RECOGNIZING WHAT THE TALK IS ABOUT: DISCUSSING REALISTIC PROBLEMS AS A MEANS OF STRATIFICATION OF PERFORMANCE

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This paper analyzes a typical classroom discussion of a realistic problem in an eighth grade mathematics classroom in a German high-streaming school. It considers performance in the classroom as not merely relying on cognitive variables but rather on social interactions. The teacher's role in these interactions is highlighted as crucial for defining the legitimate discourse and thereby the conditions for performance. As there is quantitative evidence that the performance on solving realistic mathematics problems is strongly connected to social variables, classroom discussions of these kinds of problems appear to be very fruitful for qualitative research. Therefore, a structural distinction of different discourses engaged in realistic problems is emphasized and the relation between these discourses in the discussion is outlined. Furthermore, two incidents of students' misrecognition of legitimate discourse are analyzed. The analysis shows that the teacher has at least a joint responsibility for the students' misrecognition, as he encodes his favoured ways of performing. Thus, a stratification of performance levels emerges based on the students' ability to decode implicit instructions.

PERFORMING REALISTIC MATHEMATICS PROBLEMS

In recent years there have been several studies on the performance of students in realistic mathematics problems. Roughly a distinction can be made between research on problems demanding to *make use* of the realistic contexts on the one hand and research on problems demanding to *distance* oneself from the context and to use only a few certain contextual details on the other hand. The former (e.g. Baruk, 1989, Verschaffel et al., 2000) sees the students' troubles in making sense of word problems and solving them superficially by just using trained mathematical strategies, even if there is no related sense at all. The latter states that in most situations of assessment, marking schemes are given, that value to make use of certain details of the context to be mathematized, but penalize the inclusion of realistic aspects in general (e.g. Boaler, 1994, Cooper & Dunne, 2000). These studies report that students draw too extensively on the context and fail to reckon the one certain detail which is valued by the marking scheme. Furthermore, there is evidence that this phenomenon is not equally distributed among all students, but that it is influenced by sociological variables, such as socio-economic status (Cooper & Dunne, 2000) or gender (Boaler, 1994). As Säljö and Wyndham (1993) maintain, a student's problem solving behaviour is also dependent on the situation s/he is in while solving the problem. They found that students solved a problem differently when confronted with it in a mathematics class than when confronted with the same problem in a social studies class. Accordingly, the reasons for both of the phenomena reported above can be assumed to lie in implicit assumptions students have about what is expected of them. Moreover, the distinction between the two lines of research outlined above already shows that realistic mathematics problems may entail different demands. Hence, students' performance is not only dependent on their mathematical skills, but more importantly on their skills to recognize what kind of problem solving behaviour the problem demands.

Gellert and Hümmer (2008) go further and state that a legitimate performance in a mathematics classroom is not only dependent on the broad cultural view of mathematical competence, but is socially constructed by interactions on the micro-sociological level of the classroom. Through implicit and explicit evaluation the teacher establishes criteria for a good performance.

As emphasized above, different realistic problems may pose different demands. Furthermore, legitimate mathematical performance is socially constructed within the classroom, and this construction is controlled by the teacher. Hence, in different mathematics classrooms there might be different demands for a good performance. With regards to realistic mathematics problems that means that in order to perform well, students have to find out on which level reality and mathematics shall be connected to or insulated from one another. Realistic problems are often artificial and only apparently 'real'. The performance demands are less regulated by the original context than by its recontextualization in the classroom (Gellert & Jablonka, in press). This gives rise to the question if a school-mathematics problem can be considered "realistic" or "pure mathematical" at all. A point of view, focussing more on discursive structures, as provided by Bernstein (1999) might help out.

