EXPERIENCING THE SPACE WE SHARE

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This paper offers some theoretical and practical reflection on how we share geometry and make it part of our lives and in so doing link to a shared heritage. It draws on Husserl's speculations on how geometry originated but how then it increasingly became seduced by language as a result of human attempts to capture and share its concepts. After discussing work by undergraduate students engaged in body movement exercises and other geometry it considers more generally how the truth of mathematics relates to its representation in cultural forms.

INTRODUCTION

In geometrical study we are confronted with ideal mathematical objects that nevertheless in some respects, very often, are also a function of their cultural heritage, that is, of their human construction, with respect to configurations observed in the physical world. Husserl (in Derrida, 1989, p. 173) argues that to understand geometry or any other cultural fact is to be conscious of its historicity, albeit implicitly. I take this to mean that ideal objects can only ever be accessed through technology or perceptual filters that are both time and culture specific where those technologies or filters display some historical continuity, revelatory of how they emerged from earlier manifestations. Yet our very selves have been created in a world that has a physical organisation and analytical heritage consequential to a long history of geometrical awareness. How do I fit in to the social world through participation in shared ways of organising the world? Our perceptions of the world are inevitably processed through aspects of this heritage. We cannot be geometrically naïve insofar as our subjectivity results from identifications with this shared heritage. Our physical experiences are processed through that vocabulary of set moves and analytical strategies. We have learnt some of these things in school, or through everyday life experiences, but in a fundamental sense they are also part of us, contributory as they were to our very formation. This paper offers some theoretical and practical reflection on how we share geometry and make it part of our lives and in so doing link to a shared cultural heritage.

THE ALGEBRAICISATION OF GEOMETRY

An early part of my work in mathematics education revolved around an interest in the work of Caleb Gattegno (e.g. 1988) who I had the pleasure of meeting on a few occasions. An aspect I remember particularly well is Gattegno's notion of the *algebraicisation* of geometry, or how geometrical experience is transformed, perhaps compromised, by an insistence on it being converted to symbolic form. I remember Gattegno talking about a baby pointing to a fly walking across the ceiling. Each fly position on a continuous path was associated with a particular (discrete) arm position.

But a key concern was that in school, geometrical experience generally gets converted into algebraic experience and that this results in a loss. Whilst not in anyway detracting from the importance of algebra in emergent mathematical understanding, Gattegno was keen to educate the "whole brain" where experiences of the continuity of geometry were more often fore-grounded in classroom geometry. This notion of geometry being compromised through its algebraicisation will underpin the discussion that follows. Before moving to some theoretical discussion I shall describe some practical work with students.

SHAPING UP

To explore these issues I shall recount some fun that I had with a group of students recently as a result of pursuing my interest in how we apprehend geometric phenomena. I have a weekly session with a group of first year undergraduate students (aged from 19 to undeclared middle age) preparing to be teachers of mathematics in British secondary schools. In one session we tried out various activities in which various instructions were followed that resulted in the students walking the loci of certain geometric objects: *Walk so that you are always equidistant from your partner who is standing still* (circle). *Walk so that you are at all times equidistant from your stationary partner and a wall* (parabola). *Now get in to groups of three where there are two people each standing still at some distance apart: Walk so that you remain equidistant from both partners. Walk so that you remain twice as far from one partner as you do to the other. Walk so that you can still touch a piece of loose string held firmly at each end by your two partners*. (Photographs will be available for the presentation.) In setting the task on the first occasion for some time I had some expectations, based on my own hazy memories, of some of the figures that would be generated. But given the zest and determination of this particular group of students, explorations went further than expected with some very familiar figures emerging from unexpected directions. And for the students there appeared to be a very real sense of acting out shapes and feeling them before recognising them as more or less familiar, yet perhaps now being understood differently given the novelty of the approach. The ideality of any given object cannot be apprehended in an instant, or rather, that ideality can give forth its properties in many ways, such that there comes into being a perceptual architecture that supplements the ideality with a necessarily cultural layer.

