MATHEMATICS, DEMOCRACY AND THE AESTHETIC

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Starting from Josiah Ober's etymological exploration of the origins of the term 'democracy', placed as one of a series of words identifying specific 'regime types' in ancient Greece, we shift the setting to that of mathematicians and their practices as well as mathematics classrooms, while adding in questions of the aesthetic in relation to questions of whose taste, whose judgements, whose decisions are attended to there. Ober identifies democracy as centrally having to do with power in the sense of the 'capacity to do things' and we explore different forms of power related to the notion of 'taste.' We examining to what extent questions of mathematical 'beauty' are amenable or even accessible to school students and how this key issue interacts with Ober's ideas concerning democracy as a concept.

INTRODUCTION

Relying on etymologies can prove a complex business. For one, it can seem to suggest that earlier meanings are in some sense 'truer' or 'more fundamental', as well as perhaps reflecting a conservative wish to restrain variation or divergence from traditional (even 'natural') meanings. Nevertheless, as political scientist Josiah Ober (2007) is at pains to point out in his recent informative account of the ancient Greek sense of 'democracy', on occasion there can be some benefit. He writes:

> Of course, we are not bound by any past convention, much less by the inventors' original definition: if we can devise a better meaning for a political term, it should be preferred. But if common modern uses are not particularly good, in the sense of being "descriptively accurate" or "normatively choiceworthy," then there may be some value in returning to the source. Reducing democracy to a voting rule arguably elides much of the value and potential of democracy. (p. 2)

Ober examines 'democracy' within the small semantic field of Greek terminology for regime types, many of which either have -*arche* or -*kratia* suffixes (e.g. 'monarchy', 'oligarchy', 'democracy'). Monarchy and oligarchy refer to how many are empowered in the ruling body (respectively, the one and the few), whereas the composite *demos*+*kratia* combines "a collective body" and "power", hence offering a root meaning of "the power of the people". [1] As a result of Ober's analysis, he concludes, "*kratos*, when it is used as a regime-type suffix, becomes power in the sense of strength, enablement and 'capacity to do things'" (p. 5).

Not all regime words fall into one or other of these suffix groups. One of the most interesting groups for our purposes here comprises those terms that begin 'iso-' ("same or equal"), signalling a "distributive fairness in respect of access", of which *isegoria* (used to paraphrase 'democracy') involves the right to make use of "deliberative fora: equal right to speak out on public matters and to attend to the speech of others" (p. 5). This notion alone is of considerable significance in a mathematics classroom. (For one example, see Jenny Houssart's (2001) powerful account of rival classroom discourses and a partial 'speaking out' in mathematical resistance and protest by a somewhat marginalised group of four boys.)

In attempting to display how rich a relation there is between democracy on the one hand and law, action and public goods on the other, Ober cites a passage from Demosthenes concerning a court case, pointing out how the latter "employs a rich vocabulary of strength, control, ability and protection" (p. 6). And it tempts us to wonder in what sense mathematics is or should be seen as 'a public good'. [2]

In our paper, we propose to draw on Ober's helpful re-characterisation of 'democracy' as invoking power in the sense of having the 'capacity to do things' rather than solely referring to 'majority rule'. This observation immediately motivates the question of who has, or who can assert, the power to do things in mathematics and on what basis. One interpretation of this question will take us into explicit discussions of the notions of disciplinary and human agency (e.g. Pickering, 1995) in regard to mathematics and mathematics teaching.

We simply raise this question here, however, in order to help us to re-examine the specific and complex connection between mathematics and aesthetics [3], in an educational setting, the central focus and concern of this piece. In particular, we wish to challenge directly the presumed elitism inherent in aesthetic concerns in this domain a presumption both readily and explicitly claimed by some professional mathematicians (see, e.g., Borel, 1983; Hardy, 1940; Poincaré, 1908/1956; Russell, 1967), as well as by some mathematics educators (e.g. Dreyfus and Eisenberg, 1986; Krutetskii, 1976; Silver and Metzger, 1989). It is they (the professional mathematicians) who will be our titular few (*hoi oligoi*), who perceive mathematics as an oligarchy (and also act as if it is one, with them as the oligarchs), a rule exercised among other means through the explicit notion of 'taste' (with them as the bearers of it), which we address in the next section (see also Pimm & Sinclair, 2008.)

