DEVELOPING A CRITICALMATHEMATICAL NUMERACY THROUGH *REAL* REAL-LIFE WORD PROBLEMS [1]

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INTRODUCTION: ALL MATHEMATICAL WORD PROBLEMS ARE NON-NEUTRAL

A great honour was conferred on me a few years ago when right-wing conservative Lynne V. Cheney (1998), former USA Vice-President Dick Cheney's wife, trashed my work because I stated that in mathematical texts, "A trivial application like totalling a grocery bill carries the non-neutral message that paying for food is natural" (Frankenstein, 1983, p. 328). Contrary to Cheney's claim that I would not want students to solve problems totalling grocery bills, I certainly would want students to solve such problems – comparing grocery bills in poor neighbourhoods with those in rich neighbourhoods, for example, or countless other mathematical investigations that could relate to issues of hunger and capitalism where tens of millions of tons of surplus food rot for the profit of a few (Mittal, 2002) while approximately 40 million people die from hunger and hunger-related illness every year and "available evidence indicates that up to 20,000,000 citizens [living in the USA] may be hungry at least some period of time each month." [2] (National Council of Churches, 2007). I argue that all real-life mathematical word problems contain non-numerical "hidden" messages, and that, if those problems are presented as neutral, they can stifle creative thought and questioning, by increasing the aspects of our society that people take for granted.

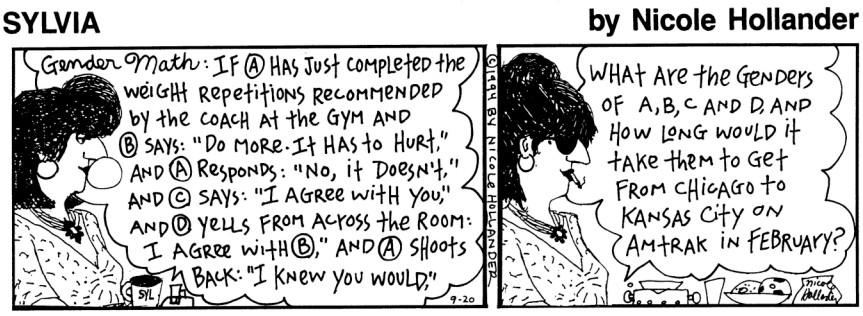
In this paper, I'll discuss some political concerns about other aspects of the non-neutral "hidden" curriculum that result from particular selections of real-life data used to create contrived and/or context-narrow word problems. Then, I'll suggest various categories of *real* real-life mathematical word problems, problems that are presented in a broad enough context for students to appreciate how understanding numbers and doing calculations can illuminate meaning in real life. In conclusion, I will discuss some pedagogical and political questions about the real-life use of *real* real-life mathematical applications, returning to the issue of the non-neutrality of knowledge, and addressing the question of teaching difficult, pessimistic perspectives.

The main goal of a criticalmathematical literacy is not to understand mathematical concepts better, although that is needed to achieve the goal. Rather it is to understand how to use mathematical ideas in struggles to make the world better. In other words, the question to be investigated about my criticalmathematical literacy curriculum is not "Do the *real* real-life mathematical word problems make the mathematics more clear?" The key research questions are "Do the *real* real-life mathematical word problems make the

social justice issues more clear?" and, "Does that clarity lead to actions for social justice?"

PROBLEMS WITH REAL-LIFE MATHEMATICAL WORD PROBLEMS

Real-Life Mathematical Word Problems Without Real Meaning



In a French study (IREM de Grenoble, 1980), a 7-year-old was asked the following question: "You have 10 red pencils in your left pocket and 10 blue pencils in your right pocket. How old are you?" When he answered: "20 years old," it was not because he didn't know that he was 7 in real life, or because he did not understand the relevant mathematical concepts. Rather it was, as Pulchalska and Semadeni (1987, p. 15) conclude, because the unwritten social contract between mathematics students and teachers stipulates that "when you solve a mathematical problem... you use the numbers given in the story... Perhaps the most important single reason why students give illogical answers to problems with irrelevant questions or irrelevant data is that those students believe mathematics does not make any sense".