In steering a particular course a student had to stay twice the distance from one partner as she was from the other. As I observed I had some vague memory that a hyperbola might be the result. Yet it eventually became clear to those present that there was just one curve and that it seemed to be closed. Yet the relative imprecision of the body movements resisted anyone achieving complete certainty as to whether it was closed and if so if its regularity suggested a circle or an ellipse. We all experienced glimpses of possibilities but remained unsure if our conjectures could be confirmed without more sustained analysis using drawings or calculations. A conceptual layer was needed to confirm intuitive assessments. But these initial moments provided exciting insights into emergent understandings, all the more intense for the person attempting to walk the path of the curve, experiencing the mathematical rules through actual bodily movements. For others there was the challenge to assume some specific perspective on the emerging locus. For the other partners this was from a fixed point.

As all of these activities involved walking on the floor the shapes constructed were all two-dimensional. Yet I was firmly caught out by one interpretation that both surprised and delighted me. With an instruction where the moving player was required to be equidistant from two stationary partners I had anticipated a straight line but surprisingly to me the moving partner, Sally, decided to stand on a chair and then on a table between her two partners. A third dimension was brought in to play where for any given distance a circle in the third dimension could be imagined. This radical departure led to an unexpected exploration later on for all of the other erstwhile two-dimensional shapes.

Together such activities provided the students with experiences of moving in space according to more or less precise instructions, more or less drawing on conventional geometrical terminology, such that continuous movement was associated with a sequence of discrete instructions. Yet like empirical science geometry comprises objects idealised by humans where the technology productive of those idealisms can never be fully separated except at the limit of our conceptualisation.

Back in the regular classroom and later at home subsequent attempts were made to capture the bodily movements in drawings and reflective writing and a new world of geometric figures were generated. Much work was carried out on the two-dimensional shapes. The mathematical objects were generally familiar once encapsulated but the routes to them made them seem somehow new, as though they were being encountered in a fresh way that made them seem different. And following the ascent of the chair and table, later developments considered how the various tasks could be extended in to three dimensions. Ellipses became eggs. Circles became balls. Lines became walls. And various bowls and saddles of infinite dimension and curious orientation also emerged. And in certain circumstances eggs could become balls or even walls.

WHAT OR WHEN IS A CIRCLE?

In a separate session several weeks later I asked the members of the same group to each write answers to the following questions without, in the first instance, sharing their thoughts with others: What is a circle? How do you imagine that circles were invented? (cf. Bradford & Brown, 2005) They then read out their thoughts for everyone to hear. Here are some of the results for the first question:

> A circle is a 2D shape, which starts and finishes at some point it is a continuous curve and has 360 degrees. Clockwise from the centre point to the curve is called the radius and the radius is the same distance to the curve all the way around the circle. We use the

> radius to calculate the area and the diameter, which is twice the radius gives us the circumference when multiplied by π.
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> A circle is a regular 2D shape, which has no straight sides. Every point on the circle is an equal distance from its centre point. This distance is called its radius. The distance around the outside (circumference) is known from the formula $2\pi r$ and the area from πr^2
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> Lots of coordinates plotted on a graph and when joined with a line it makes a circle shape. It has a centre point, and from the centre point to the edge is called the radius of the circle. Double the radius = diameter. The points can form an equation in the form $(x-a)^2 + (y-b)^2 = c^2$ where a, b is the centre point and c is the radius.