Our opening conceit, then, is to conceive of mathematics as a political regime (echoing, perhaps, Foucault's (1980) notion of a '*régime* of truth', which mathematics exemplifies *par excellence*) and to enquire, at least with regard to aesthetic considerations, what form or type of political regime is in place. In what sense, then, can mathematics be considered a *democratic* regime, one in which all (*hoi polloi*) have the capacity to do?

THE POWER OF TASTE

Taking the idea of democracy as invoking power in the sense of "capacity to do", we can read the role and nature of aesthetic considerations in mathematics in at least three different ways, corresponding to three distinct interpretations of power: power viewed as intrinsic to mathematics, power seen as the oligarchic imposition of 'good taste' and power as relational capacity. The first, well represented by subscribers to a

Platonic view of mathematics, sees power as intrinsic to the subject itself and therefore allocates agency and action to the discipline. It is, thus, the discipline itself that possesses the "capacity to do" and that capacity may then be transferred to individuals. If mathematics is good, in the sense of Plato (in terms of mathematics offering the only path to knowledge) or in bestowing successful engagement with the global economy (seen in more contemporary rhetoric), then mathematics has power, and one can obtain it by obtaining knowledge of mathematics. As Valero (2005) points out, a central problem with this view is that mathematics is not a *social* actor, and therefore can neither possess nor transfer power – at least if one subscribes to the epistemological and ontological positions of critical theories.

This intrinsic view fits well with traditional ideologies found in mathematics culture, including the notion that mathematics is somehow pure and detached from the rest of the world – that mathematics has a life of its own that might very well continue without human interference. Mathematicians such as Hardy (1940) have described some aesthetic consequences of this view, including the value of pursuing pure mathematics only and resisting the temptation of applications outside mathematics.

Hardy, as well as several of his contemporaries, has also espoused a more human (if class-driven) view of the power dynamics related to mathematics. This second view sees power in terms of the exertion of influence of one class over another, giving rise to an elitist view of mathematics and mathematical aesthetics that dominates today. If one recognises that mathematics is a discipline in which choices must be made, in which there are no objective grounds for deciding which questions or ideas count as fruitful or important, then one is forced to admit human agency in mathematics. In other words, mathematics itself cannot decide what is worth studying; mathematicians must do this. According to mathematician Henri Poincaré (1908/1956), only mathematicians are privy to the aesthetic sensibilities that enable these kinds of choices and, indeed, to any kind of mathematical creativity. [4]

There are a number of quotations that could do duty here. One of the clearest comes from John von Neumann (1947), while also attempting to offer criteria for salvation from mathematics becoming:

> more and more purely aestheticizing, more and more purely *l'art pour l'art*. This need not be bad, if the field is surrounded by co-related subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. (p. 196)

And so power can be seen as being in the hands of those who decide, those who are the arbiters of taste. Hardy was clear about his belief that only the great mathematicians could play this role. He even scoffed at the possibility of admitting those outside *hoi oligoi*: "There is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds" (p. 61).

In terms of exposition, and popularisation more specifically, Latour (1987) notes that the difficulty that mathematicians have in talking with non-mathematicians stems from the fact that mathematics "is designed to force out most people in the first place" (p. 52). Indeed, Csiszar (2003) sees this phenomenon as a "telling indicator of the discipline's tendency to exclude all but the very few" (p. 243). It is interesting that both of these commentators ascribe as much agency to "the discipline." Hardy may well deride exposition, but Latour, at least, doubts the very possibility of communication. With regard to the professional register of academic discourse as an exclusionary barrier, linguists Halliday and Martin (1993) write:

> But there is another, deeper tendency at work, a long-term trend - however faltering and backtracking - towards more democratic forms of discourse. The language of science [including that of mathematics], though forward-looking in its origins, has become increasingly anti-democratic: its arcane grammatical metaphor sets apart those who understand it and shields them from those who do not. (p. 21)

Note that they themselves attribute [**an**] anti-democratic agency to the very discourse of mathematics and science.

