REPRODUCTION AND DISTRIBUTION OF MATHEMATICAL KNOWLEDGE IN HIGHER EDUCATION: CONSTRUCTING INSIDERS AND OUTSIDERS

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The design and conduct of calculus courses has been and is an object of curricular debates and reforms. By the reconstruction of the establishment of the mathematical sub-area now called 'calculus' and its fundamental theorem as a piece of institutionalized mathematical knowledge for the purpose of its reproduction, we reconsider the notion of knowledge recontextualization within the field of knowledge production by showing that standardization of knowledge evolves in a dynamic relationship with its production. Interviews with mathematicians suggest that they, as teachers, create different recontextualization principles. In undergraduate teaching calculus they suggest to include the criteria of the field of knowledge production (e.g. proof) for the future 'insiders', while for those who will not pursue a career within this field, the 'outsiders', the criteria change towards computational efficiency.

INTRODUCTION

Calculus courses have a prominent position in undergraduate teaching in a diversity of academic areas. Steen (1988, p. xi) even claims that "calculus is a dominating presence in a number of vitally important educational and social systems". The design and conduct of such courses has been and is an object of curricular debates and reforms. As an example, in the 1980's there was a public debate in the U.S.A. about the need to improve a situation described as problematic with far reaching social and economic consequences:

> Nearly one million students study calculus each year in the United States, yet fewer than 25% of these students survive to enter the science and engineering pipeline. Calculus is the critical filter in this pipeline, [...] . The elite who survive are too poorly motivated to fill our graduate schools; too few in number to sustain the needs of American business, academe, and industry; too uniformly white, male, and middle class; and too ill-suited to meet the mathematical challenges of the next century (ibid.).

The outcome of the debate was the 'calculus reform', based on the insertion of applications and use of technology, which amounts to a weakening of the classification of the content. Similar debates are currently taking place in several countries. These debates bring to the foreground the issue of the institutionalization of knowledge for the teaching of mathematics in higher education: how is the classification established and who gains access?

By our case study of the emergence of the delineated sub-area that came to be called 'calculus' we show that the standardization of this mathematical sub-area along with its concomitant knowledge claims evolves in a dynamic relationship with its production. The historical study also points to the emergence of an independent recontextualization field for higher education out of the field of production. Interviews with researching mathematicians, some of whom are also teaching calculus courses, display that they suggest different recontextualization principles for different groups of students. This differentiation also emerges in calculus textbooks.

RECONTEXTUALIZATION IN TERTIARY EDUCATION

When discussing the structuring of pedagogic discourses, Bernstein (1990) describes a recontextualizing context operating "between" a primary context where the production of knowledge takes place, and a secondary context of knowledge reproduction. The latter is divided into four levels, i.e. pre-school, primary, secondary, and tertiary. Fields structured by the recontextualization context are defined by positions, agents, and practices, whose function is to "regulate the circulation of texts between the primary and secondary contexts" (Bernstein, 1990, p. 60). By a "principle of decontextualizing", this process of recontextualization changes the text through a delocation followed by a relocation subordinated to the rules of the field of the relocation. Once the decontextualizing principle has regulated the new ideological position of the text, a second transformation is taking place within the field of reproduction in the pedagogic process of teaching and learning.

Bernstein's theory of the pedagogic "device" is mainly concerned with the recontextualization of knowledge into the *school* curriculum, its functioning in the production, distribution, and reproduction of official knowledge and its relationship with structurally determined power relations. For the school curriculum, the discourse from the field of knowledge production (mainly universities) is recontextualized by agents in the Official Pedagogic Recontextualizing Field (OPRF). Other discourses, in particular those in the Unofficial Pedagogic Recontextualizing Field (UPRF) are also recontextualized (Bernstein, 1996). In the process ideology comes to play.

