciplines, such as mathematics, which are then used to construct the different subjects of the curriculum, such as school mathematics (Bernstein, 1990, 1996). Curricula reforms often express the intention to implement changes in the educational and communicative environment of the school and the classroom which is expressed through the Instructional Discourse (ID) and the Regulative Discourse (RD) (Bernstein, 1990). These particular forms of discourse constitute elements of framing. This concept defines the internal logic of pedagogic practices and refers to the way teaching is formed through the selection of knowledge and the strategies by which it is presented. Moreover, it refers to the shaping of the communicative and interactive pupil-teacher relationship at the micro level of the classroom.

Framing (F) is illustrated by the formula: $F = ID/RD$, revealing that the RD is the dominant discourse and affects the way the ID is implemented because it regulates the way in which knowledge is transmitted. The RD refers to the forms of hierarchical relations that emerge during the educational communication between teacher and pupils. It is these relations that determine the "control over the social base which makes this transmission possible" (Bernstein, 1996, p. 27). The ID refers to the method and practices selected for the transmission of school knowledge and includes (Bernstein, 1991, 1996) the type of knowledge that has been selected to be taught (acceptance and utilisation or not of the pupils' everyday knowledge), the selection of the communication through which to present this knowledge (selection), the sequence in which the knowledge will be presented (sequencing), the rate of expected time for the knowledge to be acquired (pacing), and the criteria by which to verify whether the pupils have acquired the knowledge being taught (criteria). The RD and ID that shape the context of the pedagogic communicative relationship at

school can vary independently of each other (Bernstein, 1991, 1996). That is to say, we may have a weak ID framing in regard to the selection of knowledge, when the school draws upon and utilises knowledge that originates in the pupils' social environment and experiences; or a strong RD framing when a teacher-centered approach to knowledge is selected.

Bernstein (1990, 1996) noted that the successful or unsuccessful acquisition of knowledge depends on the factors of the pupil's social background and the weak or strong framing implemented during the pedagogic communication in the classroom in regard to didactic knowledge management. When school mathematics centre on highlighting the esoteric mathematical domain, while paying little attention to utilising an everyday frame of reference that is familiar to the pupils, and when an attempt is made to transmit a large amount of mathematical knowledge (F++ of pace), then those who benefit are pupils from middle and higher social strata who are mostly familiar with the approach to theoretical and abstract concepts (Apple, 2000; Bernstein, 1991, 1996; Cooper, 2002; Dowling, 2002). The same can occur in the case in which, within the didactic interactive communicative relationship, the pupils are granted the freedom to act on their own in order to discover and acquire school knowledge primarily through their own efforts (Bernstein, 1991, 1996).

INQUIRING QUESTIONS – METHODOLOGY

In this paper we will explore the following research questions:

What choices do the curricula of 1982 and 2003 offer in regard to the kind of mathematical knowledge that is selected to be taught and the pace at which it will be presented upon the pupils' entry in Greek Primary School?

What differences emerge from the comparative study of the curricula of 1982 and 2003 in regard to the shaping of the didactic communicative context for the teaching of school mathematics?

Our research sources are the printed educational materials for mathematics that correspond to the 1st school trimester of first grade of Primary School and which were produced in order to implement the curriculum of 1982 (henceforth C1982) and the curriculum of 2003 (henceforth C2003). This material includes the Pupil's Book, the Teacher's Book and – only in the case of C2003 – the Pupil's Exercise Book. It should be noted that in Greek compulsory education the school year is divided into three trimesters, with the first one extending from 11 September, when lessons begin, to 10 December. The range of each subject to be taught during the school trimesters is defined by the Teacher's Books, which offer the educator teaching guidelines according to which a schedule can be drawn up for the didactic management of school knowledge.

We approached our research through the method of Content Analysis, using the sentence as our unit of analysis. The sentence includes that part of the text's content that corresponds to "*a given semantic meaning*" (Neves & Morais, 2001, p. 244).

That is to say, it transcends the grammatical rationale in approaching a text, since each unit of analysis may contain two or more phrases, the use of which produces a specific, comprehensive and clear message of mathematical knowledge. The various sentences that were located were then classified by the researchers into the different categories of analysis which emerged after the examination of the research material and taken into account in the event that there was agreement between at least two out of three judges (Vamvoukas, 1990).

In the assessment of the kind of school mathematics being taught, three cases of Instructional Discourse emerged.

ID_F-: This case involves the solving of simple exercises, such as, for example, the correspondence between children and a number of pencils. Here, mathematical knowledge is connected to the public domain (Dowling, 2002), which includes familiar and known objects that are related to the pupils' daily lives.

ID_F+: In this case the activities focus on basic mathematical elements of a specific nature which are linked to the solving of exercises or problems and which require simple mathematical operations. These activities are drawn from the esoteric domain of school mathematics (Dowling, 2002), as in the following example (Lemonidis et.al., 2006a, No. A, p. 51):

“Apostolos has 4 marbles. Ernest gave him 3 more. How many marbles does Apostolos have now?”

ID_F++: This case comprises compound exercises/activities involving the esoteric domain of mathematics through which we attempt to assess or evaluate the pupils' prior knowledge. Moreover, this ID case includes problems whose solving demands compound logical elaborations, as in the following example (Lemonidis et al., 2006a, No. B, p. 48):

“There were 5 cars in the parking lot. At noon, another 6 came and parked there. In the evening, three of the cars left. How many cars were left in the parking lot?”

Moreover, in order to assess the development of the ID which is promoted in the cases of the C1982 and the C2003, we will study the instructions contained in the Teacher's Book and thus determine the mathematical knowledge to be taught to 6-year-old pupils during the 1st school trimester. Finally, we will present the proposed method of didactic management of the school knowledge in question (selection of pedagogic-didactic communication).

Through the study of our research material (Teacher's Books), the following three analysis categories emerged in terms of the intention to form the official communicative interactive pupil-teacher relationship (Regulative Discourse) and, hence, the hierarchical relations within the classroom environment (Neves & Morais, 2001, p. 232-233):

RD_F-: Here, the emphasis is placed on the greatest possible degree of autonomy and participation on the part of the pupil in the educational process in terms of solving problems or undertaking independent projects in order to approach school knowledge. In this case, a teaching theory seems to be promoted which focuses mainly on the acquirer.