Clearly, "educating" people to accept nonsensical statements uncritically in order to "fit in" is a political problem. Moreover, it is also politically problematic even when mathematical word problems do not ask nonsensical questions, but use real-life numerical data without *real* meaning, but only as "window dressing" to practice a particular mathematical skill. First, when assumptions about what are the "natural" conditions of real life (e.g., heterosexual families) are used as the "window dressing" context for mathematical problems, students who do not fit those "natural" categories are disrespected and/or made invisible. Further, the "hidden curriculum" about what is "natural" gets reinforced, making it less likely that students will question these taken-for-granted assumptions. Second, the *real* significance of the "window-dressed" real-life data is also hidden. When no better understanding of the data is gleaned through solving the mathematics problem created from the data, using real-life data masks how other mathematical operations, as well as other non-mathematical investigations, could be performed that would illuminate those same data. It gives a "hidden curriculum" message that using mathematics is not useful in understanding the world—mathematics

is just pushing around numbers, writing them in different ways depending on what the teacher wants.

Real-Life Mathematical Word Problems Without *Real* Real Context

There are, of course, curricula that contain real-life mathematical word problems that involve using numbers to gain more information to help make real-life decisions. However, often these problems assume everyone's real-life context is the same. Underlining this point, Apple (1992, pp. 424-425) concludes that the NCTM Standards (1989) do not address "the question of *whose problem* ... by focusing on the reform of mathematics education for 'everyone', the specific problems and situations of students from groups who are in the most oppressed conditions can tend to be marginalized or largely ignored (see Secada, 1989, p. 25)." The Standards do not contain, for example, suggestions for mathematical investigations that would illustrate how the current US government's real-life de-funding of public education, through funding formulas based on property taxes, creates conditions in which the real-life implementation of the NCTM student-centered pedagogy is virtually impossible except in wealthy communities (Kozol, 1991).

Real-Life Mathematical Problems Without Enough Real Context

Those "neutral" real-life mathematical word problems that do include a real-life context like totalling grocery bills still omit the larger contexts of individual economic differences within a system where a 1997 report from the US Department of Agriculture declared that 11 million citizens, including 4 million children, "live in households categorized as moderately or severely hungry." (Sarasohn, 1997, p. 14).

Other "neutral" real-life mathematical word problems involve numerical descriptions that omit the larger contexts that created the reality of those descriptions. For example, *Multiplying People, Dividing Resources* (Zero Population Growth, 1994) contains a worksheet of real-life mathematical word problems designed to help students conceptualize large numbers. In the section on "Explanations/Applications," there is their "neutral" comment that:

> When Columbus arrived in the Americas in 1492, there were probably 5 million Native Americans living in the area of the United States, and 57 million in the two American continents. World population at that time was about 425 million, and did not reach one billion until approximately 1810. . . . In 1994, the United States has approximately 260 million people within its borders . . .

Hidden in this real-life context is the larger context of what happened to those Native Americans. Although there is some academic debate about the number of people living North of Mexico in 1492 (ranging from about 7 million to 18 million),

> There is no doubt, however, that by the close of the nineteenth century the indigenous population of the United States and Canada totalled around 250,000. In sum, during the years separating the first arrival of Europeans in the sixteenth century and the infamous massacre at Wounded Knee in the winter of 1890, between 97 and 99 percent of North America's native people were killed (Stannard, 1992, p. 432) .

REAL REAL-LIFE MATHEMATICAL WORD PROBLEMS [3]

Real real-life mathematical problems occur in broad contexts, integrated with other knowledge of the world. I (Frankenstein, 1983) contend, along with Freire (1970; Freire & Macedo, 1987) that the underlying context for critical adult education, in this case criticalmathematical literacy, is "to read and re-write the world." In that case, mathematical skills and concepts are learned in order to understand the institutional structures of our society.

Below are various categories of problems that, of course, overlap in different ways. The overarching activity is gaining a better analysis of the issue through understanding the meaning of the numbers, and gaining more knowledge about the issues through performing relevant calculations. The purpose of discussing the examples in this manner is to show many types of situations in which numbers can be used to make sense of the world, and then to make justice in the world.

Understanding the Meaning of Numbers

The *real* real-life mathematical word problems whose solutions involve understanding the meaning of numbers focus on using different kinds and arrangements of numbers (e.g., fractions, percents, graphs) to:

- describe the world
- reveal more accurate descriptions of the world
- understand the meaning of the sizes of numbers that describe the world
- understand the meanings that numbers can hide in descriptions of the world
- understand the meanings that numbers cannot convey in descriptions of the world

Understanding the meaning of the numbers is needed to understand the meaning of these situations, situations that illuminate the way our world is structured.