At institutions of higher education it is hard to identify a distinct unofficial recontextualizing field in times where there are no reform movements, and a weak influence of an official recontextualizing field on the curriculum specification; usually there are only general rules for carrying out examinations at universities. There are no clearly delineated recontextualizing fields, but individual recontextualizing agents, who are sometimes at the same time researchers. This situation has been observed by Bernstein (1990, pp. 196-198):

> The recontextualizing field brings together discourses from fields which are usually strongly classified, but rarely brings together the agents. On the whole, although there are exceptions, those who produce the original discourse, the effectors of the discourse to be recontextualized, are not agents of its recontextualization. It is important to study those cases where the producers or effectors of the discourse are also its recontextualizers.

A question to ask is therefore how the process of institutionalization of knowledge in a specific field of knowledge production is affected in cases where its members act as its recontextualizing agents. How does this situation affect the recontextualizing principles? We start with the suggestion that the discourse in undergraduate mathematics teaching is a pedagogic discourse. That is, there operate criteria of evaluation that differ from the ones operating in the activities of researching mathematicians. These criteria, together with the principles of selection, organisation, sequencing and pacing also contain a model of the learner, the teacher, and their relationship. For example, it is demonstrated by Bergqvist (2006) how success on exams in calculus courses in Sweden requires only memorizing of rules and examples. The pedagogic discourse may differ for students aiming at a career within the field of knowledge production and students enrolled in less specialised study programs, but at many places students from different programs are put in the same courses for economical reasons.

RECONTEXTUALIZING THE CALCULUS

The development of the Fundamental Theorem of Calculus (the FTC) changed the classification of sub-areas as it linked integration and differentiation. We are interested whether the recontextualization of the calculus keeps the classification or not and whether there is a dynamic between recontextualization and production in its historical development. In addition, we ask how members of the field of knowledge production see this area when they act as agents of recontextualization. For this purpose, we re-address the set of interview data and the outcomes of the historical investigation from Klisinska's (2009) study about the didactic transposition of proof.

The standardization of calculus and its fundamental theorem

The development of the statements connected with the FTC, which gave rise to a new classification of knowledge that became institutionalized, was studied with reference to original works of researchers and classical works about the history of calculus. As indicators of institutionalization we considered the reference to a sub-area or to a proposition with a common name as well as textbook or handbook appearances. Our study of the propositions related to the FTC and of the names used for basic concepts in calculus covered well-known early textbooks. As there was no independent recontextualizing field, we separated 'textbooks' from research publications by their intention to address an audience with less specialized knowledge in the area of knowledge to which the sub-area under consideration belongs. The use of a shared name (in some variation) for the theorem was a criterion for the selection of later textbooks. For the more recent textbooks, the choice resembles a "longitudinal cut" with some examples from different decades and from different countries. Only the formulation of the FTC and its proof were investigated.

Classical outlines of the history of mathematics commonly trace the ideas of calculus back to mathematicians in ancient Greece [1]. It is also common to refer to Leibniz and Newton as "inventors" of the modern calculus in a personalised history of mathematical "heroes". However, Leibniz and Newton did not invent the same calculus, and did not set out calculus as a well-defined sub-area of mathematics as they differed in problems studied, approaches taken, and methods and notations used (Boyer, 1959; Baron, 1987).

Calculus changed a lot before developing into the form that is presented in introductory courses, especially in terms of the changing criteria for the field of knowledge production (formalization and standards of proof). The early development of the limit concept was crucial for the standardization of the calculus. By using limits as the basis for definitions, Cauchy's work established new criteria. The collection of useful methods was integrated by definitions and proofs. These were complemented later in the 19^{th} century with the formal ε-δ definition of limit by Weierstrass, the definition of the Riemann-integral and a set theoretic definition of function. The recontextualized versions of the elements from these different areas make up the calculus in today's undergraduate teaching. In this development, standardization of knowledge for the purpose of teaching was one motive for the changing criteria. In the historical development, it is not easy to differentiate between criteria for the field of knowledge production and for its reproduction. Universities became the centers of both mathematical training and research, which led to the development of pure mathematics as an independent field (Jahnke, 2003), with internally socially shared knowledge codes for legitimate productions.