RD_F+: In this case a teaching theory is promoted which, despite being focused on the teacher, demands the pupil's participation in the didactic act in order to be effective. Indeed, the way in which the pupils participate in the teaching is determined by the guiding instructions given by the teacher.

RD_F++: Here, the emphasis is placed on the teacher's role as a director. This gives shape to a teaching theory that focuses exclusively on the transmitter. The teacher's authority is considerable and is expressed in an explicit way.

RESULTS – DISCUSSION

Table 1 presents the school mathematics which, according to the Teacher's Book of the C1982 and the C2003, should be taught during the 1st school trimester to 6-year-old pupils (first grade of Primary School). From studying the data in Table 1, it emerges that in the case of the C1982 a gradual, unhurried introduction of the pupils to the content of school mathematics is attempted. That is to say, in the case of the C1982, the teaching of school mathematics during the 1st trimester of the first grade was carried out at a slow pace (F- pacing) (Bernstein, 1996). Conversely, in the case of the C2003, an attempt is made to impart more mathematical knowledge and to introduce the pupils, as early as the 1st trimester, into the abstract and esoteric domain of the discipline of school mathematics (Dowling, 2002). In order to achieve this, the teaching pace of this particular knowledge is faster than that of the C1982 (F++ pacing). Moreover, in the case of the C2003, the aim is to teach, during the 1st school trimester, numbers 1-20, which in fact covers a large part of the entire first grade curriculum according to the C1982 (Ministry of Education, 1982). It should be noted here that, following the reform of the compulsory education curricula that took place in 2003, school knowledge has become increasingly demanding compared to the past, since difficult mathematical concepts from the secondary school curriculum have been moved to the last two grades of Primary School (CTCF, 2003, 3987-4008). Furthermore, part of the first grade curriculum has been moved to Kindergarten. This involves the teaching of basic mathematical concepts, as well as familiarising children with the process of counting (CTCF, 2003, 4317-4319). Therefore, it appears that in the case of the curricula of 2003 regarding compulsory education, the school knowledge that is taught is more in quantity and of a greater level of difficulty compared to the past (C1982), a fact that shapes a strong framing of pace. Nevertheless, Bernstein (1991, 1996) pointed out that the successful acquisition of school knowledge by the pupil depends, to a large extent, on the weak framing of pace; on the creation, in other words, of the conditions of learning that give the pupil the necessary time to approach, process and acquire the new

knowledge. The creation of pedagogic practices based on a strong framing of pace favours only pupils that have access to a second pedagogic context outside school, i.e. a second chance to approach and process knowledge which is offered to pupils by their family at home (Bernstein, 1991).

C1982	C2003
<ul style="list-style-type: none"> • Pre-mathematical and basic mathematical concepts • Numbers 1-5 and introduction to addition. 	<ul style="list-style-type: none"> • Orientation in space, geometrical shapes, comparison/assessment of quantities. • Numbers up to 5: addition and analysis of numbers up to 5. • Addition and analysis of numbers 6-10. • Numbers 10-20 and Coins up to 10.

Table 1: Type and presentation sequence of school mathematics knowledge

Table 2 presents the didactic choices of a communicative nature which the teacher is requested to observe in each teaching unit. From studying the data in Table 2, it emerges that, in the case of the C1982, the construction of mathematical knowledge follows two main hierarchical stages (Apostolikas et. al., 2002): The first refers to the teaching/presentation of the new mathematical topic by the teacher. The second stage is related to the solving of exercises which are included in the Pupil’s Book and aspire to help the pupils assimilate mathematical knowledge. In the case of the C2003, the process of presenting and elaborating mathematical knowledge occurs in a more complex way compared to the C1982. In particular, teaching begins at the stage of “orientation and elicitation” of new knowledge. Here, the teacher tries, by creating the appropriate teaching situations, to have the pupils themselves discover the knowledge. The stages of presenting and assimilating mathematical knowledge in the C2003 are the same as in the C1982. The new didactic action, compared to the past, is the effort to extend school mathematics to other cognitive situations. This is supported by the proposal to carry out cross thematic approaches, through which the teacher will attempt to link facets of mathematical knowledge to the content of other subjects in the curriculum. This last stage, according to the authors of the new educational material, is believed to contribute to the successful comprehension of mathematical knowledge by the pupils (Lemonidis et al., 2006b).

Didactic actions	C1982	C2003
Orientation and elicitation		X
Activities for the presentation and discovery of mathematical knowledge (Formalisation of new knowledge)	X	X
Projects & exercises for the application and assimilation of mathematical knowledge	X	X
Extent of new mathematical knowledge		X

Table 2: Didactic actions towards the construction of mathematical knowledge

From studying our research material, we have located 1311 sentences, which can be classified as follows: in the case of the C1982 there are 439 units of analysis (33.5%), while in that of the C2003 there are 872 units of analysis (66.5%).

Table 3 presents the classification of the units of analysis that refer to the type of mathematical knowledge that is contained in the mathematics textbooks used by first graders during the 1st school trimester. That is to say, it is an examination of the ID in terms of the extent to which knowledge is selected towards the formation of school mathematics either from the esoteric domain of school mathematics or from the public domain of the pupils (Dowling, 2002). From studying the data in Table 3, it emerges that the textbooks under examination differ in terms of their choices in shaping the ID, which concerns the type of knowledge ($p < 0,01$). In particular, during the 1st school trimester, according to the C1982, mathematical knowledge is presented and developed mainly through exercises and activities drawn from the pupils' experiences and familiar everyday world (64.8%), i.e. from the public domain (Dowling, 2002). Conversely, in the case of the C2003, even though mathematical knowledge is approached using elements from the public domain at a rate of 35.6%, an effort is made from the outset to introduce the pupils to abstract mathematical knowledge (F+ 62.2%).

	ID_F-	ID_F+	ID_F++	Total
C1982 (%)	162 (64.8)	83 (33.2)	5 (2.0)	250 (100)
C2003 (%)	226 (35.6)	395 (62.2)	14 (2.2)	635 (100)

Table 3: Sentences according to the type of mathematical knowledge

Table 4 shows the classification of the sentences that were located in the Teacher's Book during the 1st school trimester and which refer to didactic recommendations to the teacher towards shaping the RD, i.e. defining hierarchical relations and pedagogic communication during the didactic interaction between teacher and pupils.