HORIZONTAL AND VERTICAL DISCOURSE

> „A horizontal discourse entails a set of strategies which are local, segmentally organised, context specific and dependent, for maximizing encounters with persons and habitats." (Bernstein, 1999, p. 159)

Consequently, the validity of knowledge and strategies developed in a horizontal discourse is bound to its original contextual segments. An example for a horizontal discourse is a mundane activity, such as sharing a bowl of pasta with a friend. The validity of a sharing strategy is dependent on situational segmental variables, such as individual appetite or who is invited and who is inviting. There is no general solution to solve such a "problem". As different situations of this kind exist without having an impact on one another, this kind of discourse can be called *horizontal*.

In contrast to this, Bernstein defines vertical discourse as follows:

> „[…] vertical discourse takes the form of a coherent, explicit, and systematically principled structure, hierarchically organised as in the sciences […]"
> (Bernstein, 1999, p. 159).

In this case the validity of knowledge and strategies is dependent on knowledge of higher generality. Knowledge needs to be coherent with other context-independent knowledge of the same discipline and not with contextual segments. Hence, new knowledge can be generated from what is already known. The structure is therefore hierarchical and this kind of discourse can be called *vertical*.

In pedagogic discourses, and in school mathematics discourse in particular, horizontal discourses are often used to provide students with access to a vertical discourse (Bernstein, 1999). If one exchanges "pasta" for "pizza" in the example above, it would be likely to be found in a mathematics classroom as an entry point to the vertical discourse of fractions. Mathematics is thereby restricted to its horizontal 'origin' and students are lead to believe that mathematics is crucial to participate in such situations. Further, fractions seem to reference the situation of sharing food. Even though teachers might be suggestive of parts of pizzas and fractions being about the same thing, out of school they are still describing different things. Hence, the boundaries between horizontal and vertical discourse remain. Therefore, Dowling (1998) uncovers both the aspects of participation and of reference as myths.

As shall be shown in the following analysis, a crucial condition for students' performance in "realistic" problem solving is the ability to *recognize* the boundary and its strength between the horizontal and the vertical discourse. Using Bernstein's terminology (1996) this ability shall in the following be called the possession of *recognition rules,* while the strength of the boundaries between the two discourses shall be referred to as *classification.*

> "Where we have strong classification, the rule is: things must be kept apart. Where we have weak classification the rule is: things must be put together." (Bernstein, 1996, p. 26)

However, classification is not created by a mathematical content itself. By shifting the content into a classroom and therefore into a context of transmission and acquisition, the mathematical content is *recontextualized* into school mathematics. Consequently, something or somebody has to control this recontextualization. Again following Bernstein (1996), the strength of this control shall be referred to as *framing*.

> "Where framing is strong, the transmitter has explicit control over selection, sequence, pacing, [evaluation] criteria and the social base. Where framing is weak, the acquirer has more *apparent* control (I want to stress apparent)" (Bernstein, 1996, p. 27).

While classification (C±) connects a structural level, framing (F±) [1] can be seen as its interactional implementation. A change of classification can be reached through framing. If classification is strong, framing can furthermore be used to either explicate or blur classification modalities. The impact of classification and framing on the acquisition of the recognition rules by the students will be pointed out in the analysis and discussed in the conclusion.

THE DATA

This paper analyses a four-minute sequence in an eighth grade (age 13) mathematics classroom in a German "Gymnasium" [2]. The data is taken from the rich data corpus of the Learner's Perspective Study (LPS) [3]. In sixteen culturally diverse countries three classrooms each were videotaped for sets of ten consecutive lessons, using a three-camera approach (Clarke, 2006). Furthermore, video-stimulated interviews were conducted with students after each lesson, and a questionnaire was handed out to the teacher. The data collection in Germany was carried out in 1999. The sequence has already briefly been analyzed by Gellert & Jablonka (in press). A deeper analysis of the classroom discourse, based on the language of description by Basil Bernstein (1996) and developed further by other scholars (Singh, 2002, Morais, 2002, Tsatsaroni et al., 2004, Gellert & Jablonka, in press) has been carried out by Straehler-Pohl (in press). This work will be summarized in the section on the characterization of the classroom talk, before the focus is moved towards two specific incidents of students' misrecognition of discursive demands. The chosen sequence is the only sequence in all ten videotaped lessons that shows this class working on a realistic mathematics problem. However, the discussion of that problem and the intricacies emerging for the students can be seen as exemplary for a "Gymnasium", as Gellert & Jablonka (in press) have pointed out.