Using Numbers to Describe the World

Example:

> Although Helen Keller was blind and deaf, she fought with her spirit and her pen. When she became an active socialist, a newspaper wrote that "her mistakes spring out of the… limits of her development." This newspaper had treated her as a hero before she was openly socialist.
>
> In 1911, Helen Keller wrote to a suffragist in England: "You ask for votes for women. What good can votes do when ten-elevenths of the land of Great Britain belongs to 200,000 people and only one-eleventh of the land belongs to the other 40,000,000 people? Have your men with three millions of votes freed themselves from this injustice?" (Zinn, 1980, p. 337).

Students are asked to discuss how numbers support Helen Keller's main point and to reflect on why she sometimes uses fractions and other times uses whole numbers. Information about the politics of knowledge is presented as a context in which to set her views, including class discussions about Keller's militant answer to the editor of the *Brooklyn Eagle* (Zinn, 1980, p. 338) and about why so many children's books ignore her socialist activism (Hubbard, 2002).

Using Numbers to Reveal More Accurate Descriptions of the World

Example: Students are asked to read articles that present numbers that counter taken-for-granted assumptions that many view as "natural" facts about the world. For example, an article which shows that in spite of widespread belief that "illegal" [4] immigrants are robbing tax payers through their use of hospital emergency rooms and public education, not only do "illegal" immigrants pay sales and other such taxes, but they also pay over $6 billion in Social Security and about $1.5 billion in Medicare taxes, without collecting any of the benefits from those taxes (Porter, 2005).

Understanding the Meanings that Numbers Cannot Convey in Descriptions of the World

Example: Following this is an example of art encoding quantitative information. The numbers are the data of our world—our wars; the art allows us to understand the quantities in ways we could not understand from the numbers alone. As Toni Morrison states: "Data is not wisdom, is not knowledge" (quoted in Caiani, 1996, p. 3).

The famous memorial in Washington, D.C. by artist Maya Lin lists the names of 57,939 Americans killed during the Vietnam War. In "The other Vietnam Memorial" (Museum of Contemporary Art in Chicago, IL), Chris Burden etched 3,000,000 names onto a Rolodex-type structure, standing on its end, that fills the entire room in which it is displayed. The names represent the approximate number of Vietnamese people killed during the US war on Vietnam. Since many of their names are unknown, Burden created variations of 4000 names taken from Vietnamese telephone books. Also, the museum notes comment that by using the form of a common desktop object that functions to organize professional and social contacts, Burden underlines the unrecognized loss of Vietnamese lives in US memory.

Understanding the Calculations

The *real* real-life mathematical word problems whose calculations are an integral part of understanding a situation focus on:

- verifying/following the logic of an argument
- understanding how numerical descriptions originate
- using calculations to restate information
- using calculations to explain information
- using calculations to reveal the unstated information

The purpose underlying all the calculations is to understand better the information and the arguments, and to be able to question the decisions that were involved in choosing which numbers to use and which calculations to perform.

Understanding how Numerical Descriptions Originate (Seeing how Raw Data are Collected, Transformed, and Summarized into Numerical Descriptions of the World)

Example: Students are asked to read the excerpt below so that they are thinking about issues of how to teach and how people learn mathematics at the same time that they are learning the mathematics. Then, they are asked to: describe the study's methodology (i.e., what procedures were followed in the study, what the "raw" data consisted of, and how the raw data were transformed and summarized); re-write the findings described by creating a chart; discuss which presentation of the data is clearest, and why; list conclusions they can and cannot draw from the data; and indicate what other information they would want in order to clarify the data or strengthen and/or change their conclusions.