	RD_F-	RD_F+	RD_F++	Total
C1982 (%)	49 (25.9)	107 (56.6)	33 (17.5)	189 (100)
C2003 (%)	112 (47.3)	103 (43.4)	22 (9.3)	237 (100)

Table 4: Sentences that define the didactic interactive relations between teacher and pupils during the 1st school trimester in first grade mathematics curricula

From studying the data in Table 4, it emerges that the curricula under examination differ in terms of the hierarchical interactive relations that are promoted during the 1st school trimester of the first grade, and particularly in relation to the didactic communication between teacher and pupils ($p < 0,01$). Specifically, in the case of the C1982, the teacher is the strong factor that defines the way in which school

mathematics will be taught (F++ 17.5%, with a total rate of positive framing at 74.1%). However, the didactic recommendations contained in the Teacher's Book for the greater part of the mathematics curriculum ask of the educator to guide the pupils in order for them to participate as actors in the educational process (F+ of the RD at 56.6%). Moreover, a significant finding is that, compared to the period prior to the C1982 during which a teacher-centered way of teaching was prevalent, it can be seen in 25.9% of the activities included in the C1982 that the approach to and elaboration of mathematical knowledge requires self-activated learning on the part of the pupils. Furthermore, in the majority of the didactic recommendations of the Teacher's Book of the C2003, the educators are asked to create, as early as the 1st school trimester, the suitable conditions for the pupils' self-activation. Thus, the pupils will be able to work either individually or in groups towards discovering knowledge in an experiential way (F- 47.3%). Nevertheless, we must take into account that mathematical knowledge in the C2003 compared to the C1982 is more complex and focuses on a larger curriculum, which is mostly drawn from the esoteric abstract domain of mathematics. Also, carrying out the exercises/activities listed in the C2003 demands of pupils a greater degree of independence than in the past, a fact which perhaps favours children that come from privileged social backgrounds. The reason for this is that certain pupils may possess the necessary cultural capital (Bourdieu & Passeron, 1977) that allows them to handle theoretical mathematical knowledge with greater facility, given that they are orientated towards abstract meanings and have been socialised into taking initiatives and working independently, traits that are linked as much to the discovery and elaboration of knowledge as to the utilisation of this knowledge in order to tackle different social situations (Apple, 2000; Bernstein, 1991, 1996; Cooper, 2002; De Abreu & Cline, 2003; Dowling, 2002).

CONCLUSIONS

From the study of and comparative approach to school mathematics in the cases of the C1982 and the C2003, we arrive at the following conclusions:

- In regard to the way in which the didactic communicative approach to the C2003 is shaped, two new elements are introduced: The emphasis is placed on taking initiatives on the part of the pupils and an attempt is made to bring them in contact in an experiential way with the new knowledge they are to acquire. Moreover, an effort is made to utilise the new mathematical knowledge within the context of other disciplines and courses in the first grade of Primary School curriculum.
- During the 1st school trimester of the first grade, in the case of the C1982, a slow pace is selected for the presentation of mathematical knowledge (F- pacing). Conversely, in the case of the C2003, a fast pace is selected for the presentation of mathematical knowledge (F++ pacing), which is drawn chiefly from the esoteric domain of mathematics.

- In the case of the C1982, during the 1st school trimester, the Instruction Discourse is shaped by the selection and utilisation of knowledge that is familiar to the pupils (F- 64.8% of selection), since the goal was to gradually and unhurriedly transport them to the internal theoretical field of mathematical knowledge (Apostolikas et al., 2002). Conversely, in the case of the C2003, the Instructional Discourse is shaped by selecting exercises/activities which are drawn mainly from the esoteric mathematical domain (F+ 62.2% of selection).
- The Regulative Discourse in the case of the C1982 is shaped in a way which renders discernible the hierarchical relations in the educational process (RD: F+ 56.6% and F++ 17.5%). In the case of C2003, despite the fact that during the 1st school trimester a significant part of the curriculum requires the expression of didactic strategies through which the teacher's guiding/hierarchical role is distinguished (F+ 43.4% of the RD), an attempt is made to give pupils the space needed in order for them to approach mathematical knowledge in an experiential way (F- 47.3% of the R.D.).

The above observations give rise to important research questions regarding the social strata that benefit from the specific choices that shape contemporary school mathematics, as these choices are expressed in C2003.

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WHAT'S IN A TEXT: ENGAGING MATHEMATICS TEACHERS IN THE STUDY OF WHOLE-CLASS CONVERSATIONS

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This paper discusses an activity done in a lesson study group of beginning teachers working in high needs urban schools. The activity involved the interpretative analysis of a whole-class conversation transcript, via functional grammar tools, with focus on how the teacher conducted the interaction. As a spoken text genre, whole-class interaction poses heavy demands on teachers in terms of their ability to provide skilful conduction. After presenting the text, we describe how it was read in our study group. This description is intended to show how the guided deconstruction of whole-class interaction texts can serve to strengthen teachers' awareness of the semiotic choices they have available for shaping the joint making of such texts and the effects of these choices in supporting students' mathematical meaning-making.

INTRODUCTION

This paper discusses an activity done in a lesson study group for beginning teachers. Participants in this study group are working in high needs urban middle schools, teaching mathematics to low socio-economic status 6-8 grade students from a wide range of cultural/ethnic/linguistic backgrounds as well as levels of academic performance. The lesson study group is part of a 3-year exploratory project (NSF, DRL 0822223) wherein mathematics teaching, learning, and learning to teach are addressed from a social semiotics perspective. This project involves two sub-groups of 6 teachers, each facilitated by a PI, which meet every other week for 3-hour sessions. The focus of the in-service activity is the "Necklace Text," a whole-group conversation excerpt captured in a 6th grade class taught by an experienced teacher.

For the analysis of classroom texts we draw upon systemic functional linguistics (SFL) (Halliday, 1994) and critical discourse analysis (Fairclough, 2003). Among the tools we borrow from SFL are: language as meaning potential; context, text, register, and genre; meaning as choice; teaching as enlarging students' meaning potential; the interpersonal, ideational, and textual meta-functions of language; grammatical form, speech function, and modality; material, mental/verbal, and relational meanings; and the packing of mathematical meanings into technical wordings via nominalization and grammatical metaphor. Rather than presenting these tools to participants as a crash course in functional grammar, we introduce them as we see them fit, during our lesson study activities of analyzing classroom-generated texts and artefacts, solving of non-routine mathematics problems, and planning, discussing, and revising lessons and lesson sequences.