The students then speculated on how were circles invented:

> Circles were first invented by the Aztecs. They are widely regarded as the first astronomers of our time. They saw the shape of the moon and the sun and recreated the same image on the ground with sticks in the mud, which later became marks on walls like Egyptian hieroglyphs.
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> By God when he made the human eye – Ask him!
>
> In the days of caveman they decided it was easier than carrying certain objects to put them on a sledge type thing and pull them along. ... But when they travelled over gravely ground they realised the ground was assisting the movement. This gave them the idea of raising the sledge up off the ground and attaching large bits of gravel to the bottom. Over time they developed the axle helping the stones move and again over time the stones wore down to a circular shape.
>
> After Allah created the moon and sun they were observed by man and copied.
>
> Circles were invented when a man cut down a tree and noticed the shape of the stump was of a different shape and the logs it created were a different shape. He also noticed that it rolled easily enough and he realised this may be a good template for a new shape…

Quite apart from the humour these stories suggest some interesting social constructions. In particular, some curious historical perspectives are apparent. The definitions of circle are occasionally dependent on words or ideas derivative of circles. Indeed one of the descriptions cannot avoid using the word circle in the description of a circle. How might we have imagined circles without this linguistic and symbolic apparatus that is seemingly consequential to the supposed existence of circles? How in the present day might we see ourselves engaging with Husserl's quest to understand how geometrical configurations originated? Where and when could we possibly start? We could envisage extending the search to other mathematical objects, or indeed any empirically derived scientific object. And such an attempt would alert us to the cultural nature of each and every mathematical idea encountered in our mathematics educational quest, and of the cultural derivation of the framework that produces those ideas. Or do we encounter the situation in which some mathematicians suppose they can identify mathematical objectivity beyond culture and its history? And if we do encounter that situation how would it impact on our understandings of how humans apprehend mathematical phenomena? To what extent could one suppose a clear historical perspective on such concerns and how are such perspectives functions of particular linguistic constructions? History and our collective understandings of time are both linguistic constructions. Time is a function of the stories we tell about it (Ricoeur, 1984; 2006). People in earlier times did not understand history better than we do today. As an example, during a visit to an art galley in Venice my then seven-year old daughter Imogen was rather taken by Tintoretto's 16th century painting entitled *Creation of the animals*. But she was alarmed by an apparent omission: 'Where are the dinosaurs?' Her awareness of cultural history could detect the limits of Tintoretto's worldview that had been shaped by assumptions that have been revised in more recent years. After all dinosaurs, a twentieth-century human construction, were unknown to our earlier ancestors. Her brother Elliot, meanwhile, chipped in with a comment that he had not realised that God was a man. I speculated on the many ways in which cultural histories have been revised since the painting was created and thus on how individuals understand themselves fitting in to the world we inhabit. History and histories are revisable, for individuals and for cultures, yet residues of previous eras, and earlier conceptions of those eras, remain locked in to the genesis of later formulations. Circles are now a function of contemporary thinking and perhaps cannot any longer be understood independently of that cultural baggage. But was that ever possible? And if so, in which ways could this be possible? We have also changed as humans, such that those earlier humans could not have known circles in contemporary terms, and those earlier humans and their apprehensions could not be processed in contemporary terms. And so many other mathematical constructs would have histories and meanings rooted in different, more or less recent, intellectual circumstances. The growth of mathematical knowledge for example has much to do with market forces and how universities and individual mathematicians get funded to focus on different types of mathematical knowledge such that new and existing mathematical phenomena derive their meanings from how they now relate to this ever-expanding mathematical knowledge.

And as with the group of impressive but maybe fairly typical trainee teachers introduced above, specialising in secondary mathematics, we might speculate on how other mathematical constructions are held in place by incomplete memories of school learning and how those areas or gaps are manifested by teachers in schools working with children who, like all of us, will have specific and restricted historical and mathematical conceptions in some areas of their knowledge.

THE SEDUCTION OF LANGUAGE

Husserl sought to enquire how geometry came into being and concluded that without the anchorage of words (that is, culturally specific constructs) it was quite difficult to conceptualise.

> It is easy to see that even in [ordinary] human life, and first of all in every individual life from childhood up to maturity, the originally intuitive life, which creates its originally

> self-evident structures through activities on the basis of sense experience very quickly and in increasing measure falls victim to the *seduction of language*. Greater and greater segments of life lapse into a kind of talking and reading that is dominated purely by association; and often enough, in respect to the validities arrived at in this way, it is disappointed by subsequent experience (Husserl, in Derrida, 1989, p. 165, Husserl's emphasis).