> Sixty-six student teachers were told to teach a math concept to four pupils - two White and two Black. All the pupils were of equal, average intelligence. The student teachers were told that in each set of four, one White and one Black student was intellectually gifted, the others were labelled as average. The student teachers were monitored through a one-way mirror to see how they reinforced their students' efforts. The "superior" White pupils received two positive reinforcements for every negative one. The "average" White students received one positive reinforcement for every negative reinforcement. The "average" Black student received 1.5 negative reinforcements for each positive reinforcement, while the "superior" Black students received one positive response for every 3.5 negative ones. (Sklar, 1993, p. 53)

Using Calculations to Restate Information (Changing the Quantitative Form)

Example: Students study a letter I wrote (Frankenstein, 2002) responding to an article by Howard Zinn (2002) in which he argues that the numerical descriptions of the deaths from the US war on Afghanistan can obscure those horrors. To dramatize my argument that numbers can illuminate the meaning of data and deepen connections to our humanity, I conclude that the 12 million children who die every year from hunger "are dying faster than we can speak their names." (Frankenstein, 2002, p. 23).

Using Calculations to State the Unstated Information

Example: Students learn about percents while analyzing the following political poster in the context of the politics of language where people who constitute a majority of the world's population are referred to as "minorities." Students also see that numbers are "behind" many economic, political, and/or social issues even if there are no numbers "visible" in the picture (Figure 1).

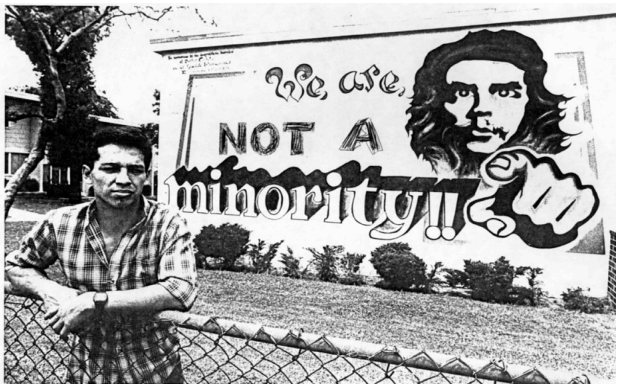


Figure 1. Los Angeles Hispanics and other recent immigrants are demanding their piece of the pie (Guardian, 1978, Mario Torero, with Zapilote, Rocky, El Lton, and Zade)

CONCLUSION: PEDAGOGICAL AND POLITICAL DIMENSIONS OF TEACHING THROUGH REAL REAL-LIFE MATHEMATICAL WORD PROBLEMS

Pedagogical Dimensions

Following ABC's 1983 airing of a film about *The Day After* a nuclear war, the network presented a panel discussion, chaired by Ted Koppel, of mostly conservative government officials and Carl Sagan, a liberal scientist. At one point, Sagan refuted the then Secretary of State Schultz's contention that the Administration was already disarming, pointing out that "its current build-up calls for an increase in the number of strategic warheads, from 9,000 to 14,000." Koppel turned to Sagan and said "… I must confess statistics leave my mind reeling and, I suspect, everybody else's too." (Manoff, 1983, p. 589)

Certainly, students need enough mathematics so that their heads do not reel from comparing the size of two numbers! As a prerequisite to accomplishing the goal of a Freirean "reading and re-writing of the world" using a criticalmathematical literacy, students need confidence that they can learn enough mathematics to use as part of understanding public and community issues. When students realize that their teacher has confidence in them and expects, with studying, that they will learn the mathematics, they can begin to let go of the negative expectations many have internalized from past mathematics learning experiences. Also, confidence is gained from analysis of the politics of language, where the label "mathematically anxious" can have contradictory effects. Naming that situation can initially reassure students that their feelings about mathematics are so common that educators have a name for them. However, the label can also focus the problem inward, "blaming the victims" and encouraging solutions directed solely *at* them. The label can direct attention away from the broader social context of how their learning got mystified, and what interests might be served by widespread mathematics "anxiety" and avoidance. And, confidence is gained from understanding the politics of knowledge that have discounted some people's knowledge and privileged others' knowledge. For example, I ask students to reflect on Freire's (Freire & Macedo, 1987) insistence that "the intellectual activity of those without power is always characterized as non-intellectual." (p. 122)

Once students are confident in their ability to learn mathematics, and motivated to reason quantitatively about public and community issues, then the question is: How much of the structure of mathematics must be demystified in order for students to be able to use numerical data for demystifying the structure of society? It is important for students to understand enough concepts behind the basic algorithms to be able to use those rules comfortably in many different situations. However, as Lange and Lange (1984) found, although mathematics education can be empowering in a more general way, it is not necessarily the best approach in working with people on specific

empowerment issues. The piece-rate workers they were organizing in the textile industry in the southern United States were struggling with a pay system made intentionally obscure. The Langes' experience was that teaching the concepts of ratios and fractions behind that rate system was not the most effective way to empower the workers in their struggle for decent pay. It was more empowering to create a slide-rule distributed by the union that did the pay calculations for the workers, making the mathematical problem disappear, so that the workers could "focus on the social and economic relations underlying the way they are treated and paid" (p. 14).

