PHILOSOPHY OF MATHEMATICS IN THE MATHEMATICS CURRICULUM. QUESTIONS AND PROBLEMS RAISED BY A CASE STUDY OF SECONDARY EDUCATION IN FLANDERS.

Karen François and Jean Paul Van Bendegem

Vrije Universiteit Brussel

This paper presents the results of an empirical investigation into the mathematics curriculum of secondary education in Flanders. The research question asks whether there is room for philosophy of mathematics within the curriculum. The method used was a screening of the curriculum with a focus on the philosophical parts. As a result we can present some initial philosophical concepts which are formulated in the general objectives of the curriculum. First we want to give an insight into the rather complex structure of the educational system in Flanders. Secondly, we want to clarify the different levels at which the mathematics curriculum is described and set out. Thirdly, we shall present the initial findings of our research. Finally we shall formulate some suggestions for a philosophy of mathematics and we will raise some questions.

INTRODUCTION

This research takes place in an inter-university research project (IUAP) in which we are looking for the relations between sciences, society, politics and the democratic constitutional state. The project has the title: "*The loyalties of knowledge. The positions and responsibilities of the sciences and of scientists in a democratic constitutional state*." Within this project, one of the key questions is the place of mathematics in this overarching alliance.

The first question is how mathematical knowledge is reproduced in our society, how mathematics is handed down from generation to generation. Obviously, education is an important way, if not one of the most important ways, to reproduce knowledge in our society. So, we transform the first question to ask whether there is room for a kind of philosophical reflection within the mathematics course. The question is not concerned with the implicit philosophy of mathematics, which is of course embedded in the curriculum, but with the way in which there is explicit room made for a philosophy of mathematics. The two questions are bound together and obviously, an answer to the second question gives us a partial answer to the first question. We shall return to this theme at the end of this paper but first, we need to explain the organisation and the structure of the Flemish education and school system.

ORGANISATION OF THE FLEMISH EDUCATIONAL SYSTEM

We are speaking of Flemish education, not only because of the difference in language in which Flemish and Walloon pupils are taught, but because of the completely separated school systems. Belgium is a federal state with three communities (the Flemish, the Walloon and the very small German community). Educational matters are under the control of these communities. Each community has the authority to decide on its own educational system and structures. It is the Flemish Community and more specifically, The Organisation for the Development of Education, which develops the curriculum that will be enforced by the Flemish parliament.

Moreover, educational freedom is provided for by the constitution. This means that we have the provision of differing schools, namely public schools, subsidized private schools and subsidized community schools which are provided by the local government. Private schools, which are mostly Catholic schools, receive public grants for as long as they are able to meet the community standards. Private schools are extremely popular in Flanders and take up to 75,5% of the pupils. The French community has a more balanced position, where 49% of the pupils attend public schools, and some 51% chose private institutions. This is a typical situation in Belgium where Flanders is largely Catholic and Walloon is primarily secular.

The differing schools	**Public** schools	**Subsidized official** schools (at the level of the local government)	**Subsidized private** schools: **Catholic** schools
Ratio of the pupils	16.3 %	8.2 %	75.5 %

Table 1: Flanders Secondary Education

It is at the level of the community that curricula are developed and these are compulsory for all schools. This is rather new in Flanders where educational freedom is limited by the law of 1990. Due to the provision of educational freedom by the constitution, the three school organisations retained some freedom in the sense that they had the potential to develop their own curriculum, which is based on the curriculum of the Flemish government. This freedom can be used in the formation of their own pedagogical methods and didactics. They also have the potential to add extra objectives and attainment targets. The curricula developed by the various school organisations must be at the level determined by law. These developments are regulated by strict inspection by the government.

Due to this double system of freedom on the one hand and compulsion on the other, we have two levels of curricula: 1) the level of the community - the curriculum as strictly enforced by law and 2) the level of the authorities of the various school systems. Schools have to integrate the attainment targets in developing their own curricula. Within education, we have three levels: 1) nursery and primary schools where basic instruction of mathematics are taught; 2) secondary education where pupils are taught mathematics ranging from the basic skills which enable them to survive in our society up to (and in the higher levels) mathematics as the purest scientific discipline; 3) high schools and universities where students are educated so that they can become mathematicians and teachers in mathematics.

The development of the content of the teaching of mathematics is positioned on four levels: 1) the level of the community - the curriculum as strictly enforced by law; 2) the level of the authorities of the different school systems - the curriculum as accepted by the government; 3) textbooks which are based on the curriculum; 4) the teacher in the classroom who has a constrained freedom. Before a teacher enters his or her classroom to teach mathematics, much about the teaching of mathematics has been debated and negotiated, written by diverse commissions and voted on in parliament.