Husserl saw geometrical understanding as being linked to an implicit awareness of its historicity, which I see as pointing to the understanding being formed through the subject's constitution with respect to the historically derived, yet still forming, discursive environment. I sit on chairs, climb stairs, wash round dishes, ride on ferris wheels, travel on trains and fly in planes. Our bodies have learnt to function and know themselves in physical environments that result from culturally situated conceptions of geometry. Derrida himself posits *the* geometric or mathematical science, whose unity is yet to come, where "The ground of this unity is the world itself … the infinite totality of possible experiences in space in general ... To pose the question of this traditional unity is to ask oneself: how, *historically,* have all geometries been, or will they be, geometries?" (p. 52). The sum total of cultural knowledge about geometry remains incomplete, but "the infinite totality of possible experiences in space in general" could never be completed. And we cannot yet know, and never will know, how reliable an indicator current knowledges are of knowledges to come. Or more prosaically, we do not know how much school knowledge as currently defined prepares the pupil for the knowledge required in later life. Geometry as an ideal field is held in place by its cultural technology which doubles as a mode of access for those learning the subject. But this technology is culture and time dependent implying a two fold task for students - learning the culturality of mathematics for social participation in that era and also access to the ideality so often seen as key in mathematical understanding.

The stories we have learnt to tell of the world often sediment into fixities that have departed from the truth they sought to capture. The stories lose their zest. And as a result truth always escapes our grasp. This can be readily understood in the context of mathematics. The geometry of Galileo is still largely true, in a sense, but its present coexistence with string theory and other contemporary geometry redefines its relationship to mathematical universality, and how we understand it fitting in as it were, and how we ourselves relate to it. But at the same time Galileo was surely formalising much that had previously been known intuitively. He could not have been the first person to notice the phenomena that he described, but perhaps his encapsulation enabled alternative modes of noticing, that shaped later thinking. History has a tendency to organise previously intuited stuff – looking back on the past or the current to project into the future. Geometry, like much knowledge from the empirical sciences, comprises human constructions but these constructions do have a ring of truth about them.

Geometry gets converted (and perhaps compromised) into particular linguistic forms for accountancy purposes or formal recognition, such as tests/exams, but so too do we as students and teachers, since, for example, we are not teachers in ourselves but teachers subject to particular cultural specifications that restrict how others read our actions and indeed how we assess our own practice.

- What is lost and what is gained by maths being forced into descriptive categories?

(And in turn, a question asked less often in mathematics education research),

- how is the learner/teacher lost (or gained) in being read through descriptive categories?

And those descriptive categories cannot come clean.

- Mathematics is always polluted in its interface with humans as a result of a human need to mediate mathematical experience for the cultural existence of mathematics to be acknowledged, whether in humans theorising, as a manifestation in the physical world or as explicitly pedagogical form (Brown, 2001).
- And we as learners, teachers and researchers are also polluted since we similarly read each other and ourselves through descriptive categories that take us away from truth (Brown & McNamara, 2005).