THE GIVEN PROBLEM

After having treated binomial formulas in several previous lessons, the teacher draws a square on the blackboard to represent a farmer's soil and then asks a LPS-researcher to pose the following problem:

Researcher:	Now the neighbour comes to the farmer and says right listen (...) that borders on this
Student:	What
Researcher:	It would be incredibly useful for my planning if I could take away one meter from your one side say the side across on the top if you'd give that to me I'd give you an additional one on the other side instead
Teacher:	So
Researcher:	And the question is would you agree to that if you were the neighbour?

Afterwards, the researcher hands over to the teacher who proceeds to lead the discussion. Obviously, there are different ways to approach this problem. Several different aspects can be taken into consideration to answer the question of fairness. These aspects can be roughly categorized in three categories: aspects that

- can be mathematized straight away (e.g. area, perimeter),
- could be mathematized with further information (e.g. quality of the ground),
- do not need any mathematization at all (e.g. social arguments).

In the student interview, the researcher formulates her view on the problem:

Researcher: With a so to speak the smallest fence with maximal area is always a square. That was … that's what this exercise actually showed [5]

The teacher himself states the aim "representing contextual problems/tasks in linear equations" in the teacher questionaire. However, an analysis of the whole sequence has shown, that the teacher's role in leading the discussion is rather coherent with the researcher's statement than with his own one (Straehler-Pohl, in press). Hence, while the problem itself is set within a horizontal discourse it aims at extracting highly generalized knowledge, clearly settled in a vertical discourse. A crucial condition for successful performance in the classroom discourse will be to follow the path leading from the horizontal into the vertical discourse. In the following section the boundary between these two discourses and the teacher's guidance on the way from one to the other will be analyzed.

CHARACTERIZING THE CLASSROOM TALK

The classroom discussion leading to the solution favoured by the teacher is about eight minutes long (or short) in total. The analysis (Straehler-Pohl, in press) has shown that the sequence consists of two parts of almost equal length, but of very different character. While the discussion remains close to the problem text and therefore includes horizontal discourse in the first part, it is purely mathematical and therefore purely vertical in the second part. In this second part of the discussion there is no evidence for students' misrecognitions of the discourse. One could conclude that this shows the teacher's success in providing access to the recognition rules. However, none of the students showing misrecognition in the first part of the discussion is participating (or even trying to do so) in the second part anymore. This paper does not focus on the second part of the discussion as it is a purely vertical discourse. Instead it aims to analyze the classification and framing of both horizontal and vertical discourse in the first part of the discussion.