While the first developments in calculus were communicated entirely within the field of knowledge production through personal communication, soon textbooks for the wider distribution of calculus appeared. The first printed textbook in differential calculus appeared in Paris in 1696, by de l'Hospital with the help of Johann Bernoulli. From the introduction (1716 edition) it becomes clear that the name 'Integral Calculus' ["Calcul integral"] was already in use. Thus, by having a specific name it had gained an 'official' status as a classified part of knowledge to which one could easily refer. However, what was signified by this name changed considerably.

The *École Polytechnique* in Paris was established to increase the number of engineers needed to maintain the new French Republic. The school was kept after the counterrevolution to serve the military. That mathematical knowledge was considered important for these purposes, is apparent from the admittance rules:

> Les élèves n'ayant obtenu leurs admission qu'après avoir satisfait à un examen sur l'arithmétique, les éléments de la géométrie et ceux de l'algèbre, c'est état de leur instruction dut être pris pour point de départ, et il fut établi que les connaissances mathématiques enseignées a l'Ècole comprendraient l'*analyse* et la *description graphique des objets* (Fourcy, 1828, p. 42).

Cauchy's *Cours d'analyse* from 1821 and *Résumé* from 1823, written for The *École Polytechnique*, were the first textbooks in which calculus appeared as an integrated body of knowledge with clear borders towards other mathematical areas. It included a proof of a proposition that looks like what now is called the FTC. We interpret the textbook as an attempt to provide access to a knowledge code promoted in the field of mathematical knowledge production. There was no different code for the initiation into the fields in which the knowledge was supposed to be applied.

Also in other early textbooks the propositions related to what is now called FTC are not named, but in the *Course d'analyse mathématiques* from 1902 by Goursat, translated from French into English already in 1904 and widely spread, the "fundamental theorem" refers to the fact that "every continuous function f(x) is the derivative of some other function". Later in the book this name is used also in the context of complex analysis. In the textbook *An introduction to the summation of differences of a function* by Groat, printed in 1902, the expression "the fundamental theorem of the integral calculus" appears, as well as the more short "fundamental theorem". In *The theory of functions of a real variable & the theory of Fourier series*, published in 1907 by Hobson, one chapter has the title "The fundamental theorem of the integral calculus for the Lebesgue integral". That the name of the theorem serves as a chapter title as well as extended to a more general application indicates a strong level of institutionalization. Wiener refers several times to "the fundamental theorem of the calculus" in *Fourier transforms in the complex domain* from 1934. That this name became standardized is evident from the classical book *What is mathematics?* from 1941, where Courant and Robbins use the chapter title "The fundamental theorem of the calculus", and in a simplification of the history write (p. 436):

> There is no separate differential calculus and integral calculus, but only one *calculus*. It was the great achievement of Leibniz and Newton to have first clearly recognized and exploited this *fundamental theorem of the calculus*.

The textbooks mentioned all contain a proof of the FTC as criteria for its status as legitimate knowledge, but there were also textbooks published at the beginning of the 20th century written for an apparently growing body of non-academic readers, as for example *Calculus made easy* by Silvano Thompson from 1910, which recontextualizes computational algorithms and mathematical notation in everyday discourse.

The development of the calculus shows that the process of institutionalization of a body of knowledge has to be seen in relation to the practices of its circulation and reproduction; there is a dynamic between the fields of production and reproduction. For example, reference to Cauchy's scholarly work is commonly made by drawing on his textbooks. That his textbooks became popular and his exposition of the calculus was generally adopted (Boyer, 1959) can be explained by the combination of its influence on the field of knowledge production through applying a knowledge code that became internally socially shared, and the relative autonomy of teaching that accounts for its circulation. In this case the fact that a producer is at the same time a recontextualizer affects both, the unmediated and the pedagogic discourse.