SFL has proven well-suited for studying science and mathematics classroom texts (Atweh, Bleicher, & Cooper, 1998; de Freitas & Zolkower, 2009; Lemke, 1990;

Morgan, 1998; Shreyar et al., 2009; Veel, 1999; Wells, 1999; Zolkower & Shreyar, 2007). To this line of research we contribute an exploratory inquiry on the effects of engaging beginning middle school mathematics teachers in the functional-grammatical study of spoken and written texts. As a unique spoken classroom genre which involves the interaction of multiple speakers positioned in asymmetric power/knowledge relationships, whole-class conversations pose heavy demands on teachers in terms of providing skilful conduction. In line with genre-based pedagogy (Martin, 1999, 2009; Rose, 2005), we guide participants to deconstruct exemplar of this text genre with the aim of developing their awareness of the range of semiotic choices they have available when conducting interaction—i.e. choices regarding diagramming, gesturing, discursive moves (e.g. evaluation, follow up, etc.), choices of grammar and vocabulary in the wording of their contributions, and so on.

Within our study, currently in midway, we are collecting audio and video data from participants' classrooms. We have constructed a rubric that identifies relevant features in the conduction of whole-group interaction which will be used to assess participant change along this dimension of their practice. We expect that our data will provide evidence that the guided reading of whole-class conversation texts strengthens teachers' ability to conduct the joint making of such texts in manners that are conducive to enlarging their students' mathematical meaning potential. The Necklace Text exemplifies the following three indicators from our rubric (Fig.1):

B.6: [The teacher] uses gesture (e.g. pointing) and demonstrative pronouns (*this, that, these, those*) and other deictic terms (e.g. *here, there*) to relate speech to the writing and diagramming that co-occurs on the blackboard or other writable surface; B.8: Her lexical and grammatical choices promote back and forth movements among material, mental/verbal, and relational processes; B.13: She makes functionally appropriate use of repetition, re-voicing, rephrasing (e.g. using targeted technical terms or, alternatively, rendering those into non-technical ones); and requests repetition and rephrasing from students.

Fig. 1

In the sections below, we sketch our theoretical framework, present the Necklace Text, and describe how the facilitator guided the reading of this text. This description is meant to illustrate our central claim, namely, that the guided deconstruction of whole-group interaction texts selected as paradigmatic instantiations of this genre strengthens teachers' awareness of their semiotic choices for shaping classroom conversations and the effects of these in promoting or hindering students' mathematical learning.

THEORETICAL FRAMEWORK

Our work relies upon a view of mathematics as mathematizing, that is, organizing subject matter with mathematical tools (Freudenthal, 1991). Following Freudenthal (1991), we conceive mathematics teaching as guiding students in *reinventing*

mathematizing. And, inspired by Vygotsky (1978) and Halliday (1993), we value skilfully conducted whole-class conversations as serving the function of ‘interpersonal gateways’ for enlarging the potential of what students can mean in the spoken, diagrammatic, and written language of mathematics.

SFL offers tools for explaining how speakers/writers use language to ‘realize’ (i.e. express, manifest) interpersonal, ideational, and textual meanings in texts (Halliday, 1994). Interpersonal meanings construe and/or maintain social relationships and perform communication roles; ideational meanings—i.e. material, behavioural, verbal, mental, existential, and relational—construe our experiences about the world around and inside of us; and textual meanings allow us to construe interpersonal and ideational meanings in texts. In whole-group conversations, ideational meanings pertain to the topic of conversation; interpersonal meanings concern the roles and relationships among the interactants; and textual meanings give cohesion and coherence to the conversation so it serves its intended purpose in the context wherein it unfolds.

The problem of how to support the development of student thinking within school contexts can be reformulated as how to initiate students into the “ways of saying: ways of meaning” (Hasan, 1996) privileged by school-taught disciplines. Among the six aforementioned types of ideational meanings, the relational ones are centrally implicated in the language of schooling (Schleppegrell, 2004) and, most conspicuously, in school mathematics (Morgan, 1998; Veel, 1999). Relational clauses, which are typically worded with the verb *to be*, relate two separate entities such that something is said to be something else, thereby realizing experience not as ‘doing’ (material) or ‘sensing’ (mental) but as ‘being.’ In other words, relational clauses make claims not about what happens or happened but about how things happen to be (Halliday, 1994). Teaching mathematics as an activity demands great skill from teachers in helping students turn material processes into relational ones as well as recover those material processes packed into relational clauses.

From an SFL perspective, analyzing a text amounts to explaining the meanings realized therein, accounting for how these meanings (as choices of grammar and vocabulary) fulfil interpersonal, ideational, and textual functions (Halliday, 1994). When analyzing a whole-class conversation text, we guide lesson study participants to consider the semiotic choices the teacher (in the text) made in her contributions to the text and relate what she said and did against the horizon of what she could have said or done.

THE NECKLACE TEXT

- 1 Teacher (T): Ok. The next one (*Draws: ●●○○●●○○●●○○●●... on the board*), the same question. What would be the colour of the 1000th bead?
- 2 Kaylan: We know that the tenth bead is black, right? The tenth bead is black... so I just timed it by 100 which got me to 1000.

3 Emily: Oh! Can I say something?

4 T: Wait, what Kaylan was doing, can anyone rephrase it in a different way?

5 Emily: I kind of used that pattern but...

6 T: No, no, no. You have to rephrase what he said in a different way first, before you can say anything else. Who can do that?

7 Carlos: What Kaylan said is that the 10th bead is black, right? So, if you times it by one hundred, you will get 1000 beads. The 1000th bead will be black.

8 Melika: So, basically he's saying that the 1000th bead will be like the 10th one.

9 T: Mm.... someone sees something yet?

10 Leah: He said that he timed it by 10 and then he did 10 groups of 100-

11 T: Wait, wait. What did Kaylan do? What was he doing here? He counted up to 10 and then MENTALLY he SNAPPED it off, right? (*Makes a hand gesture indicating cutting with scissors*) He made a cut.

