"I WAS THINKING THE WRONG THING" / "I WAS LOOKING IN A PARTICULAR WAY": IN SEARCH OF ANALYTIC TOOLS FOR STUDYING MATHEMATICAL ACTION FROM A SOCIO-POLITICAL PERSPECTIVE

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This paper focuses on the theoretical and methodological challenges of keeping the mathematical action of students in view when conducting research from a socio-political perspective. I present a theoretical perspective and associated analytic tools that are structured by the work of Fairclough in critical linguistics, but have been supplemented with the work of Morgan, Moschkovich, Sfard and Valero in mathematics education. I illustrate the use of the tools with data from a study investigating student action when solving problems with real-world contexts in an undergraduate mathematics course.

INTRODUCTION

Valero and Matos (2000) argue that the use of social, political or cultural approaches to mathematics education research, which often draw on theoretical perspectives in other fields of social science, enable us to engage with and understand aspects of mathematics education that are not necessarily offered by traditional psychological perspectives. Adler and Lerman (2003, p. 445) frame the choice of approach as an ethical one, and part of getting the description "right". They argue that certain questions cannot be asked or answered when the "zoom of the lens is tightly on mathematical activity".

Embarking on a research study related to my teaching at a higher education institution in South Africa, I was attracted by the possibilities suggested by adopting what I understood at the time to be a socio-political perspective of mathematics education and of research. In fact several features of my teaching and research practice suggested that I had no alternative. I was teaching (and at the same time wanted to research) an undergraduate mathematics course specifically designed for students identified as being disadvantaged by the enduring inequitable system of school education, with the aim of providing these students with access to and success in higher education. The unit of analysis was to be student action as they worked collaboratively to solve problems with "real-world" contexts [1]. My aim was not only to identify and describe the enabling and constraining discursive actions, but also to explain these in the light of the socio-political practices of the classroom and of the wider socio-political space.

Yet in advocating for perspectives that take into account the social, political and cultural aspects of mathematics education, Valero and Matos (2000, p. 398) acknowledge the "dilemma of mathematical specificity"; they note that such perspectives can be regarded as “non-mathematical” in the sense that going “deeply” outside of mathematics results in the mathematics tending “to vanish or to be questioned”. Sierpinska (2005, p. 229) warns that such perspectives run the risk of “discoursing the mathematics away”. This dilemma is both a theoretical and methodological issue; since these perspectives draw on other fields, the appropriate analytic tools to study the mathematical content (or what I refer to as *the action on mathematical objects* in this paper) may not have been developed. For example, Sfard (2000, p. 298) notes that while discourse analysis has been used to study the “rules and norms constituting mathematical practices”, little attention has been given to using the method for the study of mathematical content and in particular for studying mathematical objects.

In planning my study I selected the work of Fairclough (1989; 2003; 2006) in critical linguistics on the strength of its potential for linking the micro socio-political activity of the classroom with wider socio-political practices. Fairclough’s method for critical discourse analysis proved productive in studying students’ positioning and the nature of their talk, yet as my study progressed I sensed that something about the mathematics itself was enabling and constraining the students’ work, and my tools were not allowing me to view this. I was “looking in a particular way”, and getting the description “right” in Adler and Lerman’s (2003, p. 445) terms required that I further develop my way of looking to allow me to bring the student action on mathematical objects into view, while not losing sight of the socio-political nature of this action.

In this paper I present the theoretical perspective and associated methodology that have emerged after an extended interactive process, working between my empirical data and my reading of the work of Fairclough (1989; 2003; 2006), Morgan (1998), Moschkovich (2007), Sfard (2000, 2007) and Valero (2007: 2008). I then illustrate the use of my tools on a selected piece of data from my study.

THE STUDY

The study is located in a first-year university access course in mathematics at a South African university. This course forms part of an extended curriculum programme designed for students identified as disadvantaged by the schooling system. After six weeks of the academic year, this group of students is joined by those students who are performing poorly in the mainstream first-year mathematics course.

The micro-level data for the study is in the form of transcripts of video-tapes and students’ written work; two groups of students were video-taped as they worked on selected real-world problems in the regular weekly afternoon workshop (see for example the “flu virus problem” in Figure 1). The students had access to a tutor and resources such as course notes. An extract from the worked solutions, provided to students a few days after the workshop is given in Figure 2. I transcribed the video-footage to represent both the verbal and non-verbal action of the students.