In my context, my curriculum is loosely organized by a linear thread of underlying mathematical concepts (i.e., the meaning of whole numbers, then fractions, later percents, and so on). But, the lessons also involve non-linear explorations of *real* real-life public and community issues and much interdisciplinary content. However, in thinking about what numeracy citizens need to solve *real* real-life problems, I am not advocating getting rid of college preparatory mathematics. As Powell and Brantlinger (2008) argue, teaching "traditional" mathematics with understanding to students who have been marginalized from college or certain professions is another form of criticalmathematics education appropriate to that context. I would argue that all citizens need the criticalmathematics I am describing, but it does not need to replace more "traditional" mathematics.

Political Dimensions

I suspected trouble when, at a 1981 National Council of Teachers of Mathematics (NCTM) Conference, the president of the organization opened the meeting by stating that Ronald Reagan's election was great for mathematics teachers. But, I did not suspect how outraged the teachers would be by the biases in my real-life word problems. They did not accept my argument that no mathematical word problems are neutral.

A few years after my NCTM audience was furious at my biased word problems, the NCTM journal, *The Mathematics Teacher*, (March 1984, December 1984) was running multi-page spreads advertising a US Navy slide show "Math and Science: START NOW!" Toll-free phone numbers to arrange for a class presentation by a Navy representative were included. They published one critical letter that focused on the inappropriateness of the Navy starting recruiting drives in junior high school and questioned why there were no ads from government groups "whose mandate is more closely tied to social and environmental problems" (Milne, 1984). The editor answered that the Navy paid for the ad and any government agency could do likewise. He did not publish my strong critique that accepting an ad from the Navy implied:

> … a certain level of support—especially since the NCTM's Executive Director is quoted in the ad as saying "Without hesitation, we endorse the project"!! In addition, your ad policy will be skewed towards those governmental agencies with the largest advertising budgets—therefore, those agencies, such as the military, which are favored by the current administration, will also be favored

by NCTM ad policy. Finally, we did pay for the ad—not through our NCTM dues as you stated—but certainly, through our tax dollars.

One final point: the *real* real-life context illuminated by the *real* real-life mathematical word problems in my adult criticalmathematical literacy curriculum are outrageously horrible. How can these topics be taught without discouraging people and thereby stopping resistance? The context of my students' lives is such that many have been involved in our struggle to change this situation. And different groups of us have experienced some victories. However, given the resources of those in power to regroup, we wind up fighting the same battles over and over and often initial victories are overturned or co-opted. Nevertheless, those of us who are committed to the struggle for a just liberatory world keep fighting.

Audre Lorde (1988) reminds us in *A Burst of Light* that*:*

> … hope [is] a living state that propels us, open-eyed and fearful, into all the battles of our lives. And some of those battles we do not win. But some of them we do. (p. 80)

NOTES

1. A much longer version of this paper is found in Chapter 6 "Developing a Criticalmathematical Numeracy through Real Real-life Word Problems," pp. 111-130 in Verschaffel, L., Greer, B., Van Dooren, W. & Mukhopadhyay, S. (Eds.) (2009). Words and Worlds: Modelling Verbal Descriptions of Situations. Rotterdam, The Netherlands: Sense.
2. Related to the politics of language, The Progressive (2007, p.11) cites a Washington Post article indicating that the United States Department of Agriculture will no longer use the word "hunger" to describe people who cannot get enough food to eat; instead these people will be described in official government documents as having "very low food security."
3. Due to space limitations the examples are presented in an abbreviated form, and I am not giving examples for each category. I am developing these and others into a collection of columns for various websites and newsletters. Since June 2008, they have been appearing in Numeracy Briefing, edited by Europe Singh. For more information contact them at numeracy@basicskillsbulletin.co.uk. If any reader is interested in syndicating these columns, free of charge, contact me at marilyn.frankenstein@umb.edu
4. I use quotes around illegal to draw attention to who gets to make the laws that determine who is "legal" and who is "illegal".

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