12 Nyree: Yes!

13 T: Ok, so, let's make a cut. Let's do that. Let's make a cut. (*Draws a vertical line between the 10th and the 11th bead: ●●○○●●○○●●|○○●●...*). But, what happens after you cut that?

14 Emily: But look at this! (*Walks toward the board*) This (*points to the first black bead*) is starting with a black but now (*points to the 11th white bead*) it starts with a white. They don't start with the same color. See?

15 T: So, if that's the case, can we multiply it by 100 and say it will be the same?

16 Chorus: No!

17 Melika: No, because the results will change.

18 Emily: Can I say something?

19 T: Can she say something?

20 Chorus: No!

21 T: Alright, Nyree first, and then you.

22 Nyree: Like... you know, like... we said before that half of the 1000 beads is 500, right? But then, like he said, if you cut it up to 10, the other one will start with white, but if half of 1000 is 500... What if we separate it from the 5th one?

23 T: Separate it from the 5th one? That's interesting. If I have to snap it off... if I want to cut this very long necklace into chunks of little, little sections (*makes a hand gesture indicating cuts on the drawn necklace*), how should I cut it?

24 Chorus: By 4!/By 5s!

25 T: By 5 or by 4? I have two suggestions so far (*With different colour markers, draws vertical lines on the necklace to signal the two proposed cuts*).

- 26 Leah: By 4. If you cut it by 5s, each group has the same amount of color beads but the two groups are different so--
- 27 T: So, wait. This is what Leah suggests. She suggests we chunk it like this (*points to chunks of 4*) because--
- 28 Emily: That's what I was going to say! You see? You don't let me say anything!
- 29 T: (*Smiling at Emily and making a hand gesture signalling that she waits*). What do you think now, Nyree, should we cut it by 4s or by 5s?
- 30 Nyree: By four!
- 31 Emily: Yes. I think you should cut it by 4s!
- 32 T: Ok, Emily.
- 33 Emily: (*Walks to the board*). I noticed the pattern right here, that every fourth bead is a white bead (*pointing to the 4th, the 8th, and the 12th bead*). This set of 4, right here, the last one is always white. So, I know that... it's a multiple of 1000 so...
- 34 T: Wait, wait, wait. Four is a MULTIPLE of 1000 or?
- 35 Emily: A factor, a factor of 1000.
- 36 T: Oh! Ok, Melika.
- 37 Melika: I think you should cut it by 4s.
- 38 Cathy: No. I think you should cut it by 5s.
- 39 Melika: If you cut it by 5s, then you would be ending with a black bead.
- 40 T: Also... are these the same, though? Are these chunks the same? (*Pointing to the chunks of 5 on the necklace*)
- 41 Chorus: Yes! No!
- 42 T: Black black white white black.... Black white white black black. Are they the same?
- 43 Chorus: No
- 44 T: So, would that be alright?
- 45 Chorus: No!

GUIDING THE INTERPRETATIVE ANALYSIS OF THE TEXT

In prior sessions, the study of other texts (cf. de Freitas & Zolkower, this volume) served to exemplify technical SFL-related vocabulary: turn (change of speaker); clause (smallest linguistic expression with meaning in itself); speech function (questions, statements, and commands); grammatical form (interrogative, declarative, and imperative); and congruency (alignment between the grammatical form of a clause and its speech function). Of the six participants in the sub-group in which the Necklace Text was read, 3 had been in the project from the start (Dolores, Sarah, and Nyoka) and the other 3 (Jason, Karin, and Michelle) were newcomers.

Before reading the text, the group explored necklaces with repeating and recursive patterns—e.g. BBWWBBWWBBBBWWBBBBWWBB..., and BWB BBBWWWWBBBBBB—in light of the question: *Imagine we were to continue this necklace, following the same pattern. What would be the colour of the 1000th bead?*, and then identified the mathematics embedded therein (odd and even numbers, factors and multiples, and remainders; repeating vs. recursive patterns; and structuring, symbolizing, generalizing, and predicting). The facilitator (BZ) contextualized the text by offering the following information: the teacher (Ms. L) had 16 years of teaching experience; the conversation under analysis involved a heterogeneous 6th grade class of 22, with Latino, African-American, and Asian background students exhibiting a wide range of academic performance levels; the lesson took place in March; the academic performance levels of the students who participated vocally in the conversation were as follows: High: Kaylan and Leah; Medium: Carlos and Emily; and Low: Nyree and Melika, and Cathy (special needs).

The questions that framed the text analysis were: What is the teacher doing and saying? What is the grammatical form and speech function of her contributions? How does she guide the students in making mathematical meanings? If you were the teacher in this class, what would you have done or said differently and why?

First, participants were instructed to read the entire text and write down what they found interesting. After ten minutes, BZ focused everyone's attention to the number of turns in the text (45) and the distribution of student and teacher turns—teacher: 19; students: 26, with 7 students who spoke on their own and 6 chorus responses. It was noted that the number of vocally participating students was about one third of the class, in approx. 15 minutes. The dominance of the present verb tense—a total of 84 verb occurrences; past: 10; future: 5; present (simple or continuous): 69—was read as grammatical evidence that the conversation was not about reporting on work previously done but consisted of “thinking aloud together,” in the moment, and under the teacher's guidance (Zolkower and Shreyar, 2007; Shreyar, Zolkower and Pérez, 2009). Next, the presence of diagrams and italics (for non-verbal actions) was read as indicative of the multimodality of the text.

With regards to turns 18-21, Sarah asked: “Do they have like... a deal in this class, like Emily can't talk until like four other people talk? [...] I think that's great. I'm definitely going to try that in my classroom.” BZ: “Most likely, this rule was established at the beginning of the year and has now become implicit. Definitely the teacher doesn't want any kid to dominate. She wants to control the discourse. Although she wants everyone to participate and appreciates what people have to say, she does not want them to say whatever they want at any moment.” How did Ms. L. control the conversation? Offering a few examples from the text, BZ underlined that she did so via interrogatives (yes/no as well as ‘wh’ or open) and imperatives and that, for the most part, there was congruency between the grammatical form of her clauses and their speech function. Nyoka noted the presence of ‘if... then’ questions

in teacher ([15], [23]) and student contributions ([7], [22], [26], [39]), and this was read as evidence of logical-mathematical thinking.