The capacity to do, and to write about it in approved ways, in this case becomes a capacity exerted by one small group of people (the 'owners', perhaps, in a Marxist account of this power struggle) to influence a much larger one (the 'workers'). Alfred North Whitehead talked about the "literary superstition," which views the aesthetic appreciation of mathematics as being a "monomania confined to a few eccentrics in each generation" (quoted in Hardy, 1940, p. 85). In this view, aesthetic values are seen as residing with the power of few great men, and the influence of these men is thought to extend to the wider community of mathematicians. This in turn suggests that the aesthetic values of the discipline are somehow shared and consistent throughout the discipline – a belief still held by some. In other words, membership in the oligarchy would imply to some kind of indoctrination or at least inculcation.

This assumption was actually empirically tested in the 1990s by David Wells, who asked the readers of *The Mathematical Intelligencer* to rate theorems according to their mathematical beauty. He drew a number of inferences from the seventy-six responses, many from noted mathematicians, mostly from North America. First, mathematicians do not always agree on their aesthetic judgements – at least not in terms of evaluating the *beauty* of theorems. Instead, aesthetic appreciation, even in mathematics, is contextual, historical and subjective.

In addition to this empirical evidence against a class-driven view of power in mathematics, Valero (2005) points to a more philosophical concern, namely that this view of power can become highly destructive in the sense that few opportunities become available for challenging the existing power dynamics. Vithal and Skovsmose (1997) see this concern playing out in the field of ethnomathematics which, while endeavouring to broaden views of what mathematics is across different cultures and historical periods, can do nothing to challenge the existing power structure in which this question (about what counts as ‘real’ mathematics) has already been decided – by the members of *hoi oligoi*.

An alternative to the class-driven view of power has emerged from Foucault’s work, one reflected centrally in the way disciplinary regimes become installed in institutions, including regimes of truth:

The important thing here, I believe, is that truth isn't outside power, or lacking in power […] truth isn't the reward of free spirits, the child of protracted solitude, nor the privilege of those who have succeeded in liberating themselves. Truth is a thing of this world: it is produced only by virtue of multiple forms of constraint. And it induces regular effects of power. […] (1980, p. 131)

> ‘Truth’ is to be understood as a system of ordered procedures for the production, regulation, distribution, circulation and operation of statements. ‘Truth’ is linked in a circular relation with systems of power which produce and sustain it, and to effects of power which it induces and which extend it. A ‘régime’ of truth. (1980, p. 133)

His ideas challenge the notion that power is intrinsic to and permanent within disciplines or social actors. Instead, power is seen as a relational capacity of social actors (Valero, 2005), one that is constantly transforming through practice and discourse. In other words, the power dynamics at play in the current mathematical regime cannot simply be the result of the influence of *hoi oligoi*, but must arise out of the participation of various actors in social practices and in the construction and institutionalisation of discourse.

This poststructuralist view of power affords interesting opportunities to understand why mathematical aesthetics has consistently been seen as an elitist enterprise within contemporary cultures. In particular, it challenges us to seek alternative interpretations of some of the consequences of this latter view, which include the following: that the discipline of mathematics, both in academic institutions and in education, is privileged over other disciplines in the humanities and sciences; that mathematics does not have the kind of critical interface between ‘consumers’ and ‘producers’ that other aesthetically-rich disciplines such as art and literature have (elitist views do exist in these fields too, although not to such an extent); that children are deemed incapable of aesthetic appreciation in mathematics.

Each of these positions could be explained in terms of the class-driven or even the objectivist view of power, but such explanations, in addition to being problematic, are also fundamentally without hope. In the following section, we delve into just one of these positions more closely and investigate some of the relations of power that are practiced and constructed in our culture that have led to it. In so doing, we follow Ober’s lead in seeking forms of the “capacity to do” that are more relational and transformative, as well as more widely accorded.