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

b) What are the units of ?

c) What does mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

d) What does mean in practical terms? Give the correct units.

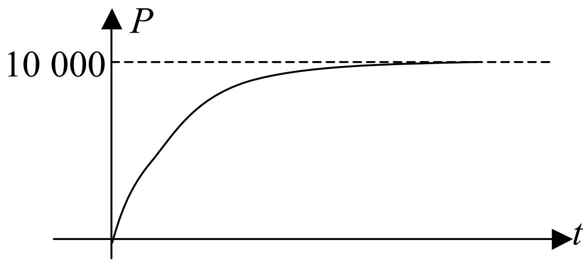
e) What does mean in practical terms? Explain why can never be negative.

f) What is ? Give a short reason for your answer.

g) What is ? Give a reason for your answer.

Figure 1: "Flu virus problem", Question 6, Workshop 10, 2007 Resource Book, p. 54

(a)



(g) . Eventually the number of people who have caught the flu becomes (very nearly) constant at 10 000, so the rate of new infections is 0 (see graph).

Figure 2: Selected solutions to the "flu virus problem", Questions 6(a) and (g) [2]

THEORETICAL FRAMEWORK: A SOCIO-POLITICAL PERSPECTIVE OF MATHEMATICS EDUCATION

According to Fairclough (2003), a *social practice* is associated with certain activities, participants, social relations, objects, position in time and space, values and discourse. Any institution or organisation is characterised by a particular *network* of social practices, a network that is constantly shifting. Drawing on the work of Valero (2007) and Moschkovich (2007) I use the concept of *mathematical discourse practices* which differ "across communities, times, settings and purposes" (Moschkovich, 2007, p. 27) as a broad term for the network of social practices in which mathematics teaching and learning is given meaning, for example, school or undergraduate classroom mathematical activity, teacher education, etc.

My reading of the work on mathematical discourse by Morgan (1998), Moschkovich (2007) and Sfard (2007), together with my empirical work, has led me to conceptualise mathematical discourse practices as characterised by certain *ways of acting mathematically*; ways of talking/writing/representing, ways of attending (of looking and listening), ways of making links and establishing relationships, ways of arguing, ways of evaluating, ways of interacting socially and discursively, and ways of identifying oneself and others.

This conceptualisation of particular ways of acting mathematically that may differ across mathematical discourse practices points to why these practices are not only social, but also political. Drawing on the work of Foucault, Valero (2007; 2008) argues that power is distributed when people participate in social practices, and she defines *power* as the capacity of people to position themselves in relation to what is valued in the practices. So power can be seen to manifest in mathematics classroom activity in two ways. Firstly, this activity is embedded in a network of socio-political practices, practices in which particular ways of acting are valued. Secondly, it is through the interaction of participants in the classroom that power is (re)produced.

The socio-political perceptive of learning proposed in this paper draws on a socio-cultural perspective of learning which views learning mathematics as coming to participate in the discourse of a community that practises the mathematics (e.g. Sfard, 2007). Since mathematics education is inherently political, becoming a participant not only involves grappling with the content and skills of the community, but also determining what ways of acting mathematically are valued and negotiating one's identity and position in that community.

METHODOLOGY

In this section I explain the structure of and use of the analytic framework presented in Table 1. This framework derives its overall structure from the work of Fairclough, but is supplemented with work by Morgan (1998), Moschkovich (2007) and Sfard (2000; 2007). Fairclough (2003, p. 26-27) identifies three ways in which meaning is constructed by text and these are represented in column 2 of Table 1. Firstly, *representation* refers to how the text represents the classroom mathematical activity, for example, the ways of representing, making links etc. *Action* refers to how text enacts relations between participants and between other texts. Thirdly, *identification* refers to how text identifies people and their values. These three meanings are not distinct, but are separated for analytic purposes only.