METHOD

In our research, we have concentrated on the curriculum of secondary education. Secondary education has four forms: general, technical, art and vocational secondary education. The four forms of education are not organised separately in the first stage. From the second stage, they are organised separately. In the first grade, there is an A class which gives access to the general, technical and art secondary education. There is also a B class, which only gives access to vocational secondary education.

At the level of the development of the content of the teaching of mathematics, we focused on the curriculum as developed by the community because this level of the curriculum is strictly enforced by law for every school (system).

The central research question is whether there is room for philosophy of mathematics within the curriculum of secondary education. The method used is a screening of the text of the complete curriculum. We placed “philosophy” in inverted commas because we needed to use a very broad interpretation of philosophy (in fact all non-technical aspects of the math curriculum) so as to have some paragraphs in the curriculum. We need to point out that there are two parts of the curriculum where “philosophical” issues can be found. Part one is the view on mathematics in education in general; part two is the attainment targets.

After screening the curriculum, “philosophical” fragments are listed according to 1) part within the curriculum (general view versus attainment targets) 2) grade within secondary education (grade I is 12-14 years old, grade II is 14-16 years old and grade III is 16-18 years old) and 3) type of education.

FINDINGS

In a first table (Table 2) we present the results for the first part of the curriculum (the general view) for all grades and for all types of education.

Grade	Type of education			
I	A type General			B type Vocational
II	ASO General	TSO Technical	KSO Art	BSO Vocational
III	ASO General	TSO Technical	KSO Art	BSO Vocational

Table 2: General View in the Curriculum: Overview

A first result we can present is the fact that we only find philosophical issues in the general type of education (marked in grey) and not at the level of vocational education (or the B type in the first grade).

In the following table (Table 3) we present the detailed results of the screening for the first part of the curriculum (the general view) for all grades and for the general types of education (since there are no issues at the level of vocational education).

Grade	Type of education
I	A type
	Ontological proposition: The proposition that mathematics is abstract and formal and that mathematics has no connection with reality, up to a certain degree. **Appreciation**: Pupils must be encouraged to see the beauty and the perfection of a geometric figure, the clarity of a well reasoned argument and the elegance of a formula. **The cultural and dynamic meanings of mathematics:** The pupils should experience that mathematics has a practical use, and that it has an educative and aesthetic value. The history of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. Mathematics in the past developed via many cultures. Due to the emphasis on this development, pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are:** Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.

II	**ASO: general**
	The ontological proposition: is absent **Appreciation:** In addition: when the commission determined the selection of the goals, they took into account, the effect of the development of a relationship with mathematics. **The cultural and dynamic meanings of mathematics:** (more abstract) The pupils should experience that mathematics has a practical use, and that it has an educative and aesthetic value. Attention to the development of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. In this manner pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are:** Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.
II	**TSO en KSO: technical and art**
	Idem II ASO
III	**ASO: general**
	Idem II ASO; in addition to the previous goals: Pupils can gain an insight into the contribution that mathematics has: in the development of the exact and human sciences, and of art, critical thinking, and technique.
III	**TSO en KSO: technical and art** The text marked in grey is omitted at this level.
	Idem II ASO **The cultural and dynamic meanings of mathematics:** (a partial interpretation) The pupils should have an experience that mathematics has a practical use, and that it has an educative and aesthetic value. Attention to the development of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. In this manner pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are**: (one goal has been dropped) Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.

Table 3: General View in the Curriculum: Details

Now we will move on to the second part of the curriculum: the attainment targets. In Table 4 we first present a general overview of possible locations for "philosophical" issues.

Grade	**Type of education**			
I	**A type** **General**			**B type** **vocational**
II	**ASO** **General**	**TSO** **technical**	**KSO** **art**	**BSO** **vocational**
III	**ASO** **general**	**TSO** **technical**	**KSO** **art**	**BSO** **vocational**

Table 4: Attainment Targets: Overview

As one will see there are no philosophical (historical or cultural) goals formulated, either for the B type, or for the first grade. The philosophical issues (marked in grey) are reserved only for the second and third grades of general education.

In Table 5 we present the results in detail for the second part of the curriculum, namely the attainment targets, for grade II and III (since there are no "philosophical" issues in grade I) and for the general type of education (since there are no issues at the level of vocational education).