IF MATHEMATICS IS AN ART, WHERE ARE ITS CRITICS?

Many mathematicians have advanced the claim that their discipline is more properly an art than a science. They cite several reasons, most of them related to aesthetics. For example, the mathematician John Sullivan (1925/1956) claimed that mathematicians are impelled by the same incentives as artists, citing as evidence the fact that the "literature of mathematics is full of aesthetic terms" and that many mathematicians are "less interested in results than in the beauty of the methods" (p. 2020) by which those results are found. There is also the argument that, unlike with the sciences, mathematics does not have to compare itself against an outside reality – thus, the implication being mathematicians have choice and freedom when it comes to selecting their objects of interest (though see note [2] about an unease with these terms). Sullivan described mathematics as the product of a free, creative imagination and argued that it is just as "subjective" as the other arts.

These characteristics that mathematics supposedly share with the arts – creativity and free choice, as well as the use of "aesthetic terms" – may sound alluring to non-mathematician, who can recognise them as familiar in other (less exclusive) experiences. Tell a mathematics-fearing artist that the discipline is really about ambiguity (Byers, 2007), creativity and freedom, and their ears will likely perk up. However, these very characteristics only serve to remove the accessibility of mathematics from the non-mathematician further since, like the aesthetic sensibilities of Poincaré, they only belong to a privileged few.

It may be more fruitful to consider the differences between mathematics and the arts in understanding the power dynamics involved. Indeed, the philosopher Thomas Tymoczko (1993) may well have pointed out the most operative difference between aesthetic judgements in mathematics and those at work in the arts: the mathematics community does not have many 'mathematics critics' to parallel the strong role played by art critics in appreciating, interpreting and arguing about the aesthetic merit of artistic products. Mathematics may well be a discipline of freedom and creativity, but individual mathematicians have to engage in what Pickering (1995) calls "a dance of agency" between their own agency and that of the discipline – radical, revolutionary new ideas must still find expression, connection and interaction within the accepted forms of the discipline. This is true for art as well; Picasso's cubism only existed and flourished through its opposition to and promotion of other artistic movements. But in the arts there are critics to interpret and negotiate the meaning and place of creative new products.

In mathematics, however, virtually no one stands on that border between the productive and interpretive aspects of creative work for mathematics (see, for instance, Corfield, 2002, on Lakatos's legacy in this regard). This is not just a problem for non-mathematicians, who have little help in assessing the importance of new developments in mathematics; it has been problematic within mathematics itself. Gödel's famous incompleteness theorem almost passed unnoticed when it was first announced in 1930; that it was not is thanks to von Neumann who first appreciated its significance. This kind of thing is bound to happen in any creative discipline – other mathematicians were blinded by their philosophical commitments to formalism and positivism – but without any mechanisms to aid in interpretation, mathematics closes itself off from others, and sometimes from itself.

For Leo Corry (2003, 2006), this border becomes a distinction between the *body* and the *image* of mathematics, which he sees as forming "two interconnected layers of mathematical knowledge" (2006, p. 135). While the body includes "questions directly related to the subject matter of any given mathematical discipline: theorems, proofs, techniques, open problems", the images "refer to, and help elucidating, questions arising from the body of knowledge but which in general are not part of, and cannot be settled within, the body of knowledge itself" (p. 135). These questions might include, as they did for Picasso's audience, the questions that mathematicians David Henderson and Daina Taimina (2006) recognise as often going unanswered: "Why is it true?", "Where did it come from?", "How did you see it?" (p. 66).

In that the image involves questions that cannot be settled from within the body, Corry contends that the black letters and symbols on white pages that constitute formal mathematical texts cannot bring forth forms and colours that constitute the image of mathematics. The same can be said with respect to the arts: questions of categorisation and importance, for instance, are settled by outside commentators, most notably art critics who employ aesthetic notions and devices.