Identifying these three meanings in the text involves a detailed line-by-line analysis of the transcript. The tools that I am using are summarised in column 3 of Table 1, and I explain their use with reference to line 482 of Transcript 1. Six students (Jane, Lulama, Darren, Hanah, Shae, and Jeff [3]) are solving question (g) of the flu virus problem shown in Figure 1. By the time the Tutor joins the group the students have agreed that the limit does not exist (lines 472 and 473, Transcript 1). In line 479 the Tutor suggests that the students should be using their graph of the function *P,* constructed in question (a) (see Figure 2). Shae responds, "No but that's of, that is not of the dash" (line 482).

Table 1: Analytic framework

Level of socio-political practice	"Meaning" of the text	Identifying features of the text
discourse as a relatively stable way of representing	*Representation*: What ways of acting mathematically are included / excluded / given significance? 1. Ways of attending 2. Ways of making links 3. Ways of arguing 4. Ways of evaluating 5. Ways of talking and writing 6. Ways of representing	Focal analysis: *1. attended focus* *2. pronounced focus* *3. intended focus* Critical discourse analysis: For example, naming, pronouns, reference relations, mood, modality
genre is a relatively stable way of acting communicatively	*Action*: What action is the text performing in constituting relations (both social and textual)? 1. Ways of interacting socially 2. Ways of making textual links	
style is a relatively stable way of being	*Identification*: How does the text identify people, and their attitudes and values? 1. Ways of identifying oneself and others	

472	Tutor:	Okay, I see two answers saying that ... *P* dash ... *t* as *t* tends to infinity is ... not defined or [does not exist] ((*He stretches across and points to Shae's Resource Book*))
473	Shae:	[Ja, it does not exist] ((*He looks up at the Tutor who is standing at his shoulder*))
474	Tutor:	Okay, [[well]] do you all have that?
475	Jeff:	[[Because it'll]]
476	Lulama/ Hanah:	Ja ((*Lulama, Darren and Hanah nod their heads*))
477	Tutor:	What is your reasoning behind that?
478	Jeff:	Because the graph ... it is s ... such a steep graph that it's tending more towards infinity ... than ... ((*All the others look at Jeff and then at Tutor, who has got down on his haunches next to the desk*))

479	Tutor:	Okay ... well can I, where is the graph?
480	Jeff:	Do we have to go and draw it? ((*He turns his page back*))
481	Tutor:	No ... you have already drawn it
482	Shae:	No but that's of, that is not of the dash ((*He looks at the graph for question (a) in his answer book*))

Transcript 1: Group 1, Question 6(g), lines 472 to 482 [4]

Sfard (2000; 2007) motivates for the use of *focal analysis* as a tool by arguing that since mathematical objects are abstract entities with no concrete referents, we use language and representations to talk and write about them. It is hard, therefore, to distinguish the object itself from the language and other forms of representation. Sfard provides three tools for studying the *discursive focus* of mathematical activity.

Firstly, the *pronounced focus* refers to the words the student uses when identifying "the object of her or his attention" (Sfard, 2000, p. 304). In line 482 Shae pronounces, "That's of, that is not of the dash". Having identified the pronounced focus I am able to do a critical discourse analysis of the words, for example, I note that Shae gives negative feedback ("No") in the form of a description of the two graphs and he names the derivative using the symbol used to represent it ("the dash"). For this critical discourse analysis I use a list of textual features suggested by Fairclough (1989, 2003), but supplemented with work by Janks (2005), McCormick (2005) and Morgan (1998). Secondly, the *attended focus* is what the student is "looking at, listening to" when speaking (Sfard, 2000, p. 304). In line 482 Shae is attending to the Tutor's reference to the graph in line 479, to the actual graph he has drawn for question (a), and possibly the visual image of the graph of the derivative that he and his classmates have been using. Lastly, the *intended focus* is what Sfard (2000, p. 304) describes as "the whole cluster of experiences" that are evoked by the pronounced focus and the attended focus. I use the pronounced and attended foci as clues to identify what meaning the speaker may be making; in line 482 it seems that Shae is arguing, "The graph for question (a) is the graph of the function P and not the graph of the derivative function P'."

Column 1 of Table 1 is what allows me to link the micro-level classroom activity with the macro socio-political practices. Fairclough (2003, p. 28) states that *discourse*, *genre* and *style* (as given in column 1 of Table 1) are relatively stable ways of representing, acting, and identifying respectively, that operate on the level of socio-political practice. By asking which discourses, genres and styles are articulated in the text it is possible to make a link to these wider socio-political practices.