By drawing a square on the board before the introduction of the problem, the teacher creates a setting, which can be identified as a school-mathematical setting instead of a realistic out of school-setting (C+). However, it is not explicit to the students, if the square is meant to represent a major frame of reference or just a sketch to support the comprehension of the following problem (F-). In contrast to the square as an institutionalized signifier, the language the teacher is using in the discussion is often mundane. This shows that he is weakly controlling the boundaries between the horizontal (mundane) and the vertical discourse (F-). The classification between these discourses is not clear yet. The weak framing achieved through using mundane language could be implemented intentionally to weaken the boundaries (C-), but it could just as well be an unintended blurring of existent boundaries (C+). Different students' answers, which can be identified as horizontal discourse, are rejected by the teacher (see the two examples in the next section). This shows that boundaries between these two kinds of discourse are, in fact, strong, and at the same time they are put into a hierarchy (C+). Answers that draw on horizontal discourse by using the sketch for visual arguments or by asking questions about details of the problem's context are ironically rejected by the teacher or are labelled "irrelevant". The analysis clearly shows that the teacher aims at leaving the horizontal discourse behind as fast as possible and at launching into the 'more relevant' vertical discourse, which he values as legitimate. This becomes very obvious when two students are giving similar, yet not identical answers regarding the decrease of the area by one square-meter. The first student (as the teacher and the researcher before) uses mundane language. The second student makes the same argument, but he uses mathematical terms such as "one m times X" and "one m times X minus one in brackets". While the teacher's comment on the first answer is a short mumbled "um" followed by his looking for further answers, he acknowledges the second answer with an "aha" underlined by a smile on his face. Now he seems to feel confident in having reached the point where he can proceed the discussion on a strictly vertical level. This shows that horizontal and vertical discourse are strongly classified (C+). The criteria for a good performance do not only include drawing on vertical arguments, but also formulating them in the *language* of a vertical discourse. This crucial criterion is not stated explicitly, but the students have to extract it from the subtle differences in the teacher's reactions. The teacher uses mundane language throughout the discussion and thereby creates the impression of weak classification. As classification de facto stays strong, this is a case of weak framing (F-). In summary, the sequence is characterized by a strong classification of horizontal and vertical dicourse (C+). Incoherently, the teacher is using a weak framing (F-) over the boundaries and by that making the classification invisible. Hence, the students have to *recognize* the implicit boundaries established by the teacher in his inconsistent interactions to have their performance *acknowledged* by him. Two situations, where students fail to recognize these boundaries and the teacher's reactions to these students' utterances shall be discussed below.

TWO SITUATIONS OF MISRECOGNITION

Student 1: No well because that he's got less (...) You can you can see that in the drawing already that you get less of the property then

Student 2: So I now somehow well I would decline the offer because that is after all (...)

Teacher: Wait now I haven't really understood that now there's such a murmur

Student 2: (...) I would decline it because somehow um you've got a piece you don't have a corner like that (...) property goes around that

Teacher: Well you got uh which one's the new property now. That's no square anymore now but it's still also a rectangle after all so you would decline it because because you'd rather have a square than a rectangle

Student 3: That's great

Teacher: No, right?

Both student 1 and student 2 argue relying on the sketch on the board:

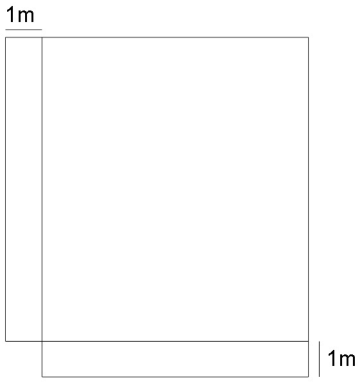

Figure 1. Sketch of the farmer's ground as drawn by the teacher.

Such arguments are bound to the segment of this singular sketch and therefore they are part of a horizontal discourse. Student 1 argues that she can see that the area will decrease, relying on her visual judgement. Visual judgement, however, might be a legitimate strategy for a farmer to solve problems but not for mathematicians. Student 2's argument is different: She misinterprets the sketch and argues from a practical viewpoint, seeing a problem in having an irregular area. Again, this might be an important argument for a farmer needing to steer his tractor around a corner, but not for a mathematician, arguing in the vertical discourse of general areas. So in the end, both students are drawing on different arguments, a visual argument of area sizes on the one hand and a practical argument of shapes on the other hand. Both of these arguments are cases of horizontal discourse. The teacher's ironical reaction shows that he considers none of these answers legitimate. In Bernstein's words: both students do not have access to the recognition rules. Hence, at this moment both students are not in the position to perform well in the discourse. In his ironical comment on preferring squares over rectangles, the teacher introduces vertical discourse and by that gives a hint as to what kind of talk is considered legitimate. In the chronological order of the talk, the teacher seems to react to Student 2's answer. But regarding the content of the answer his reaction is pretty vague and rather seems to refer to Student 1's answer. In the end, the delay in time and the vagueness of the reaction puts the students in charge of finding out, what to change to make their answers legitimate and, thus, to perform well.