An interview study: producers as recontextualizing agents

For the interview study, the principle of selecting extreme or atypical cases was used in the selection of eleven mathematicians from universities in Canada and Sweden. They have diverse backgrounds in education in different countries and work in different, mostly unrelated, areas of research in mathematics, and have varying teaching experiences. In the interviews the first question aimed to reveal their most spontaneous conceptions of the FTC. Then they were presented eleven examples of formulations of the FTC from textbooks from different countries and periods in history, and asked which they considered closest to the FTC as they understood it. Other questions raised issues of significance, meaning and reference of the FTC, as well as issues linked to the teaching of the theorem.

When answering the first question, six of the interviewees used the word ‘inverse’ when describing the FTC, saying that differentiation and integration are ‘inverse processes’, ‘inverse operations’, or simply ‘inverses’. In only one interview it was pointed out that the integral should be defined as a limit of a sum, and not simply as an antiderivative, to make the FTC interesting. Three persons mentioned versions of the FTC for multivariable functions or with weaker assumptions. In general, the interviewees had a quite informal approach to the formulation of the theorem without providing assumptions or a remark about possible parts of the theorem. Only those with the experience in teaching calculus, were more careful. The criteria for the pedagogic discourse seem to be more strict than those used in these conversations.

The big variation of the answers to the question where eleven formulations of the theorem were given reveals substantial differences in how the mathematicians evaluated the given formulations as matching their own views. Reasons given for accepting or not accepting a formulation often indicated that a personal view was expressed.

When discussing issues related to teaching, different ‘versions’ of the calculus and of the FTC for different groups of students were mentioned by nine interviewees. Some point to a difference for students in different tracks, others differentiate between levels of calculus courses, and some refer to different needs and dispositions of students in the same course:

> I think you have to adapt it [teaching] to the kind of students you are working with. If you have an honours class [ ] you can go a bit further. But really, if they had as good understanding of the basic principles as Newton and Leibniz did, that would be extraordinary (I6).
>
> We do it [the FTC] for honours class [...] one teaches the FTC in a way by attempting to I don’t want to say trivialize it so much as reduce its scope and make it manageable (I4).
>
> I am saying that it is different how I am teaching at the first year calculus and it is different when I have a real analysis, where I am proving the theorems. Very big difference. Completely different kind and way of thinking about the theorem (I10).
>
> It depends. We are talking about the first course, I guess. If they understand in an intuitive way that will be enough for them (I8).

In the first course the FTC is presented as “a recipe” for how to find the function that satisfies some requirements, followed by a number of almost the same exercises. Then, “nothing is beautiful with fundamental theorems of calculus”:

> Why we should explain for first year students that this theorem is beautiful? Maybe we should not? Maybe it is a recipe and that's all. And why we should, why we should force people outside the university to understand that some theorems are beautiful? Should we? I don't believe so (I10).

Engineering students are by some interviewees seen as a group who do not have to gain access to the criteria of the field of knowledge production:

> And for engineering students: It's just engineers. They would never think that something is beautiful in mathematics (I10).
>
> Here at the University of Technology we usually don't spend much time on the theoretical parts of the foundation of the subject (I11).

A distinction between students who might grasp the FTC and its proof and those who would not was commonly made in terms of their working habits and intellectual dispositions:

> I feel I could do more [...] if there was some very good student who gets A+ no matter what, if there was some students who don't work and get F no matter what. But there are some students in between then I could help (I3).
>
> With a bright bunch of students, I would like to go into it a bit more hoping that some of it will stick so that when they come back to it later on they'll have something to build on (I6).
>
> And then I would prove it but not according to Newton [...] I would use a straight forward proof with whatever; the limits and you know the mean value theorem [...] And maybe, maybe some people will understand the proof, maybe (I2).