The facilitator described turns 4 and 6 as requests for rephrasing, the former realized with low and the latter with high modality (“can anyone rephrase...”; “you have to rephrase...”). Participants offered differing interpretations of the rationale behind this request: “to reinforce students’ literacy skills,” “to check that they understand,” “maybe if they hear it again someone would catch the error,” “to make sure everyone is paying attention.” These interpretations, which speak volumes of the purpose participants attribute to whole-group interaction, seem at odds with the view of this conversation as a thinking aloud together text.

With respect to turn 9, Nyoka wondered: “I’m just was curious as to what motivated her to say that and to say it in that way.” Sarah: “It’s like she’s challenging other senses: what do you *see*?” BZ: “In looking ahead at her turns 11 and 13, it seems that Ms. L. means: Let’s take a closer look at the necklace so we can see that we can’t just infer from the fact that the 10th bead is black that the 1000th bead will be black as well.” It was noted next that, in turn 11, the teacher reacted to Leah’s rephrasing with a question realized with the verb *to do* (material process) and answered it herself, a hand gesture, a mark on the diagrammed necklace and an interrogative question demanding a process of *happening*.

Regarding turn 14, participants commented that Emily was finally was allowed to speak and that now her contribution fitted well in the text. BZ commented that, in turn 15, the teacher used an ‘if... then’ clause to complete Emily’s statement by making explicit what she had left out. Yet, in Nyoka’s view, this question was “too leading.” BZ: “Yes. We can read it as a statement in the grammatical form of a question, not a true question, as if saying: Are you all with me now?” Karin wondered, with regards to student turn 17, “Why isn’t the teacher asking her what she means?” Jason responded: “Maybe she was going to, but Emily got in the way.” Sarah: “If you look at Emily’s use of pronouns: *I* and *me*, in turn 28, the teacher is trying to elicit the entire class to remind her, this is not just for me, this is what you’re doing for the entire class. But her language is saying: you, the teacher, you are not letting me talk.” BZ: “How do we have a conversation in which *I/me*, *he/she*, *his/her*, and the individual *you* are less important than what *we* are doing, here and now, together?” Nyoka, who teaches in an all-girls middle school, shared: “I have a room full of Emilys [...]. They sit in triads and I just let them talk to each other... like they’re gonna talk to each other anyway. I don’t want them all yelling at me. So, I say: Take a moment and talk to each other about the math, what is going on here?”

With regards to Nyree’s suggestion (turn 22) to “separate it from the 5th one,” Sarah observed: “You can tell that Nyree is listening. She got the point that the chunks have to start with the same colour.” When BZ highlighted that, in turn 23, the teacher contributed the most important question, Jason observed that Ms. L. interrupted Leah [26-27] and wondered why she did so. Nyoka: “Because what she’s about to say is

going to go over everyone else's head." BZ: "Yes. Her interruption served to chunk Leah's argument, which she worded with two 'if... then' conditionals: if you cut it by 5s, if you cut it by 4s. Just as the teacher was chunking the necklace, she was also chunking the talk, to distribute it throughout the class."

Participants' attention was next focused on the verb choices throughout the text. The initial question, "What *is* the colour of the 1000 bead?" requested a relational process. In response to Kaylan's suggestion, the teacher asked: "Someone *sees* something yet?" whereby *sees* can be read as calling for a mental or a behavioural process (*seeing* as realizing or as viewing or noticing). Later on, Ms. L. reworded Kaylan's contribution as a process of doing ("What did Kaylan *do*?"), mentally snapping off the necklace in groups of 10, and inviting the class to "make a cut." Dolores noted that, in turn 42, the teacher's reading aloud of the pattern allowed the students a chance to "hear" the repeating chunk. As to turns 40, 42, and 44, Sarah commented: "Instead of asking the same question twice, three times, I would have asked: If you say they are the same, how are they the same?"

The group then was led to reconstruct the mathematics in the Necklace Text as a sequence of relational clauses: *What is the relationship between 4 and 1000? How can I use this relationship to predict what colour would the 1000th bead be? The length of the repeating pattern is 4. 1000 is equal to 4 times 250. A 1000th bead necklace with that pattern will be made of 250 identical chunks, each of length 4. The colour of the last bead in the repeating pattern is black. Therefore, the 1000th bead will be black.* Moving away from the text, the discussion ended with the exercise of defining, via relational clauses, mathematical notions (nouns) volunteered by the participants ('slope,' 'square root,' 'order of operations') and then unpacking their meaning by recovering the processes (verbs) encapsulated in them.

Below (Fig. 2) are participants' comments on the Necklace Text (end-of-the session written feedback form).

"The way they use language is so concise. And there's so much math in the dialogue without anyone talking a lot." "I was impressed with how the teacher controlled the discourse so it stayed within the students' zone of proximal development." "Using action verbs like cutting, chunking, breaking apart, creates greater meaning for mathematical ideas. The teacher created opportunities for students to move between material (doing) and relational (being) processes." "The conversation moved very smoothly, even when the teacher corrected a vocabulary error. I like how she does not evaluate right away what students are saying. Sometimes I find this very difficult to avoid." "You can use necklaces for teaching multiples and factors and developing number sense. There is also algebra in them: odd and even, using letter symbols, generalizing, making predictions, and so on."

Fig. 2

As described above, throughout the activity, participants were guided to describe and explain the semiotic choices made by an experienced teacher working in a

heterogeneous classroom and to consider the effect of these choices in supporting movements back and forth between doing, happening, looking, seeing, noticing, saying, having, and being. Engaging with the Necklace Text served as a productive encounter with a paradigmatic text of the genre of whole-class interaction which functioned as an exemplar of how teachers and students can think aloud together, mathematically and about mathematics.

The documentation of our lesson study sessions is yielding evidence that engaging beginning teachers in the functional-grammatical study of whole-class conversations shifts their perspective and attitude away from just commenting on the texts towards first analyzing and only then evaluating them, with an eye towards imagining alternative teacher contributions. As expected, this is not an immediate and uniform process; instead, it is one that happens rather slowly, at different paces and in a different manner for each of the group participants.