DISCUSSION

How might a mathematical object be understood given its changing relations with the social apparatus that locates it? What alternatives might we have? What or how or when does a mathematical object signify? How do we understand the apprehension of such signification? Yet how different are mathematical objects to other objects? And how is it decided that certain objects are defined mathematically? Such questions are central to the task of mathematics education research. I have speculated on how notions of the circle, as an example of a mathematical concept, are developed, transmitted and transformed through the need to traverse cultural and historical perspectives. The objectivity of this mathematical concept was shown to be far from stable, although it would be difficult to achieve clear consensus on how mathematical objectivity is understood. If we take a circle as an example of a mathematical object, how might we understand its original conception as an object and how have apprehensions of circles evolved as circles acquired so much historical and cultural baggage as they have been progressively used more as elements in building constructions of the world around us? The original coining of the term circle to capture some apprehended aspect of the world has now become a common primitive in shaping the world thereafter. In Badiou's philosophy the term circle would originally have been "counted as a one" (e.g. the set of points obeying the relation obeying the relation $x^2+y^2=1$, or the points passed through by a boy on a roundabout) but thereafter became a member of other sets of objects (e.g. regular shapes {triangles, ellipses, squares, circles etc}) seen as making up the world and utilised in organising our apprehension of the world (e.g. Badiou, 2009). As proposed by one of my students, perhaps early man looked at the moon or the sun and saw the two objects displaying similar characteristics, characteristics that may have also been seen in other naturally occurring objects (for example, berries, oranges, eyes, etc). The similarity was eventually given a name, circle, or sphere. Yet uptake of such terms would be different across cultural groups according to how the terms intervened in everyday living or were included in the intellectual life of the cultures. And as different aggregations of such objects shape our wider apprehensions of life the formative impact of "circle" continues to evolve and operate in diverse ways. Yet increasingly such usage conceals its original historical contingency as an arbitrary construction from the past, more or less motivated by empirical observation, against which we could perhaps understand aspects of the wider world in a different way. As a wider example I am sure that many aspects of the mathematics produced by the ancient Egyptians retain validity today, yet the meaning of these valid elements now need to be put alongside more sophisticated or contemporary mathematics such as that produced by Newton, Einstein or Hawking. Any supposed universality of the Egyptian conceptions would be disrupted by later developments.

This later concern opens the wider question of how do we define the limits of mathematics and how does the assumption of any frame result in an adjustment to the meaning of the constituent terms? It would indeed be difficult to achieve consensus on how such limits could be drawn. And in the analysis so far mathematical meaning has been considered as though this could be decided by being clear about definitions of what constitutes mathematics. Yet the meaning also depends on how it is apprehended. People are diverse in character and any individual can be understood through a variety of social filters. And we need to make a further decision as to whether we privilege the individual or the social filter as the frame of analysis. Moreover this decision introduces yet a further layer whereby we ask the question as to where the meaning is located, in the object, in the apprehension (however that is located) or somewhere between.

And is a circle a good example of mathematical objects more generally? Most people can immediately apprehend a circle. It is a widely recognised cultural object. Yet there would be a considerable variety of meanings brought to it as indicated. But many mathematical objects or entities or definitions require rather more specialist training to even apprehend their existence, let alone their finer qualities. Depending on how we make sense of the mathematical field the conception of a mathematical object could be understood as being represented in many entities; writing a quadratic function, producing a set of axioms, following a statistical procedure, demonstrating rotational symmetry, showing topological equivalence, etc. As an example, a mathematical generalisation reached through some investigation could be thought of as a mathematical object. Mathematics education research, especially where it is conceived of as a corrective to a fault in the system that has produced hordes of failing students, is in the business of enabling students to better apprehend and use socially derived mathematical apparatus and draws on social interactive processes that locate but also transform the objects concerned. Given this focus mathematical objects are recast as pedagogical objects that result in the specifically mathematical definitions becoming implicated in socially governed processes. The meaning of the mathematical objects is necessarily a function of the relationships within such social settings. The truth of mathematics is constructed, preserved and disseminated through apparatus that is necessarily cultural and hence temporal. The truth of mathematics is never substantial except in its cultural manifestations (Badiou, 2009), manifestations that derive from and feed history but never fully locate truth. Geometry as a field comprising ideal objects is held in place in the collective memory through the technologies that have been developed to access it, or perhaps in the school context, those technologies used to formally assess understanding of it. Truth, in any eternal sense, is beyond that technology, yet accessed through this continually evolving technology. Truth is located and to some extent preserved thorough its crude indicators but potentially at a cost to the profundity of the understanding achieved.

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