A SAMPLE ANALYSIS

In this section I present an analysis of the action of the group of six students (Jane, Lulama, Darren, Hanah, Shae, and Jeff) as they solve question (g) of the flu virus problem. I describe the action with reference to the socio-political practices of the classroom, and attempt to link this action to wider socio-political practices. I support my argument with quotes from Transcript 1 and also draw on my knowledge of the wider set of data.

As represented in much of the transcript, Shae and Jeff work ahead of the other four students. Shae looks at the derivative $P'(t)$ in the limit expression, links this expression to the task context, and identifies the limit with the maximum value of the derivative function; he pronounces that the answer is 10 000 “cause it could be 10000 people that catch it per day that would be the maximum amount” (lines 346 and 347). This link between the limit and the maximum value of the function is regarded as a common notion of the limit (e.g. Cornu, 1991), thus identifying this action with that of other undergraduate mathematics students.

Instead of making a link to the graph in question (a) as is valued in the worked solutions (see Figure 2), Shae and Jeff attend to the graph of the derivative. This is confirmed in lines 479 to 482 of Transcript 1 when there is confusion about which graph is being attended to and Shae explains the difference. Jeff proposes a vertical straight line, the “steep graph” referred to in line 478 of Transcript 1 (he demonstrates the graph in the air with his hand), a decision that may be cued by Shae’s argument in line 346 that 10 000 people can catch the disease in one day.

The graph proposed by Jeff becomes the graph that the students attend to from this point, and the other students do not interrogate his representation of the derivative. I have identified three possible explanations for the absence of any interrogation of this representation. Firstly, it is possible that the use of gesture to represent the graph as opposed to a physical drawing, a common feature of the data, may prevent the other students from focusing on the representation. Secondly, the easy acceptance of Jeff’s proposal may be linked to the promotion of co-operative group work as a genre in school mathematics and in the course itself. Adler (1997) suggests that a participatory-inquiry approach, in which students work together and are encouraged to value one another’s contributions, can inadvertently constrain rather than enable mathematical activity. Thirdly, it is possible to explain this acceptance of the vertical line graph with reference to the power relations in the group. Jeff and Shae, both first-language English speakers who joined the class from the mainstream mathematics class, identify themselves as the authorities in the group. They are the first to volunteer possible solutions (although often tentative solutions) and verbalise their evolving ideas publicly. In addition, the other students position Shae and Jeff as the authorities by consistently appealing to them for assistance and feedback.

Attention to the vertical line graph demonstrated by Jeff seems to set up links that are constraining. Darren makes a link to his lecture notes for that day by paging back in his book; the lecture topic was the limit definition of the derivative and the lecturer presented cases where the derivative does not exist, for example, at the point where the tangent to the graph is vertical. It also emerges in a later discussion that Darren has possibly not yet looked at the required limit expression, $\boxed{\quad\quad}$, in question (g).

Yet together, Darren and Jeff argue that the gradient of a vertical line “does not exist” and that vertical line graph is “not differentiable” / “non-diffable”, and hence the limit in question (g) does not exist. The link to Darren’s lecture notes and their language use suggests they are drawing on the Course discourse in their argument. Unlike Shae’s earlier attempt to ground his argument in the task context (as in lines 346 and 347), Darren and Jeff arrive at their answer without reference to the task context. This absence of a meaningful link between the mathematical content and the task context may be a result of their experience of the discourse of school mathematical word problems; for in such problems they need only “pretend that” the situation described in the task context exists (Gerofsky, 1996, p. 40).

An analysis of how the students talk also points to why they do not appear to critically interrogate one another’s reasoning. Although Jeff’s vertical line graph represents the derivative in the limit expression , this is not pronounced in the conversation and only emerges in line 482 (Transcript 1) in discussion with the Tutor. Yet the students’ argument suggests that their focus is on the gradient (the derivative) of this vertical line. The students’ tendency to use the pronoun “it” to reference different concepts rather than explicitly naming them may constrain them from evaluating their own arguments and from focusing appropriately. For example, in his explanation to the Tutor in line 478 (Transcript 1), Jeff uses “it” for the vertical line graph and possibly “it’s” to reference the tangent to this graph.