Grade	**Type of education**
II	**ASO: general**
	Pupils can give examples of the contribution of mathematics to art.[1]
II	**TSO en KSO: technical and art**
	Pupils will gain appreciation for mathematics (possibilities and limitations) in confrontation with the cultural, historical and scientific aspects of mathematics.[2]
III	**ASO: general**
	Idem II ASO; in addition to the previous goals: Pupils can gain an insight into the contribution that mathematics has: in the development of the exact and human sciences, and of art, critical thinking, and technique.[3]
III	**TSO en KSO: technical and art**
	Pupils can give examples of the application of mathematics in other courses and in society in general.[4]

Table 5: Attainment Targets: Detail

CONCLUSIONS

As a first general remark we have to conclude that the first part of the curriculum, the general overview, contains more philosophical issues than the second part which contains the attainment targets. As teachers are more focused on part two, because the attainment targets are the criteria for the evaluation of pupils, we have to conclude that there is little room for a philosophy of mathematics within the curriculum of math education. There are some initial formulations at the level of the general overview in the curriculum which are not completely translated into the attainment targets.

A second general remark is the fact that there is no room for "philosophy" of mathematics within the vocational type of education. Here we want to remark the difference between vocational and general education. On one hand, we can say that mathematics in vocational education is completely embedded in a modular system and attention is paid to core skills. On the other hand we must say that pupils are prepared for specific (professional) occupations, for personal and social functioning, in order to survive in our society. Access to higher education is theoretically possible but in practice impossible. Mathematics in general education is an independent course. General education provides a strong base for higher education (e.g., university.)

Using the distinction Alan Bishop (1988) has introduced between the small m and the large M of mathematics, where the small m stands for basic mathematical competence such as: counting, locating, measuring, designing, playing and explaining, and the large M stands for mathematics as the Western scientific discipline, we can conclude that pupils in vocational mathematics are taught the small m and pupils in general education are taught the large M. The more general the education, the larger the M, and the higher the status in society.

A third general remark is the fact that –at the level of the attainment targets- there is no "philosophy" of mathematics included in the first grade of general education and there is little room for it when we look at technical and art education. Also here we have to conclude that the more general the education is, the larger the M, and the higher the status in society is.

Maintaining the difference between vocational and general education, we can conclude that, for an explicit philosophy, there is very little space in general education and there is none at all in vocational education.

In as far as an implicit philosophy can be identified, it seems to us that it is mostly a rather absolutist view that is present, seeing mathematical truth as absolute and certain, and connected with some humanistic values.

In support of our claim that the curriculum presents the absolutist view, we want to refer to the following arguments:

- there is no room to discuss the status of mathematics,

- the status is very clear and rather static,
- there is no philosophy at all in vocational education,
- the larger the M, the higher the status in society,
- the appreciation for mathematics that pupils are encouraged to gain is seen as the highest form of motivation,
- experience-based learning is only used to gain the interest and to motivate disinterested pupils, to help them to gain appreciation for mathematics with the truly large M.

As to the humanistic values, we observed the following:

- there is only a small space for philosophy in education in general,
- there is some limited attention given to “the possibilities and the limitations of mathematics”, although in the curriculum it is placed between brackets,
- some attention is given to the applications of mathematics,
- there is some limited attention to historical and cultural components (where in addition most of the space is filled with art).

The challenge we wish to propose (and at the same time the source for the questions we would like to raise) is to show how an implicit philosophy can be made explicit and how the implicit philosophy can be modified. In short, what philosophical topics could have a place in the curriculum compared to the present implicit philosophy?

We would like to end with five questions for further research.

(1) Is there room for an explicit philosophy of mathematics in higher education, at the university, and in teacher training? (see, e.g. the work of Ernest (1994), Ernest (1998)).

(2) If so, what kind of philosophical approach? Should one stress the fallibility of mathematical knowledge, should one stress the social nature of mathematics, or should one stress the curious mechanisms that have led to such a strong consensus among mathematics (see, e.g., the work of Heintz (2000)).

(3) Related to (2), what should be the role of ethnomathematics in western school curricula? Is the distinction between big M mathematics and small m mathematics productive, interesting, provocative, necessary?

(4) In the confrontation with culture at large, how can we move beyond the stereotypical associations between mathematics and the arts. Do we always need to refer to Escher? Are there really no other possibilities? Is there no mathematics in the work of Jackson Pollock, to give but one possible alternative?

(5) Given that all the above questions can to some extent be answered, what should the teacher do in the classroom? How should these ideas, views and confrontations be implemented? In short, what are good practices for teachers?

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NOTES

[1] Attainment target 8 of the I. General attainment targets.

[2] Attainment target 11* of the I. General attainment targets. The * says for ‘attitude’.

[3] Attainment target 19 formulated in ‘Mathematics and Culture’.

[4] Attainment target 7 of the I. General attainment targets.