The following subsequent answer by Student 4 shows that the teacher (at least for this student) did not succeed to provide access to the recognition rules through his comment:

Student 4: (...) I mean um wanting to take away something on one side and add something on the other side. Where is the neighbour's garden at all? Is it on the left hand side or below it?

Student 5: All around.

Teacher:	It is all around.
Student 4:	That's a little illogical isn't it?
Student:	Well why
Student6:	Oh
Teacher:	That is now that is now uh
Student:	Boy
Teacher:	Uninteresting our question uh whether the one concerned well if he should swap whether whether that's favorable for him or whether uh well whether it doesn't matter or that's all this is about whether for that farmer who's making trouble here in here whether for him uh yes

Student 4 seems to have problems imagining the given problem in reality. She, therefore, asks for further information. This shows that she is still 'stuck' in the horizontal discourse of reality, while student 5 seems to have already recognized, that the problem is not about a real neighbour who needs to be positioned somewhere. He is even able to translate the vertical fact that this is not relevant into the language of the horizontal discourse as practiced by Student 4: "It's all around." The teacher, who seems impressed by this performance of Student 5 acknowledges it: "It's all around." So, while communicating meanings of a vertical discourse, they translate them into - and by that participate in - the horizontal discourse. As this translation is invisible for Student 4, she rightly doubts their answers. Being surrounded by the property of a single neighbour would be a very unlikely situation for a real farmer. However, the teacher states that her objection is not of interest. To explain why, he again engages in the horizontal discourse of contextualized language: "*that* farmer who's making trouble *here* in *here*". By that, he encodes his implicit actual message. To perform well in the further discussion, students need to be able to decode this message. It is pretty obvious in the sequence above, that not all students are able to do so. The possession of the recognition rules is a major prerequisite for good performance. At the same time the teacher is not explicating but rather encoding them. This creates a situation, stratifying students into those who have the ability to decode implicit evaluation criteria and those who have not. Hence, hierarchies among students emerge.

CONCLUSION

From the point of view of mathematics education, dealing with realistic problems can have different aims. These problems can be discussed to use mathematics for a serious examination of realistic problems, as favoured by scholars advocating of mathematical literacy (Gellert, Jablonka & Keitel, 2001). A different, yet not less respected, school advocates the use of mathematically rich realistic situations as entry-points for students into a self-employed construction of complex mathematical ideas, as postulated by Realistic Mathematics Education (RME) (e.g. van den

Heuvel-Panhuizen, 1996). These are just two of the many diverse, but respected stances on realistic problems. For teachers, this creates a slightly obscure situation. They might not always know which aim to follow and which problem to choose for their aims. The inconsistent situation of strong classification and weak framing analysed above can be regarded as a documentation of such a situation. While the teacher consciously aims at "representing contextual problems/tasks in linear equations" the practiced discourse is taking a different course. The problem's inherent aim is rather the development of a complex mathematical insight (see the researcher's statement above). So it is no wonder, that a situation is created, where the teacher's aims (C+) and actions (F-) are not coherent anymore. At the same time, the analysis has shown that students' performances are highly dependent on their ability to decode the teacher's aims in this classroom discussion. Hence, the unjust situation is created where students need to read an implicit code (of C+) out of the teacher's incoherent behavior (of F-) to be able to perform in the classroom. In the end, the condition for performance is rather the ability to read the teacher's codes than the competence of either "representing contextual problems/tasks in linear equations" or using mathematics to solve real problems (mathematical literacy) or to develop mathematical ideas out of real situations (RME). This is an inappropriate situation and it is likely that it is not desired by the teacher himself.

NOTES

1. The strength of classification will be indicated by C+ and C-. The same applies to framing: F+ and F-.

2. The German school-system is separating students into three streams after primary school. "Gymnasium" is the highest stream and graduation from it allows students to enter university.

3. See http://www.lps.iccr.edu.au/

4. The researcher refers to rectangular areas. By the smallest fence she means the smallest perimeter.

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