Thus, in the interviews a distinction was made between different students, which was crucial for the way they should or can be taught. Most beginning students and engineering students, as well as those described as not having the right dispositions, were seen as needing mainly access to the computational aspects of the FTC and of calculus. In contrast, for special groups of students and for the more advanced course, the proposed discourse could be described by a knowledge code belonging to the field of production. In their reasoning the interviewees rely on common sense discourses about the needs and dispositions of the students rather than on a recontextualised version of a discourse from pedagogy or psychology. In an interview question about how they would attempt to explain the FTC to beginners (where a situation was outlined with initial work on a problem using a velocity graph), the uniformity of the answers shows that the proposed pedagogic discourse remains at an informal level, relying on visual impression, their "intuition", and on approximations not described by means of specialised mathematical language. This can be seen as an expression of a view that the students imagined here, need not be invited to take part in the internal code for mathematical knowledge production. These answers, as well as views expressed in the previous quotes, represent an insider-outsider perspective where only a small community is seen to be able to take part in and get something out of theoretical work in mathematics, while only technical skills need to be taught to the ‘outsiders’.

OUTCOMES AND DISCUSSION

According to Bernstein (1990, p. 60), there are at least four recontextualizing fields involved in the shaping of the school curriculum: official educational authorities, university departments of education, specialized media of education, and fields not specialized in educational discourse but able to influence it. The data presented in this paper suggest that when the distance between producers and transmitters of knowledge is reduced the influence form these fields is also reduced.

However, an important outcome of our findings is that the fact whether an agent from the field of knowledge production at the same time is a recontextualizing agent as such does not make a difference for the criteria that are transmitted in pedagogic discourse. There is a difference in this scenario in the historical and in the present context of higher education. While in the historical context the criteria for the two discourses (the unmediated and the mediated) match or are developed in dynamic relationship between producers and transmitters, in the present context the mathematicians suggest a switch as soon as pedagogic discourse is at issue. That is, there seems to be a recontextualizing principle that constitutes insiders and outsiders.

Another outcome in relation to the process of knowledge recontextualization is the fact that as textbook writers for higher resp. tertiary education, researchers contributed and still contribute to the institutionalization of the mathematical sub-area of calculus. In the historical account some examples were given where a person relocates outcomes of her/ his own knowledge production into the field of reproduction. When producing a textbook, the text was by other scholars brought back to the field of knowledge production, contributing both to the standardization of knowledge and to further developing the criteria for knowledge production. At present time when persons from the field of production of mathematical knowledge teach undergraduate calculus courses, the knowledge to be transmitted is not resulting from their own knowledge production. As recontextualizing agents for tertiary mathematics education, their recontextualizing principles seem to construct ‘insiders’ and ‘outsiders’, based on a metaphor of participation for the former, responding to the interests internal to the field of knowledge production, and on the alleged applicability for the latter, responding to perceived external interests (from study programmes in which mathematics is a service-subject) only in the utility of mathematics. Some rely on a discourse that naturalises the distinctiveness and inaccessibility of mathematical knowledge.

It is not evident why a massification of higher education should lead to the abandonment of transmitting the code for knowledge production (by leaving out the proof, for example). This might as well be a coincidence of two separate developments. However, the move is vindicated by an ideology of the exclusiveness of the code (by mathematicians) and fed by the demand for applicability of academic knowledge on the side of technical instrumentalists who become more influential in tertiary education. Arguments of applicability of the knowledge for the outsiders can be interpreted as a means to make the students complicent of the strategy. On the other hand, the computational version of undergraduate calculus courses can also be seen as an outcome of a progressive agenda attempting to make it more accessible and relevant. In the 'calculus reform' in the U.S.A. that made calculus more applied and computer based, reference was commonly made to its use value. "Calculus now is more important than ever", because:

> the most serious reality we face today is the need to harness science and technology for economic growth. And harnessing science and technology for economic growth means harnessing calculus. (Steen, 1988, p. 6)

It is worth considering that the power relations in the structure of academic fields related to mathematics are not aligned with the simple distinction between pure and applied mathematical sciences. The distinction has always been a battleground for ideology. It is not clear that the cultural capital of being educated as a theoretical mathematician is easily exchangeable into economic capital. From this perspective, there is no need to restrict access to studies in pure mathematics. Keeping this classificatory principle and the symbolic capital of being a theoretical mathematician still remains possible by means of policing, most prominent with strategies that draw on knower characteristics and pointing to the distinctiveness of the objects of interest in the field of knowledge production. Some of the interviewee's comments might be interpreted in this light.