CONCLUSION

The quality of mathematics education in general and, in particular, that of students from low SES and other non-dominant social groups may be improved by creating opportunities for teachers to learn how to shape whole-class conversations as interpersonal gateways for students to learn how to think mathematically by thinking aloud with their peers under the teacher's guidance. This professional development model involves guided deconstruction of paradigmatic texts of that genre, with opportunities for teachers to participate in the joint making of such texts as they interact with peers and facilitator around non-routine problems. These interconnected experiences of joint text deconstruction and construction prepare teachers to support the making of spoken and written texts in their own classrooms (Martin, 1999).

As we continue generating and processing project data, we expect to demonstrate that our approach equips beginning teachers with semiotic tools for enacting hybrid pedagogies in their classrooms. Gleaning from Bernstein's (1990) theory of pedagogical discourse and its applications to teacher education (Morais et al., 2001), by this we mean engaging students in framing, solving, and reflecting on non-routine—hence, weakly classified—mathematics problems yet offering waves of strong and weak framing that guide their interactions with teachers, peers, and texts.

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Racism as Gazing Bodies: From ‘body-color’ epistemology to epistemic violence
A response to: Not-so-strange bedfellows: Racial projects and the mathematics education enterprise

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Danny Martin commences his lecture on ‘*racial projects and the mathematics education enterprise*’ by pointing out how ‘racism’ still affects any attempt to work out a social equity agenda for mathematics education in both educational institutions and pedagogical practices. He observes how globalization tends to transform the institution of ‘university’ from a social project to a market force that re-distributes financial investment of public funds. As such, the primary focus moves from generating innovative knowledge towards providing highly skilled and well trained work force, whilst, at the same time, its democratizing role aims at promoting opportunities for social, political and economic mobility. Danny Martin proceeds to relate this ‘factory image’ of the university to mathematics education programs. He points out, based on Nielsen (2003), how current mathematics education programs adhere to a range of ideological agendas that vary from critical to neoliberal. Such agendas seem to get involved into a continuously diverse endeavor of prescribing, theorizing or even dominating and colonializing what should be the interconnections amongst mathematics, mathematics education curricula, and societal needs. He asks: ‘What sort of project is mathematics education?’, and ‘Whose interests are being served by this project?’ Trying to account for these questions, Danny Martin returns to examining issues of social justice and equity where ‘race’ and ‘racism’ become the central axis for his investigation. Reviewing a number of research projects focusing on social justice and mathematics education, he concludes that although most scholars provide compelling critique to the fact that mathematics education and mathematical knowledge have increasingly been put in service to neoliberal and neoconservative agendas, they do not provide compelling analysis of race and racism. In short, although race is still an essential marker for excluding and marginalizing individuals within mathematics education practices, it has not been taken, yet, seriously into consideration.

I believe that Danny Martin has set up an important mission for himself not only as an academic within the field of mathematics education, but also as an active member of his local community in Chicago, US. I take seriously the internal motives gearing Danny Martin’s work for they can lead to a more sensitive engagement with issues of social justice. I will, thus, turn towards responding to his lecture drawing on the field of technoscience and considering a (post)colonial and feminist perspective (see Haraway, 1989, Harding, 1998, Spivak, 1999). From this optic, issues of race, gender and science are not seen separate but interconnected. Although over the years (post)colonial and feminist scholars have tried to explore and unravel potential links that could initiate a dialogue -still their claims are open for further critique (see Spivak, 1999). Next to differences, a basic agreement is that ‘race’ (and racism) is socially constructed in (post)colonial discourses. In this realm, it is interesting to note how ‘race’, historically, has been evolved into a ‘tool’ at the hands of both ‘soft’ and ‘hard’ scientists –sociologists and anthropologists, but also biologists, zoologists and physicists (for more details concerning the move towards postcolonial feminist science studies see Chronaki, 2008).

A first departure in such a travel could be to account about racism as the practice of ‘gazing bodies’ –a practice highly mediated by discourses related to ‘color’ as is indicated by the metaphor of ‘white institutions’, offered by Danny Martin. Color becomes an essential material indicator that captures the gaze and penetrates consciousness via perception. It is easy to assume that what we ‘see’ is what it ‘is’ –as a representational view of mind might imply (Hall, 1997). Therefore a ‘black’ person, whatever his/her personal history and agency might be, runs the danger for being locked within stereotypical (and hegemonic) discourses of ‘blackness’. The ‘black’ then becomes exotic, oriental and characterized as ‘other’. Said (1978) explains that the ‘orient’ occupied a marginal discursive position since for centuries it was constructed by colonials as the inferior feminine or racial other. The ‘orient’ is always in need to be studied and displayed, to be disciplined and civilized. The ‘representational view of mind’ coupled with ‘orientalism’ can easily confirm a ‘body-color’ epistemology –a search of knowing that is mainly driven by ‘gazing bodies’ through/as stereotypic representations and by reproducing hegemonic discourses of subject agency.

Gazing bodies and specifically colored bodies has a long history in anthropological research but also in biology as, science historian, Londa Schiebinger argues in her book entitled *‘The mind has no sex: women in the origins of modern science’*. Londa Schiebinger (2000) discusses the shameful case of ‘Hottentot Venus’ a woman from Southern Africa, named Saartjie (or Sarah) Bartmann, who was brought to Europe and displayed naked as a female body in either freak-shows or museums. She was made an object of sexual and scientific investigation and her body provided part of evidence for constituting modern biology. Londa Schiebinger (2000) explains:

‘In the spring of 1815 she was summoned to the Jardin du Roi by a commission of zoologists and physiologists, where she was examined for three days. Henri de Blainville, professor at the Museum d’ Histoire Naturelle in the Jardin du Roi, set out his purposes in observing her: (1) to provide a detailed comparison of the woman with the lowliest race of humans (the Negro) and the highest type of apes (the orangutan); (2) to provide the most complete possible description of the anomalies of her genitalia. This investigation required that Sarah Bartmann strip naked in the austere rooms of the museum in front of at least three formally dressed men’ (p. 29).

Sarah Bartmann died nine months later from ‘inflammation’ at the age of twenty-six and her dead body was brought to the museum for further examination and display. Parts of her body –like the many apes whose skeletons and skin were sold or donated to natural history museums- were preserved in formalin and made available for purchase as a souvenir. In this case gender traits, by means of a ‘black’ woman, were persistently invoked to explain purported racial superiority of mainly the white, middle-class man. It was only until 1994, after the African National Congress victory, that Nelson Mandela asked formally the French Government to return her remains. Today, Sarah Bartmann has become a symbol of colonial history -known as the daughter of South Africa (see http://en.wikipedia.org/wiki/Saartjie_Baartman).