Furthermore, while the students look at the derivative function P' in the limit expression , they do not appear to look at the symbols in this expression. It is possible that Hanah tries to draw the attention of Darren and Jeff to this when she argues, “But they are talking about the days … time” (line 367). However, this pronouncement is not attended to by the other students, possibly because Hanah tends to position herself outside of the group by working as an individual and not entering into the group discussions. In an individual interview she indicated that she felt “intimidated” when working in the group.

When the Tutor approaches the group he reads the answers aloud from the students’ written work as in line 472. But the way that he reads the full answers by linking the mathematical symbols to their meaning in words and his insistence that the students explain their reasoning appear to be enabling. Jeff responds to the Tutor’s challenge to explain his graph, but then pauses, as he repeats the Tutor’s phrase “t tends to infinity” (line 472, Transcript 1): “We thought that, okay, it will be... °t tends to infinity° ((*looking up at the ceiling*)), okay wait, I’m thinking of the wrong thing” (line 485). From this point the change in time features in the students’ talk, suggesting that they are attending to the symbols . For example, pointing to the problem text Shae says, “this is equal to the amount of people over time … that is the increase … °per day°” (line 494). The link that the Tutor makes between the limit expression and the graph for question (a) also appears to be enabling, evidenced by the fact the Jeff rapidly comes up with a correct answer of “nought” (line 502) for (g) and indicates with his hands that he is visualizing an appropriate graph. Hanah's naming of the expression as "the rate of change at infinity" (line 510a) suggests that she is now looking at the expression as an object with meaning, rather than looking at separate parts such as and $P'(t)$. She also tries to explain her claim that the answer is zero by drawing on the task context.

CONCLUSION

Gómez (2008) argues that the methodological procedures involved in mathematics education research are usually only described in doctoral dissertations and tend to refer to methodologies that have already been developed. In this paper I have presented some of my personal journey in developing an analytic framework for my study. I have suggested that my original socio-political framework and associated tools did not mean, in Jeff's words, that I was "thinking the wrong thing" (line 485), but rather that I was "looking in a particular way". While my initial tools were useful in identifying important aspects of the student action, I needed to do some "mathematical work" in order to bring the student action on the mathematical objects into view.

The analysis presented here suggests that the students' ways of talking, representing, making links, identifying themselves and one another, and interacting socially and discursively may constrain their attempts at solving question (g) of the flu virus problem. In contrast, the Tutor talks in such a way that he makes links between the mathematical symbols, the description of these symbols in words, and the appropriate graphical representations. His approach is enabling in that he allows the students space to talk and to work with the links he has set up. I argue that, with the Tutor's support, Shae, Jeff and Hanah are able to position themselves appropriately in relation to the valued ways of acting mathematically for this problem. Yet there are absent voices in the data presented here; my analysis of the wider data set suggests that Lulama is positioned outside much of the group discussion, despite his ongoing attempts to participate.

By addressing the "mathematical specificity" (Valero & Matos, 2000, p. 398) of the student action from a socio-political perspective as described here, I argue that I am able to explain this action in terms of different ways of acting mathematically, rather than with reference to the cognitive mathematical ability of the individual students. So rather than viewing students' action as "thinking the wrong way", I can argue that they are "looking/linking/talking/etc. in a particular way", and that these mathematical ways may not be of value in the particular mathematical discourse practice in which they are engaged.

NOTES

1. I use the term "real-world" as a label for a group of problems in the Course. Developing a description of this group of problems is part of the wider study as reported in Le Roux (2008a, 2008b).

2. In the wider study I argue that in some cases the construction of the problem and/or the worked solution can be viewed as constraining.

3. The names of the students have been changed. The names Jane (gracious), Lulama (gentle and kind), Darren (great), Hanah (grace), Shae (gift) and Jeff (gift of peace) have been selected to acknowledge my admiration for these students and my gratitude for their willingness to take part in the study.

4. The transcription notation used in the study is based on Jefferson notation; three dots "…" indicates a short pause, square brackets around [text] or [[text]] indicates overlapping text, italicised text in double round brackets ((*text*)) presents the non-verbal action, underlined <u>text</u> indicates emphasis or stress, and °text° indicates speech said quieter than normal.

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