In our analysis then, mathematical elitism with respect to aesthetic issues arises from an instructional lacuna that could be filled by the creation of mathematical image-makers or mathematical critics. Like the art critic who helps the interested citizen understand the challenging art of the day – Barnett Newman, John Cage, Damien Hirst – so the mathematical critic would help the non-mathematician citizen understand why, for example, mathematicians continue to take the unproven Riemann hypothesis as true – using it to prove other results – even though high school geometry inculcates the idea that no result, not even the most obvious one, should be accepted without proof. Some may counter that it is easier to explain Newman's brushstrokes than it is to explain the techniques used in mathematics to prove results. (See Mazur, 2003, and Tahta, 2004, for a related discussion around issues of mathematics and poetry.) But aesthetic decisions are rarely about specific thorny techniques; rather, they relate to issues of the body: why is this true or important, where does it come from and how can you help me see it?

IS[E]GORIA AS A GOAL OF MATHEMATICS EDUCATION

In conclusion, we return to the opening notions of democracy, and in particular *isegoria*, the 'equal right to speak out on public matters'. Does it make sense to see a mathematics classroom as a deliberative forum? If we are thinking in terms of schooling for democracy in mathematics, and democracy is taken in the sense of Ober's formulation of 'capacity to do', then what forms of mathematics education should students encounter? That right does not need to be exercised all the time, but it needs to be genuinely felt as a possibility.

Secondly, recalling Noddings' (1994) view of mathematics needing to be made optional once again (past a certain point) for progress to be made, who, then, will see mathematics as an activity, a discipline, a way of thinking, that they can *do* something with? This is not 'utility' seen in terms of a naïve external utility, the stuff of much contemporary debate, nor the equally problematic 'mathematics give[**s**] access to power' lobby, but a view that mathematics can do something for me, in a humanistic sense, one that repays the careful attention and deep engagement it may require. One that may expose students to a fundamental sense and experience of equality, the 'iso-' root of a number of words that connect to political regimes, and provide them another sense of human commonality. In addition, mathematics can provide aesthetic experiences with which students can shape themselves and orient themselves toward a culture of action and thought that is at least as old as the written.

Lastly, we wish to highlight the tradition and style of mathematics that masks the aesthetic values that underlie judgements at the same time as presuming their universality. This double concealment makes it harder to criticise such decisions and judgments even as we strive to make them manifest. At the heart of the oligarchy is a hierarchy – another key -*arche* term – where *hieron* refers to the sacred. That mathematics has a sacred writing is unarguable. To what extent the members of the congregation – *hoi polloi* – are able to engage with and criticise the sacred writing of mathematics remains to be seen.

NOTES

1. *Demos* meaning "the people" actually referred only to native, adult, male freeborn citizens of a *polis* (state), so is, of course, not exactly all-embracing. Also 'aristokratia' (*hoi aristoi*, the excellent) is another label for a form of governing 'power' that has resonance in mathematics.

2. A distinction between private and public goods is helpful in this context, where public goods, such as environment and culture, health and education, should be decided upon in the public realm. In her Massey lectures, Janice Gross Stein (2002) deconstructs notions of 'efficiency' and 'choice' in the latter two arenas, signalling how these have become ends in themselves rather than powerful means to contested ends. Choice is not a public good. Or, as our colleague Eric Love has eloquently framed it: "People don't want choice; they want what they want."

3. There is a tradition in mathematics of writing in terms of aesthetics. As will become clearer in this paper, we use 'aesthetic' in the following way, though unsurprisingly perhaps, this is not how everyone sees it. For us, aesthetic considerations concern *what* to attend to (the problems, elements, objects), *how* to attend to them (the means, principles, techniques, methods) and *why* they are worth attending to (in pursuit of the beautiful, the good, the right, the useful, the ideal, the perfect or, simply, the true). We have deliberately framed this specification in general terms, so that it applies equally well to mathematics as to art, historically the realm of much discussion of things aesthetic.

4. This stance is reflected in the fact that the Fields medal (equivalent for mathematics to the Nobel prize) can only be awarded to mathematicians under the age of 40.

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