What is transmitted to the insiders and also is reflected in the classical books about the history of calculus, is based on a view that the acceptance and rejection of mathematical theorems is primarily, or even usually, a matter of evidence or reason that follow an internal logic of rational evolution of knowledge towards more "exactness". The concomitant view of the development of mathematical knowledge corresponds to 'Science as rational knowledge' in Callon's (1995) classification of models that account for the dynamics of science. In this model the actors are the researchers themselves restricted to their role as researchers. According to Callon, this model inherits a "tragic beauty" in that "it is the scientists and scientists alone who have to choose which statements to preserve and which to discard" (p. 36). In relation to the account above, the tragic beauty is its ignorance of the social history of mathematical knowledge.

A CONCLUDING REMARK

The study and our discussion make it obvious that the calculus and the FTC as part of a modern undergraduate programme can be analysed as being purely a social construction. The above discussion points to the space left for ideology in this construction. However, the knowledge transmitted in such a curriculum is not fully arbitrary either. One could exchange the example from Boyle's Law to the FTC in

Young's (2005) observation (and perhaps also exchange "the Chinese" into another cultural group):

> However, what is distinct about the formal knowledge that can be acquired through schooling and that therefore needs to be the basis of the curriculum in any country is (a) the conceptual capacities it offers to those who acquire it, (b) its autonomy from the contexts in which it is developed (the Chinese are interested in Boyle's Law but not in the gentry culture of which Boyle was a part). (p. 14)

NOTES

1. See Juschkewitsch and Rosenfeld (1963) for alternative interpretations of the historical roots of calculus.

REFERENCES

Baron, M. E. (1987). *The origins of the infinitesimal calculus.* New York: Dover.

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics in Sweden. *Journal of Mathematical Behavior, 26*(4), 348-370.

Bernstein, B. (1990). *The structuring of pedagogic discourse. Class, codes and control: Volume IV*. Routledge: London.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique.* London: Taylor & Francis.

Boyer, C. B. (1959). *The history of the calculus and its conceptual development.* New York: Dover Publications, Inc.

Callon, M. (1995). Four models for the dynamics of science. In S. Jasanoff, G. Markle, J., & T. Pinch (Eds.), *Handbook of science and technology studies* (pp. 29-63). Thousand Oaks, CA: Sage Publications.

Fourcy, A. (1828). *Historie de l'École Polytéchnique.* A Paris, Chez l'Auteur, A l'Ecole Polytéchnique.

Jahnke, H. N. (Ed.). (2003). *A history of analysis*. Providence, RI: American Mathematical Society.

Juschkewitsch, A., & Rosenfeld, B. (1963). *Die Mathematik der Länder des Ostens im Mittelalter*. Berlin: Deutscher Verlag der Wissenschaften.

Klisinska, A. (2009). The Fundamental Theorem of Calculus: A study into the didactic transposition of proof. Dissertation, Luleå University of Technology.

Steen, L. A. (Ed.). (1988). Calculus for a new century: A pump not a filter. Papers presented at a colloquium (Washington, October 28-29, 1987). *MAA Notes, 8*.

Young, M. (2005). The knowledge question and the future of education in South Africa: a reply to Michelson's 'On trust, desire and the sacred: a response to Johan Muller's Reclaiming Knowledge'. *Journal of Education*, *36*, 7-17.