Sarah Bartman’s story is just an exemplary of how race and gender have become the ‘material’ for developing science itself at the foreground of colonialism. Race and gender are being discussed by Nancy Leys Stepan (1986) as a powerful analogy for

science that as she argues occupied a strategic place in scientific theorizing about human variation in the nineteenth and twentieth centuries. The traces of this argument can be found in examples from anthropometric, medical and embryological studies where the focus has been the measuring of human and animal skeletons (see Gould, 1981). Such studies provide evidence of black men and women's low brain weights and deficient brain structures as compared to men from varied cultures or even to animals. Woman, thus, was observed to share with Negroes the primitive traits of a narrow, childlike and delicate skull found in lower cases, so different from the more robust and rounded heads characteristic of males of superior races. Evolutionary biology making use of such evidence provides the analogy of woman as being the 'conservative element' to the man's 'progressive' (Ellis, 1926). Donna Haraway (1989) provides additional evidence for the tacit implications of 'scientific orientalism' through her studies in animal sociology in the context of primatology discipline. Whilst primatology might appear to be about animal communities it has become responsible for legitimizing a colonial perspective on projects where 'white' dominance becomes recontextualised. Haraway observes how scientific claims for connections between *social functionalism* and *physiological functionalism* have emerged 'naturally' and the related scientific outcomes become easily re-applied in areas such as medical, educational and industrial management or even military and administration (for a further discussion see Chronaki, 2008).

Racism today is based on a strong image of a 'collective identity' of some sort (i.e. ethnicity, religion, gender, ideology, knowledge hierarchies, scientific competences etc) that serves to inscribe a strong distinction amongst 'we' and 'others' – a distinction that reflects precisely a 'chromatic' or 'body-color' epistemology. This epistemology is linked to a fixed and static notion of representing knowledge hierarchies and subject agency. Gazing, visualizing and categorizing provide a rigid adherence to stereotypic images of cultural identity and scientific knowledge. As such, certain subject positioning(s) become excluded, marginalized and silenced producing *epistemic violence*.

Whilst epistemology theorises the origin, nature, methods and limits of knowledge, 'episteme' has been defined by Foucault (1970) as a 'unitary body of theory' which tends to privilege some knowledges whilst subjugating certain others and ranking them low in its hierarchical paradigm. According to Spivak (1999), epistemic violence results when in colonial and postcolonial discourse, the subaltern¹ is silenced by both colonial or indigenous patriarchal structures. Gayatri Spivak (cited in Harasym, 1990) argues how epistemic violence is easily ignored when the 'us' and 'them' division creates a clear distance between the 'object of race' and the 'subject of racism'. She explains how current discourses of anti-racism approaching race simply in terms of skin color can replicate similar structures to the ones used to produce epistemic violence in colonialism. A rigid adherence to body-color epistemology can severely limit an anti-racism theorizing because, as Spivak explains, it:

¹ Gramsci has originally coined the term 'subaltern' in order to address the economically dispossessed, and today Ranajit Guha reappropriates Gramsci's term in an effort to locate and re-establish a voice or collective locus of agency in postcolonial India. In her essay "Can the Subaltern Speak?", Spivak acknowledges the importance of understanding the 'subaltern' standpoint but also criticizes the efforts of certain subaltern studies emphasis towards creating a 'collective voice' through westernised mediating practices (see Chronaki, under publication).

'...obliges us to ignore the fact that in countries which are recognized as Third World countries, there is a great deal of oppression, class oppression, sex oppression, going in terms of the collusion between comprador capitalist and that very white world. The international division of labor does not operate in terms of good whites, bad whites and blacks. A simple chromatism obliges you to be blind to this particular issue because once again it is present in excess. I was trying to show how our lives, even as we produce this chromatism discourse of anti-racism, are being constructed by that international division of labor, and its latest manifestations were in fact the responsibility of class-differentiated non-white people in the Third World, using the indigenous structures of patriarchy and the established structures of capitalism. To simply foreclose or ignore the international division of labor because that's complicit with our own production, in the interests of the black-white division as representing the problem, is a foreclosure of neo-colonialism operated by chromatism race-analysis (cited in Harasym, 1990, p. 126).

Coming back to the lecture, although Danny Martin is not entirely satisfied with a 'factory image' of the university and of university mathematics education, and with respect to the highly contested meanings invested in words such as highly skilled, well educated, democratizing, race, racism and racialism, his work reveals a determination towards unraveling the structural constraints and affordances that could transform university mathematics education into a social justice project. A social justice project that would include (instead of exclude) marginalized minorities within US context such as black people (African, Latino or Indian American), and a social project that would create a dialogue amongst 'we' and 'others' aiming to bridge inequalities. But, at this stage, one needs to pose and think: Whose interests should that 'social project' serve? And, who counts for its success? And in what measure or whose's measure? In other words: Do all black people should want to be included in the same social project? Do they all perform the same politics? Do all black people favor a similar agenda for their mathematics education? Taking into account Spivak's critique of (post)colonial discourses of anti-racism one needs to re-consider not only the colonial (and postcolonial or global) order, but also the indigenous structures of patriarchy and capitalism which affect epistemological assumptions of subject agency and knowledge politics as they are performed at the level of curricula planning and implementation.

During the last three decades mathematics education is heavily concerned with how issues of multiculturalism and multilingualism affect access to mathematical sciences. However, we tend to forget how notions of cultural and linguistic diversity are being inscribed in bodies -bodies with flesh and color but also bodies with history and agency. Bodies have been largely naturalized and silenced. Bodies could be not only numbers of black people, but full-fleshed subjectivities. Instead of trying to overcome 'complexity' at the expense of a more generic language that treats 'body' as insignificant or easily replaceable by 'language', 'symbolism', 'color' etc., we could place more emphasis on 'reading' the body as framing a multiplicity of materialities, meanings and ethics. Theorizing the 'body' has been an important endeavour in the fields of philosophy, cultural studies and feminist theory, and the 'body' metaphor can be utilized to enable us imagine alternative ways on how mathematics, as school technoscience, becomes recontextualised in education (see Chronaki, 2008).

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